Magnetic power spectrum in a dynamo model of Jupiter

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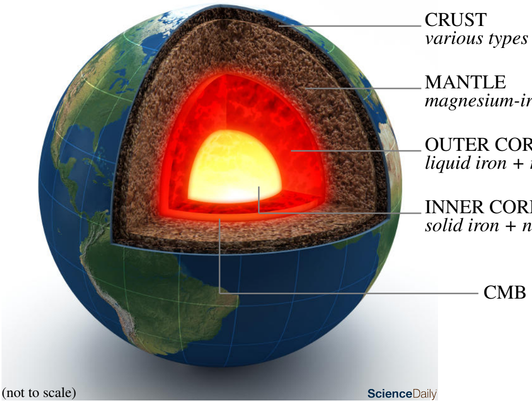
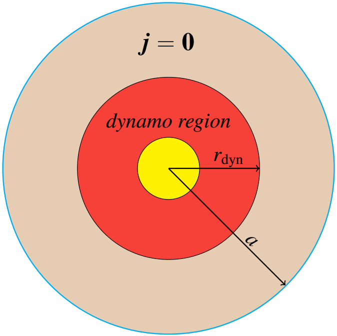
Let's start on Earth . . .



- **core-mantle boundary** (CMB): *sharp boundary* between the non-conducting mantle and the conducting outer core
- location of CMB r_{dyn}: the depth at which dynamo action starts
- seismic waves observation gives $r_{\text{dyn}} \approx 3486$ km
- another way to estimate r_{dyn}: magnetic power spectrum

Gauss coefficients g_{lm} and h_{lm}

- Outside the dynamo region, $r_{\text{dyn}} < r < a$:



$$\boxed{\boldsymbol{j} = \mathbf{0}}$$

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{j} = \mathbf{0} \implies \boldsymbol{B} = -\nabla\Psi$$

$$\nabla \cdot \boldsymbol{B} = 0 \implies \nabla^2\Psi = 0$$

a = *radius of Earth*

- Consider only internal sources,

$$\Psi(r,\theta,\phi) = a \sum_{l=1}^{\infty} \sum_{m=0}^{l} \left(\frac{a}{r}\right)^{l+1} \hat{P}_{lm}(\cos\theta)(g_{lm}\cos m\phi + h_{lm}\sin m\phi)$$

$\hat{P}_{lm}$: *Schmidt's semi-normalised associated Legendre polynomials*

- g_{lm} and h_{lm} are determined from magnetic field measurements on the surface ($r \approx a$)

The Lowes spectrum

- Average magnetic energy over a spherical surface of radius r

$$E_B(r) = \frac{1}{2\mu_0}\frac{1}{4\pi}\oint |\boldsymbol{B}(r,\theta,\phi)|^2 \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi$$

- Inside the source-free region $r_{\mathrm{dyn}} < r < a$,

$$2\mu_0 E_B(r) = \sum_{l=1}^{\infty} \left(\frac{a}{r}\right)^{2l+4} (l+1) \sum_{m=0}^{l} \left(g_{lm}^2 + h_{lm}^2\right)$$

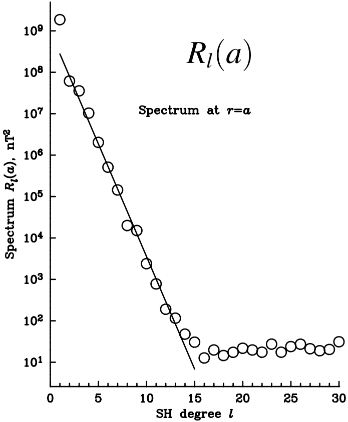
- **Lowes spectrum** (magnetic energy as a function of l):

$$R_l(r) = \left(\frac{a}{r}\right)^{2l+4} (l+1) \sum_{m=0}^{l} \left(g_{lm}^2 + h_{lm}^2\right)$$

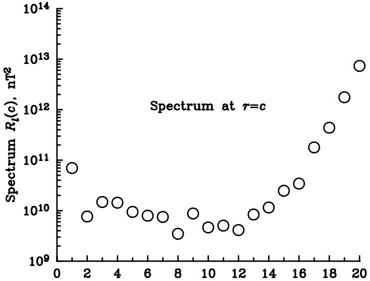
$$= \left(\frac{a}{r}\right)^{2l+4} R_l(a) \qquad \text{(downward continuation)}$$

Estimate CMB using the Lowes spectrum

$$R_l(a)$$



$$R_l(c) = \left(\frac{a}{c}\right)^{2l+4} R_l(a)$$



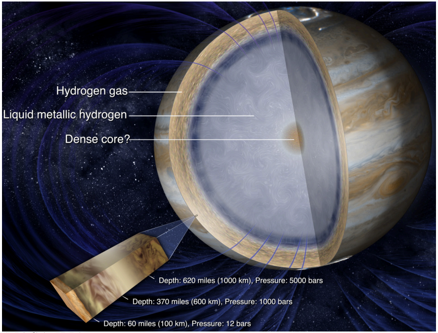
(Robert Parker, UCSD)

$a =$ Earth's radius

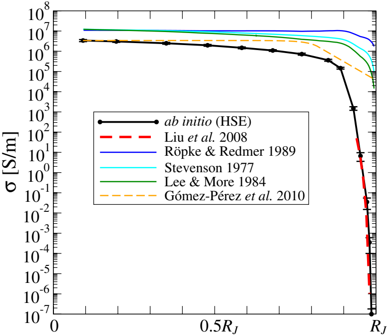
$c = 3486$ km $\approx 0.55a$

- **white noise source hypothesis**: turbulence in the core leads to even distribution of magnetic energy across different scales l
- $R_l(c)$ independent of $l \implies c \approx r_{\text{dyn}}$

Interior structure of Jupiter



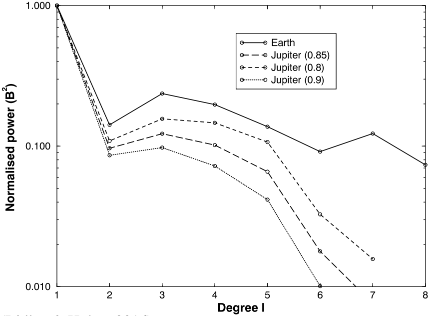
(NASA JPL)



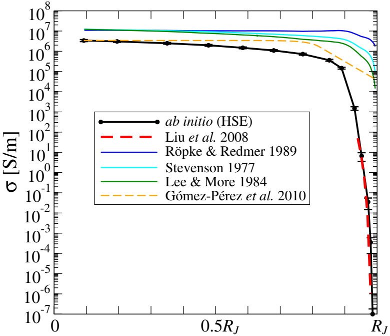
(French et al. 2012)

- low temperature and pressure near surface ⇒ gaseous molecular H/He
- extremely high temperature and pressure inside ⇒ liquid metallic H
- core?
- **conductivity** $\sigma(r)$ varies smoothly with radius r

Lowes spectrum for Jupiter: observations



(Ridley & Holme 2016)



(French et al. 2012)

- g_{lm} and h_{lm} for $l_{\rm max} \sim 4-7$ computed from magnetic measurements of various missions (e.g. Pioneer 10 & 11, Voyager 1 & 2, JUNO: $l_{\rm max} = 12$)
- downward continuation to a flat Lowes spectrum $\Rightarrow 0.73R_J \lesssim r_{\rm dyn} \lesssim 0.90R_J$
- For a continuous conductivity profile $\sigma(r)$:
  - dynamo action in transition region?
  - dynamo radius $r_{\rm dyn}$ well-defined?
  - estimation of $r_{\rm dyn}$ from Lowes spectrum reliable?

A numerical model of Jupiter

- spherical shell of radius ratio $r_{\mathrm{in}}/r_{\mathrm{out}} = 0.0963$ (small core)
- anelastic: linearise about a hydrostatic adiabatic basic state $(\bar{\rho}, \bar{T}, \bar{p}, \dots)$
- rotating fluid with electrical conductivity $\sigma(r)$ forced by buoyancy
- convection driven by secular cooling of the planet
- dimensionless numbers: Ra, Pm, Ek, Pr

$$\nabla \cdot (\bar{\rho}\boldsymbol{u}) = 0$$

$$\frac{Ek}{Pm}\left[\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u}\right] + 2\hat{\boldsymbol{z}} \times \boldsymbol{u} = -\nabla\Pi' + \frac{1}{\bar{\rho}}(\nabla \times \boldsymbol{B}) \times \boldsymbol{B} - \left(\frac{EkRaPm}{Pr}\right) S \frac{\mathrm{d}\bar{T}}{\mathrm{d}r}\hat{\boldsymbol{r}} + Ek\frac{\boldsymbol{F}_\nu}{\bar{\rho}}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B}) - \nabla \times (\eta \nabla \times \boldsymbol{B})$$

$$\bar{\rho}\bar{T}\left(\frac{\partial S}{\partial t} + \boldsymbol{u}\cdot\nabla S\right) + \frac{Pm}{Pr}\nabla\cdot\boldsymbol{\mathcal{F}}_Q = \frac{Pr}{RaPm}\left(Q_\nu + \frac{1}{Ek}Q_J\right) + \frac{Pm}{Pr}H_S$$

Boundary conditions: no-slip at r_{in} and stress-free at r_{out}, $S(r_{\mathrm{in}}) = 1$ and $S(r_{\mathrm{out}}) = 0$, electrically insulating outside $r_{\mathrm{in}} < r < r_{\mathrm{out}}$

(Jones 2014)

A numerical model of Jupiter

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- rotating fluid with electrical conductivity $\sigma(r)$ forced by buoyancy
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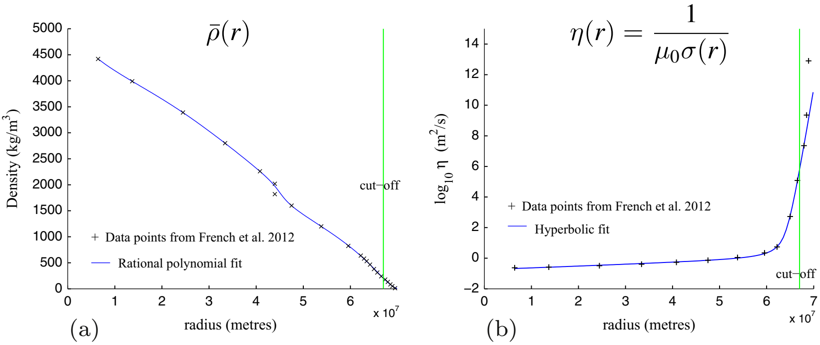
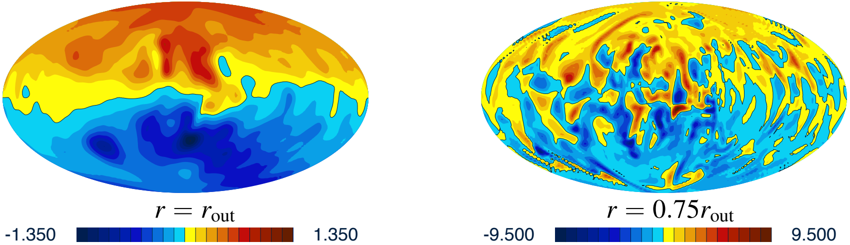
- dimensionless numbers: Ra, Pm, Ek, Pr
- **a Jupiter basic state:**

C.A. Jones / Icarus 241 (2014) 148–159

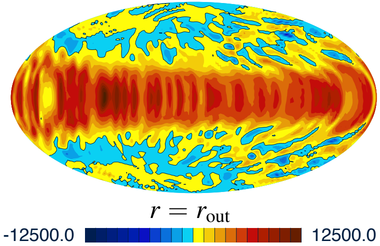


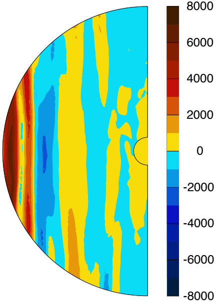
$Ra = 2 \times 10^7,\ Ek = 1.5 \times 10^{-5},\ Pm = 3,\ Pr = 0.1$

radial magnetic field

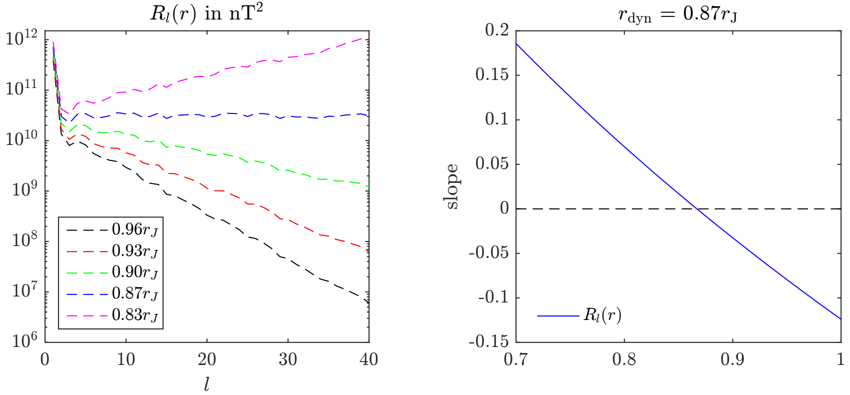


zonal velocity





Dynamo radius from Lowes spectrum

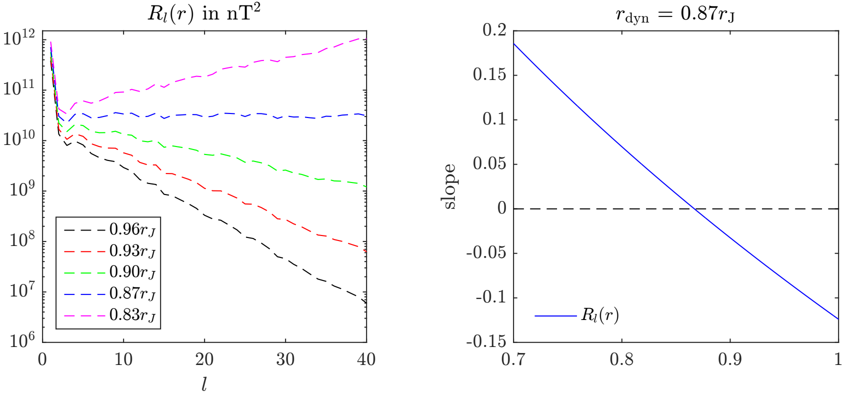


- B_r at the surface $r = r_{\rm out} \implies g_{lm}, h_{lm} \implies R_l(r_{\rm out})$
- downward continuation into 'source-free' $\boldsymbol{j} = \boldsymbol{0}$ region:

$$R_l(r) = \left(\frac{r_{\rm out}}{r}\right)^{2l+4} R_l(r_{\rm out})$$

- $r_{\rm dyn} = 0.87r_J$, how reliable is this estimate?

Magnetic power spectrum, $F_l(r)$

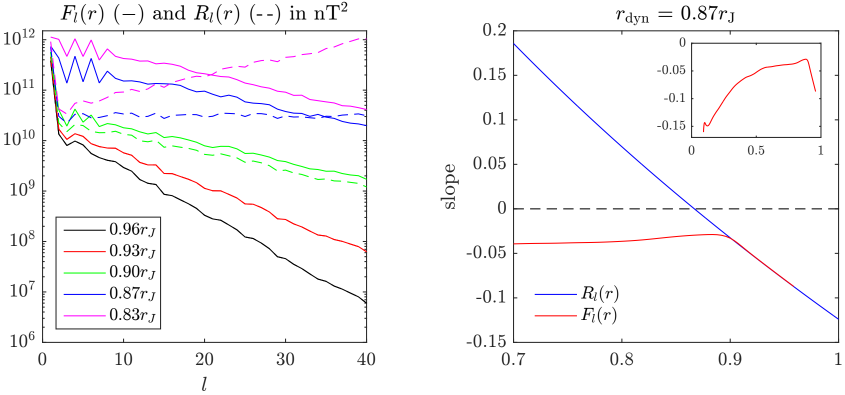


$$2\mu_0 E_B(r) = \frac{1}{4\pi} \oint |\boldsymbol{B}(r,\theta,\phi)|^2 \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi$$

$$= \sum_{l=1}^{\infty} F_l(r)$$

$$\boldsymbol{j} = \boldsymbol{0} \text{ exactly } \implies R_l(r) = F_l(r)$$

$R_l(r)$ versus $F_l(r)$



- $\boldsymbol{j} \neq \mathbf{0}$ (R_l deviates from F_l) starting at about $0.9 r_J$
- Lowes spectrum R_l prediction deeper than actual $r_{\rm dyn}$
- numerical model produces $r_{\rm dyn}$ consistently with observations
- transition layer (moderate σ) not contributes to dynamo action
- $F_l(r) \sim$ flat in a large range $0.5 r_J < r < 0.9 r_J$: white-noise source