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Effects of a guided-field on particle diffusion in magnetohydrodynamic turbulence

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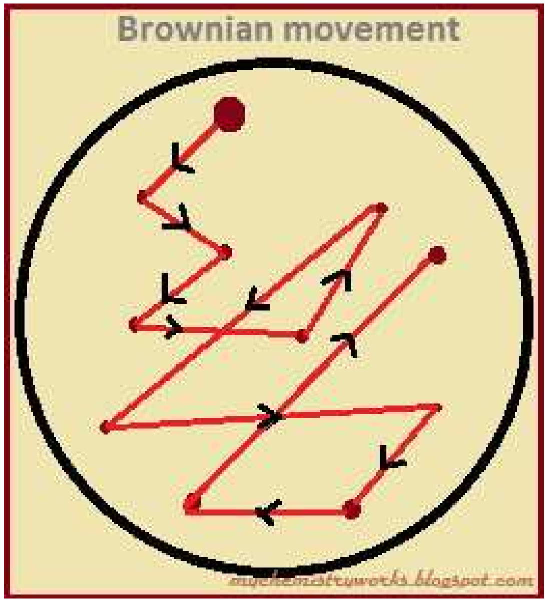
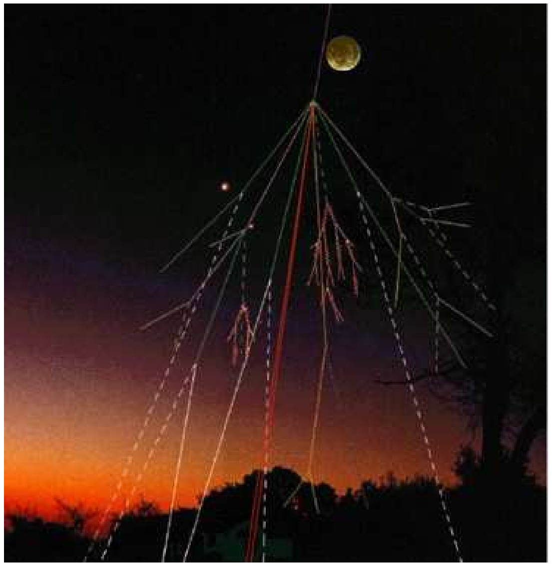
Particle transport in fluids





- Brownian motion observed under the microscope
- dispersion of pollutants in the atmosphere
- cosmic ray propagation through the interstellar medium
- tracing particle trajectories gives alternative view of the structure of the fluid flow — the Lagrangian viewpoint

Single-particle turbulent diffusion

- mean squared displacement:

$$\langle|\Delta\vec{X}(t)|^2\rangle\,, \quad \Delta\vec{X}(t)=\vec{X}(t)-\vec{X}(0)$$

- Taylor's formula (1921) for large t:

$$\vec{X}(t)=\vec{X}(0)+\int_0^t \mathrm{d}\tau\,\vec{V}(\tau)$$

$$\langle|\Delta\vec{X}(t)|^2]\rangle = 2\,t\int_0^\infty \mathrm{d}\tau\,\langle\vec{V}(\tau)\cdot\vec{V}(0)\rangle = 2tD$$

assume system is homogeneous and stationary and the integral exists

- Lagrangian velocity correlation:

$$C_L(\tau)=\langle\vec{V}(\tau)\cdot\vec{V}(0)\rangle$$

- diffusion coefficient:

$$D=\int_0^\infty \mathrm{d}\tau\,\langle\vec{V}(\tau)\cdot\vec{V}(0)\rangle$$

Field-guided MHD turbulence + tracers

- Motion of a electrically conducting fluid:

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{1}{\rho_0}\nabla p + (\nabla \times \vec{B}) \times \vec{B} + \nu \nabla^2 \vec{u} + \vec{f}$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{u} \times \vec{B}) + \eta \nabla^2 \vec{B}$$

$$\nabla \cdot \vec{u} = \nabla \cdot \vec{B} = 0$$

$\vec{f}$: isotropic random forcing at the largest scales

- Field-guided MHD turbulence:

$$\vec{B}(\vec{x}, t) = B_0 \hat{z} + \vec{b}(\vec{x}, t)$$

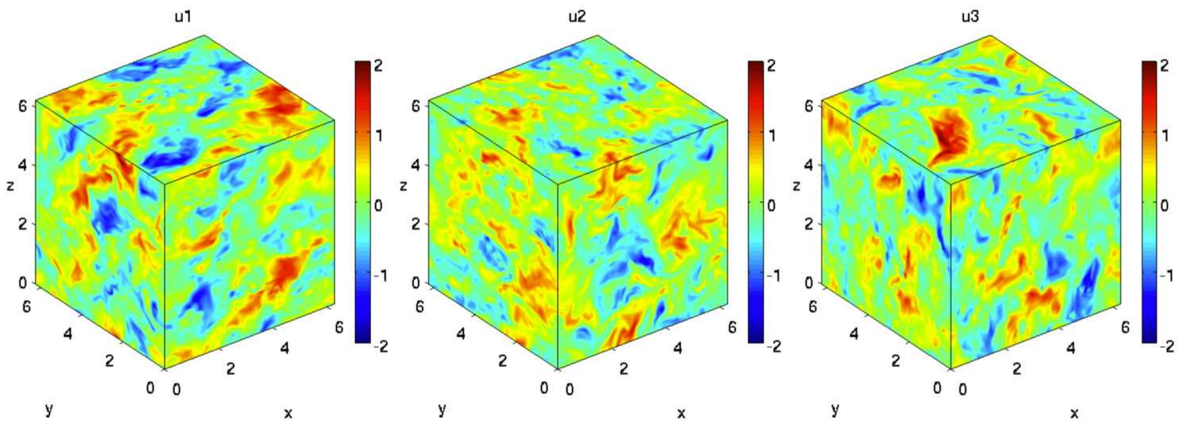
- Evolution of passive tracer particles:

$$\frac{\mathrm{d}\vec{X}(t)}{\mathrm{d}t} = \vec{V}(t) = \vec{u}(\vec{X}(t), t)$$

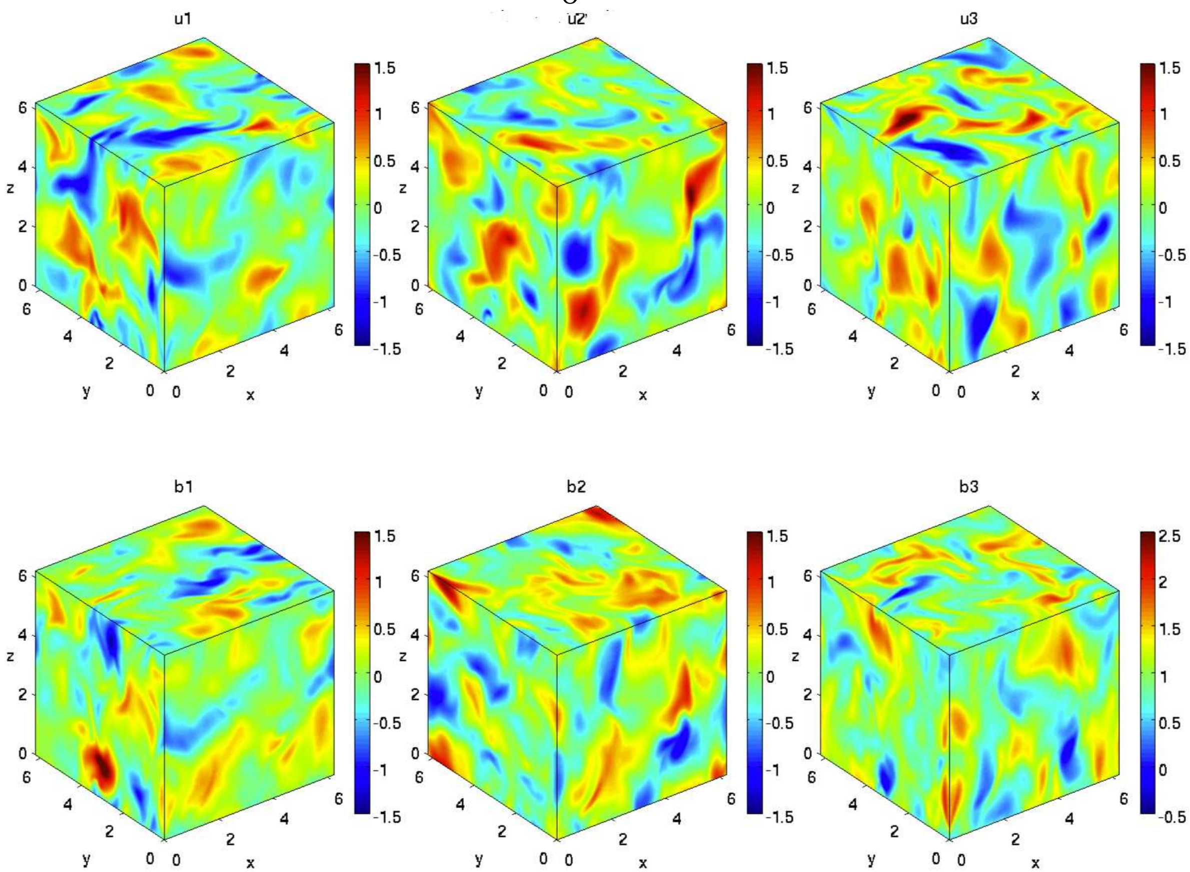
$$\vec{X}(0) = \vec{\alpha}$$

Typical velocity and magnetic fields

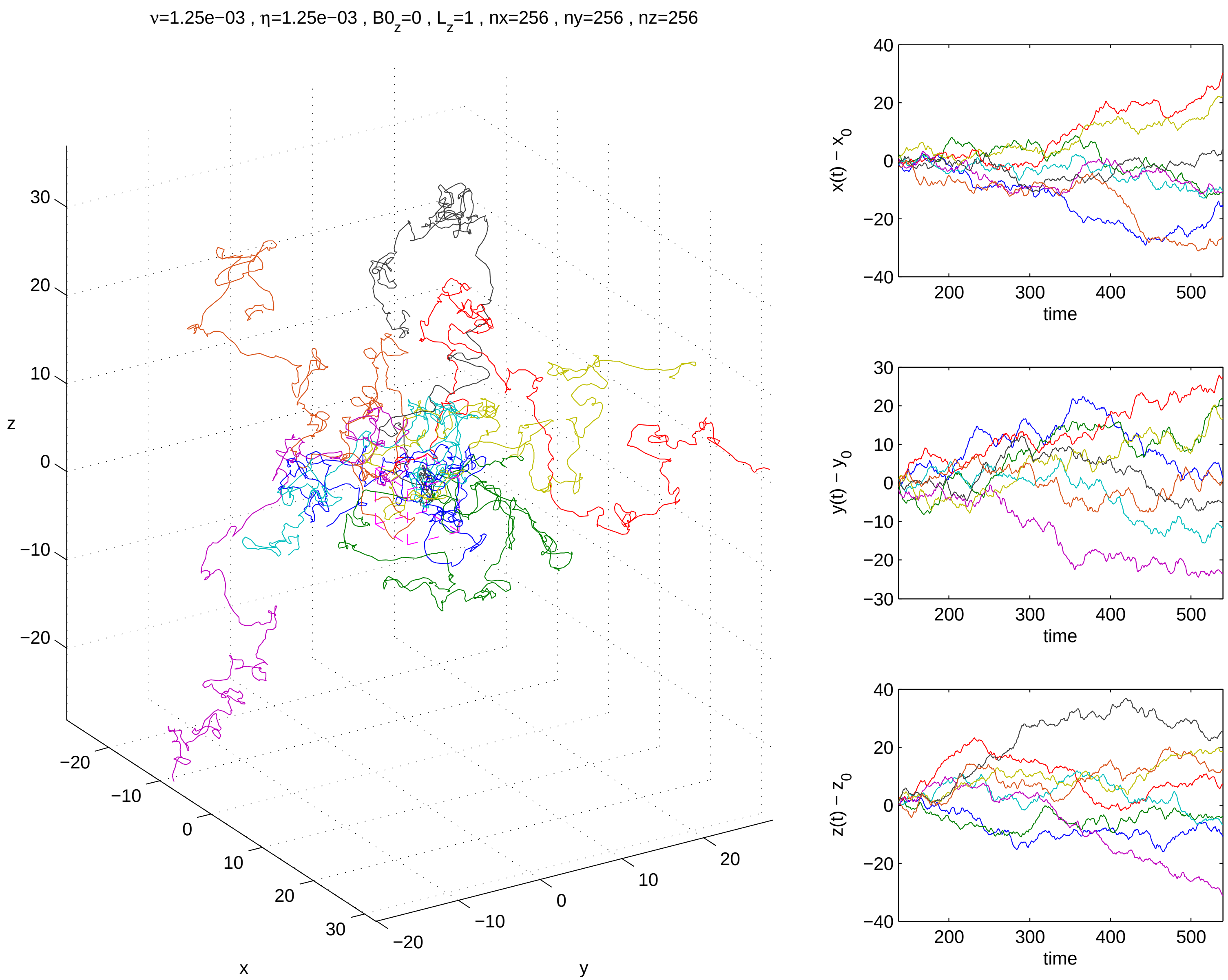
hydrodynamic case $(\nu = \eta \sim 10^{-3})$



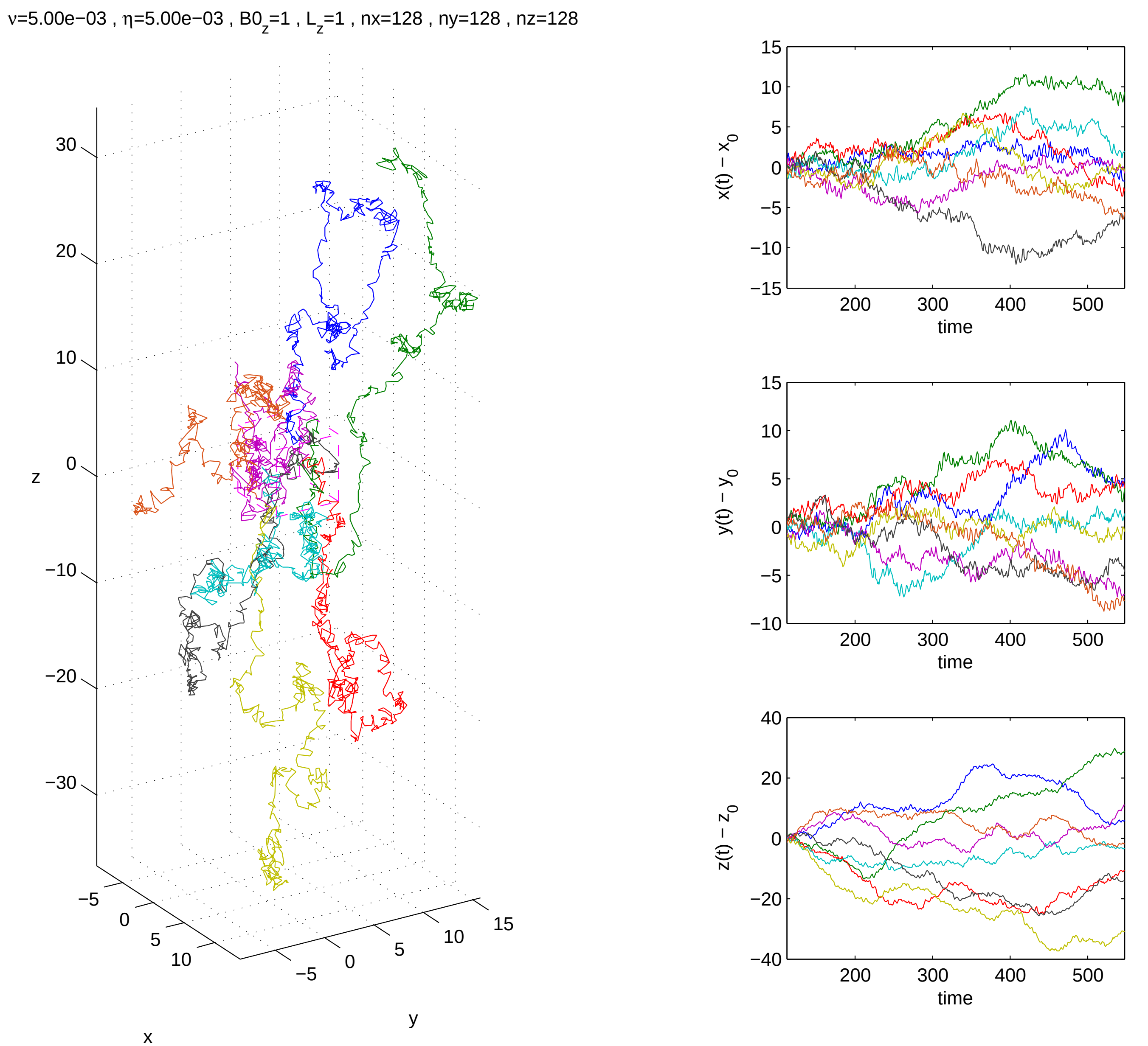
$B_0 = 1$



The hydrodynamic case

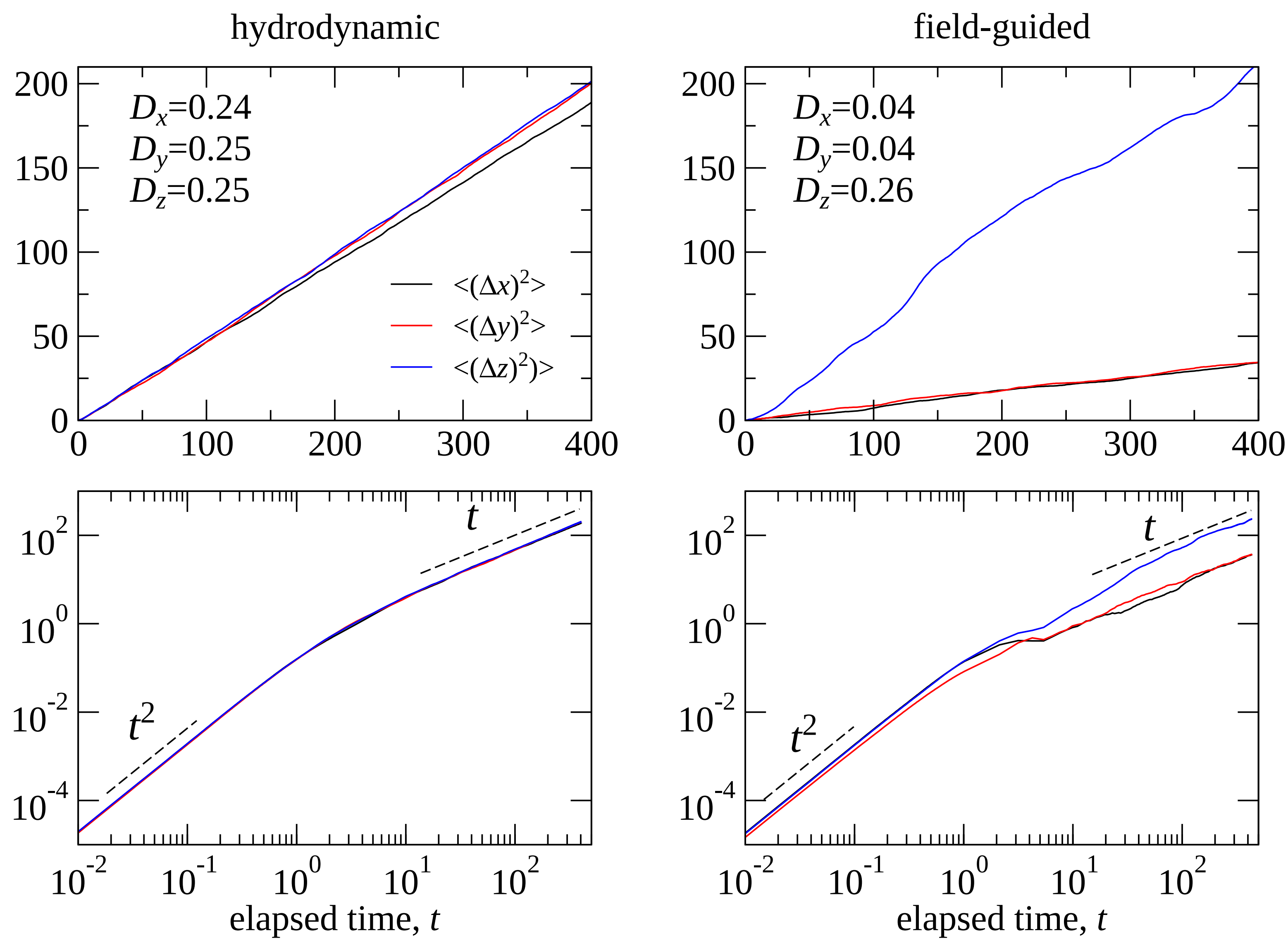


The field-guided case $(B_0 = 1)$



- transport suppressed in the field-perpendicular direction!

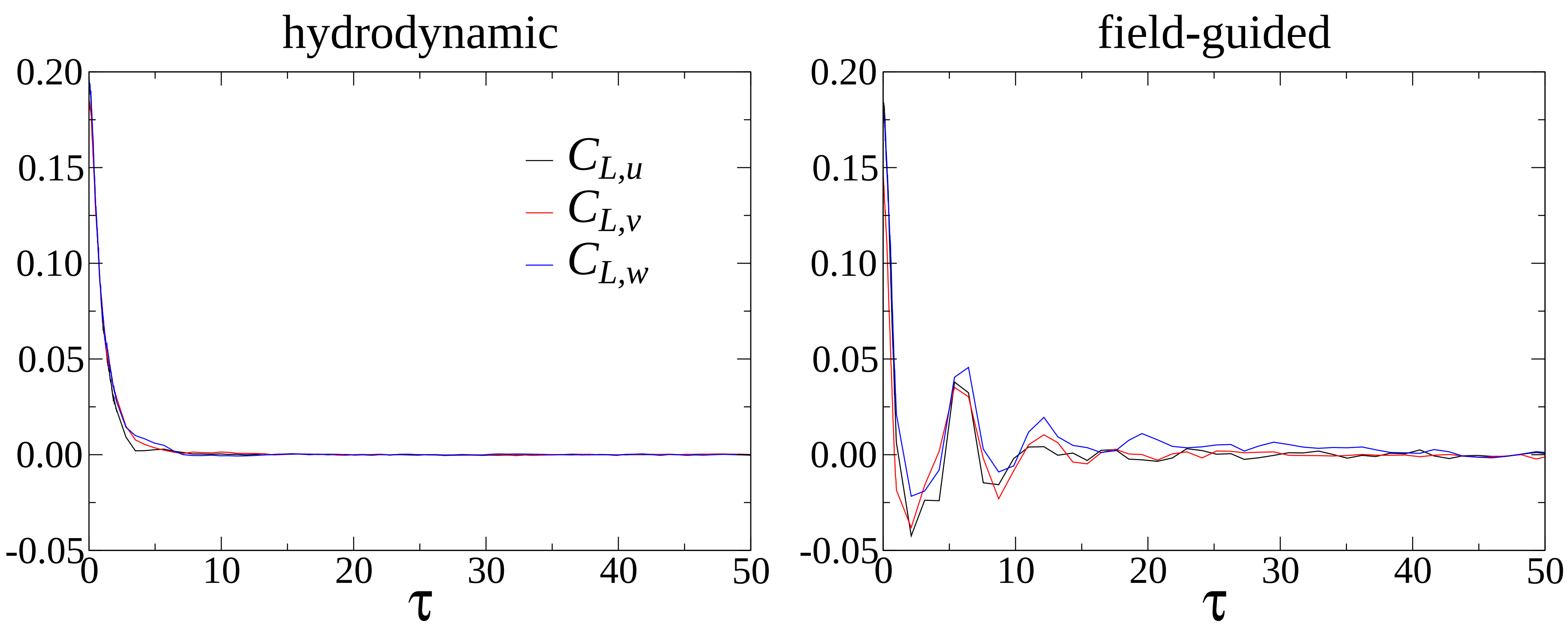
Scaling of mean-squared displacement



- ballistic limit: $\sim t^2$ at small time
- diffusive scaling: $\sim t$ at large time, $\langle(\Delta x)^2\rangle \sim 2D_x t$, etc

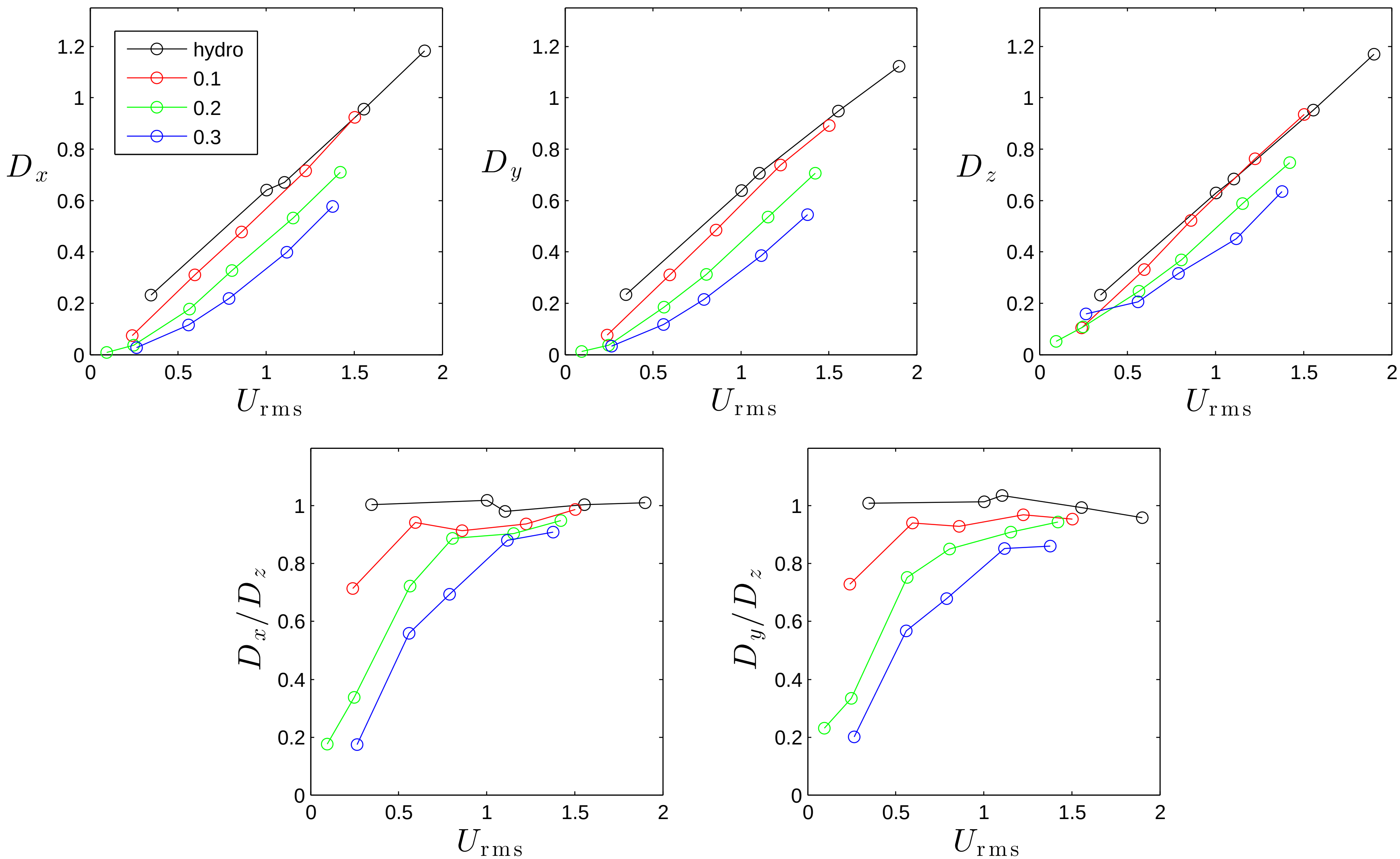
Lagrangian velocity correlation function

$$C_L(\tau) = \langle \vec{V}(\tau) \cdot \vec{V}(0) \rangle$$



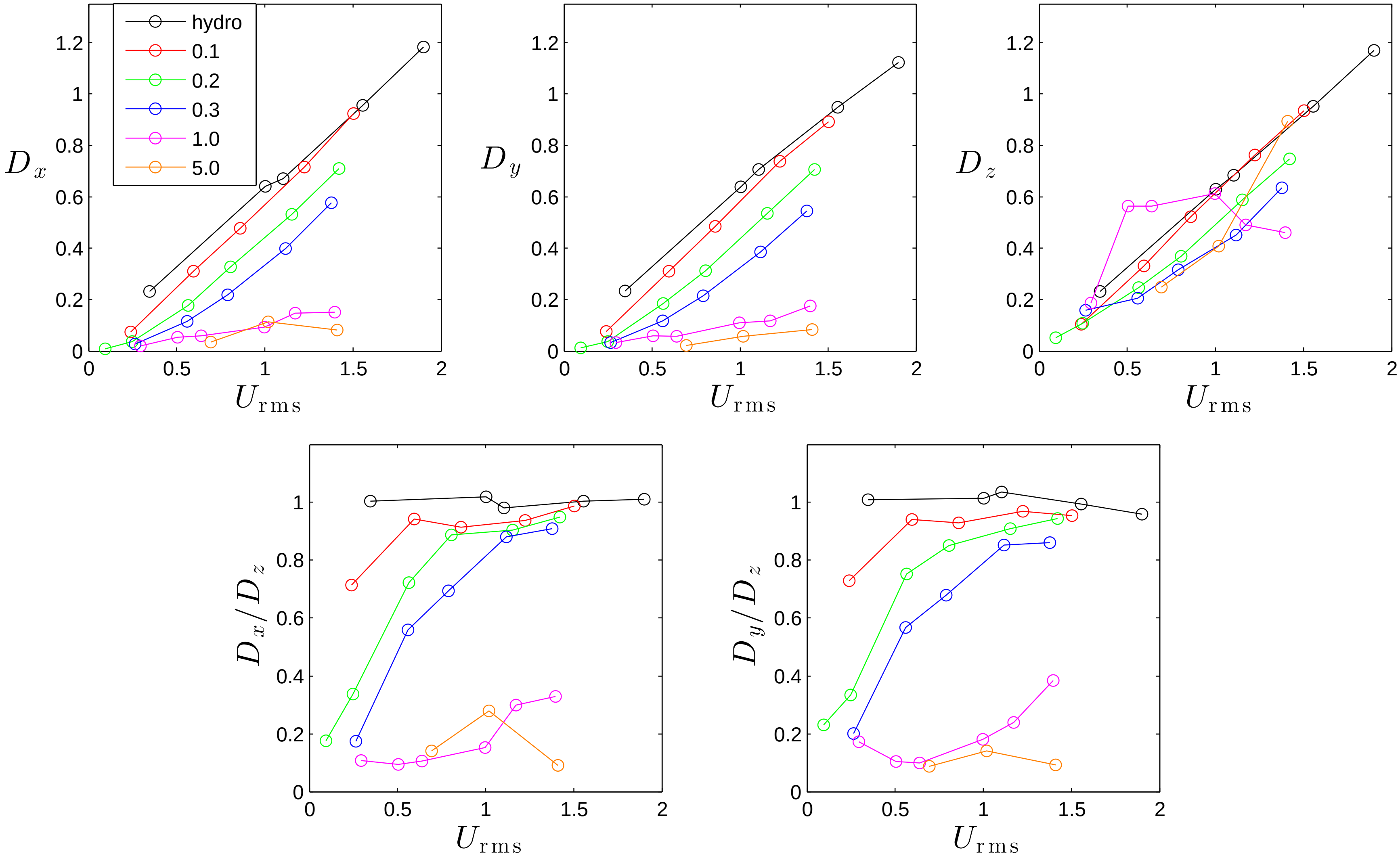
- hydrodynamic: $\sim \exp(-\tau)$, short correlation time
- field-guided: oscillatory, long correlation time
- how things depend on the guided-field strength B_0?

Diffusivity at different (weak) $B_0 \lesssim U_{\rm rms}$



- diffusion is reduced by B_0, including the z-direction
- anisotropic suppression: $D_x, D_y \lesssim D_z$
- strong $U_{\rm rms} (\gtrsim B_0)$ reduces the anisotropy in D's

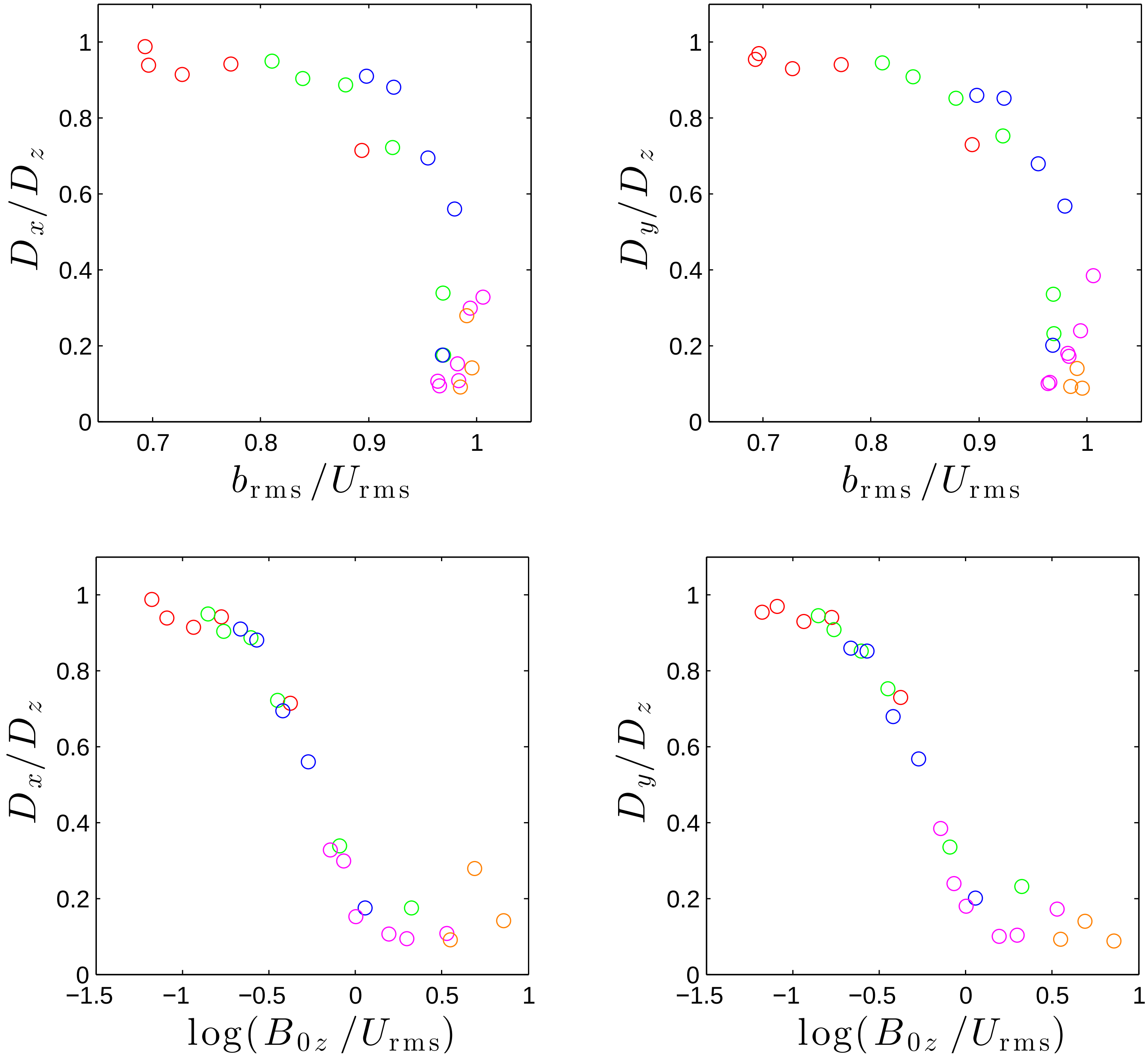
Diffusivity at different B_0



At strong guided-field strength, $B_0 \gtrsim U_{\rm rms}$

- D_x, D_y are strong suppressed, anomalous behavior of D_z
- D_x/D_z , $D_y/D_z \ll 1$ for the values of $U_{\rm rms}$ studied

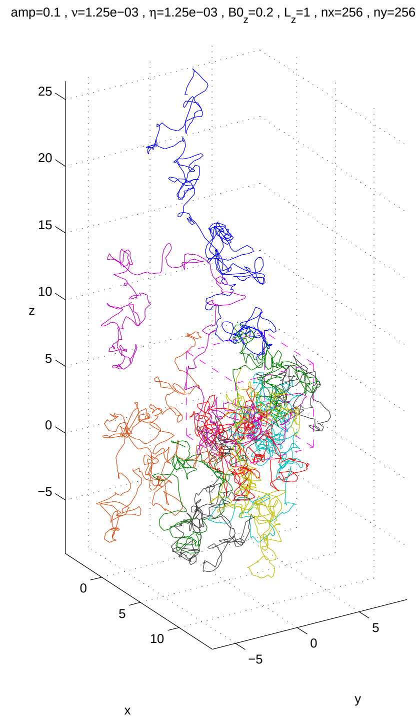
Anisotropic turbulent diffusion

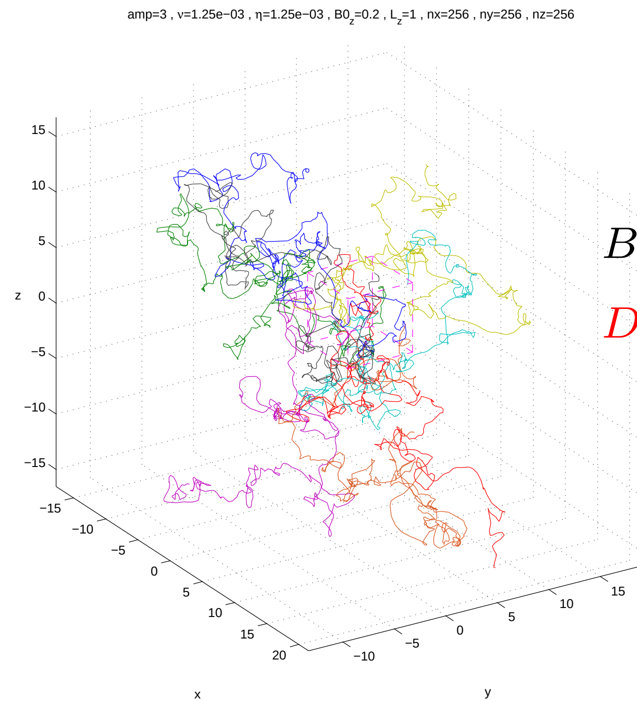


Particle trajectories

$B_0 = 0.2, U_{\rm rms} = 0.25$

$D_x/D_z = 0.34$



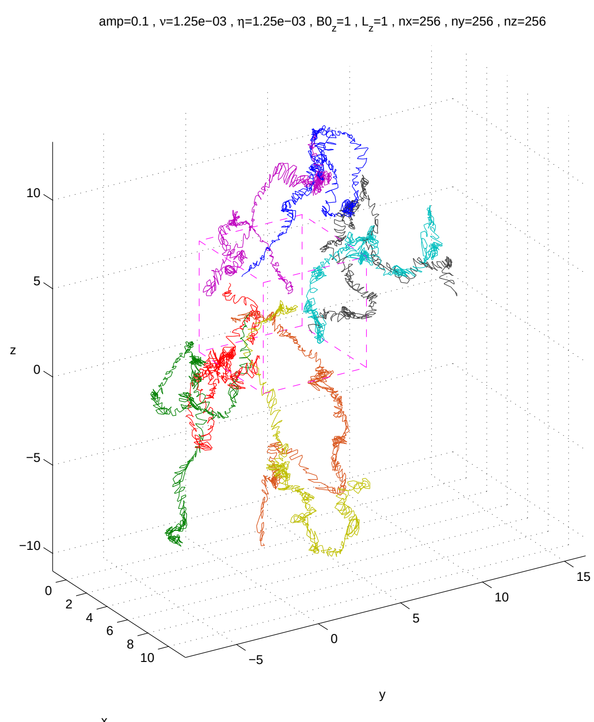


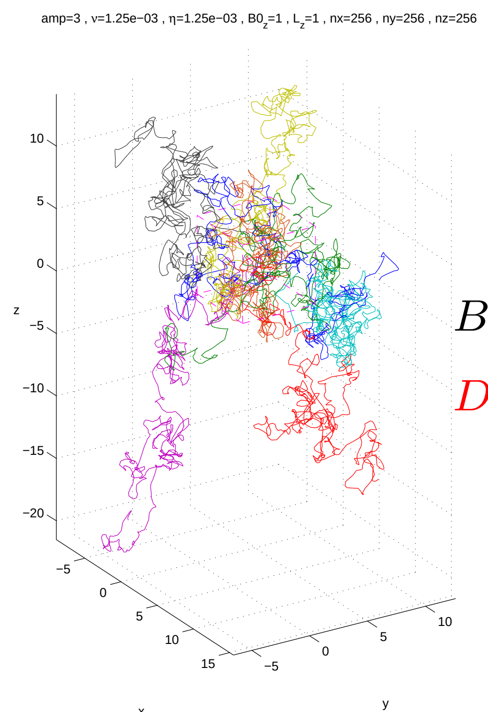
$B_0 = 0.2, U_{\rm rms} = 1.42$

$D_x/D_z = 0.95$

$B_0 = 1.0, U_{\rm rms} = 0.29$

$D_x/D_z = 0.24$

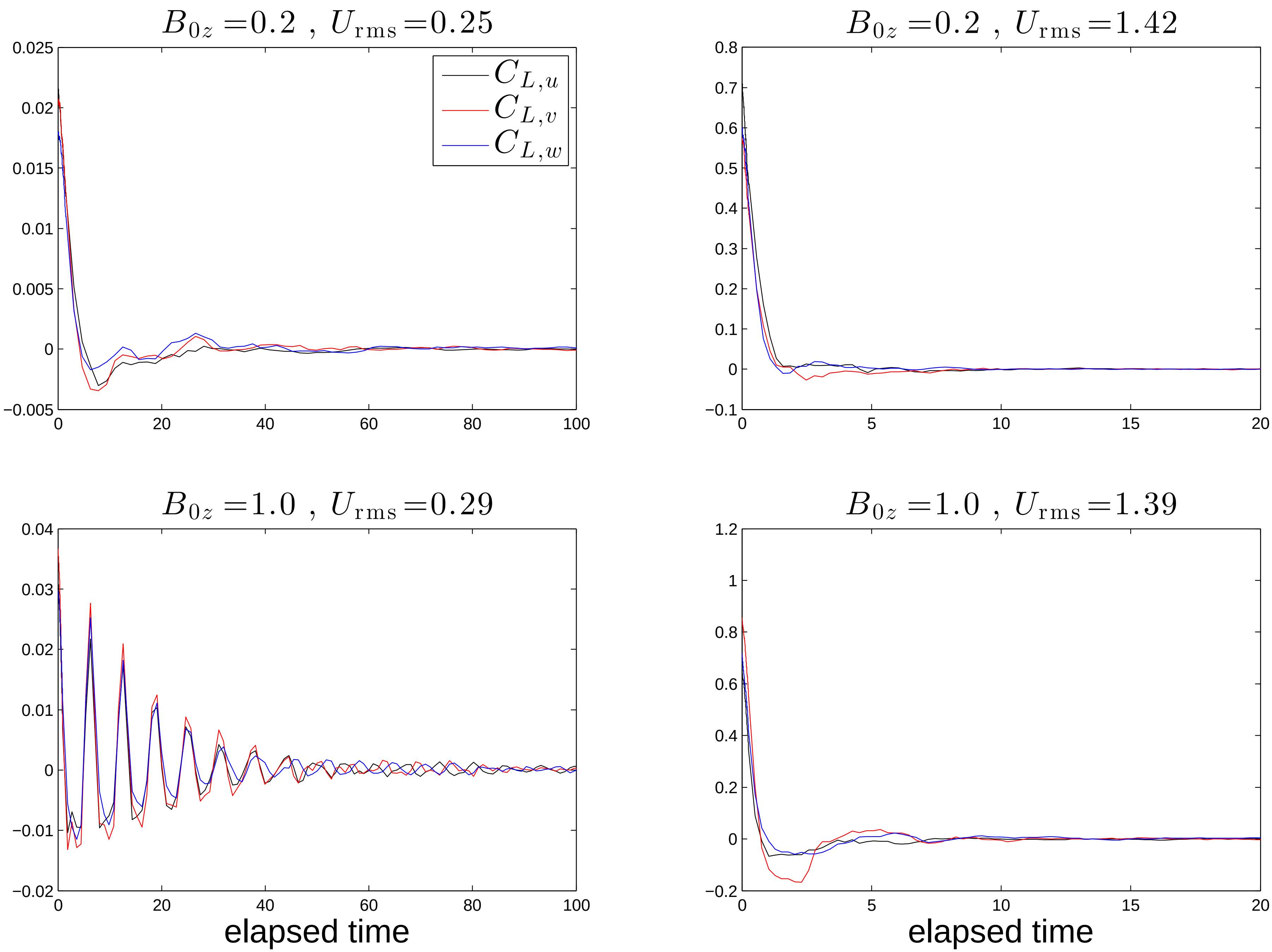




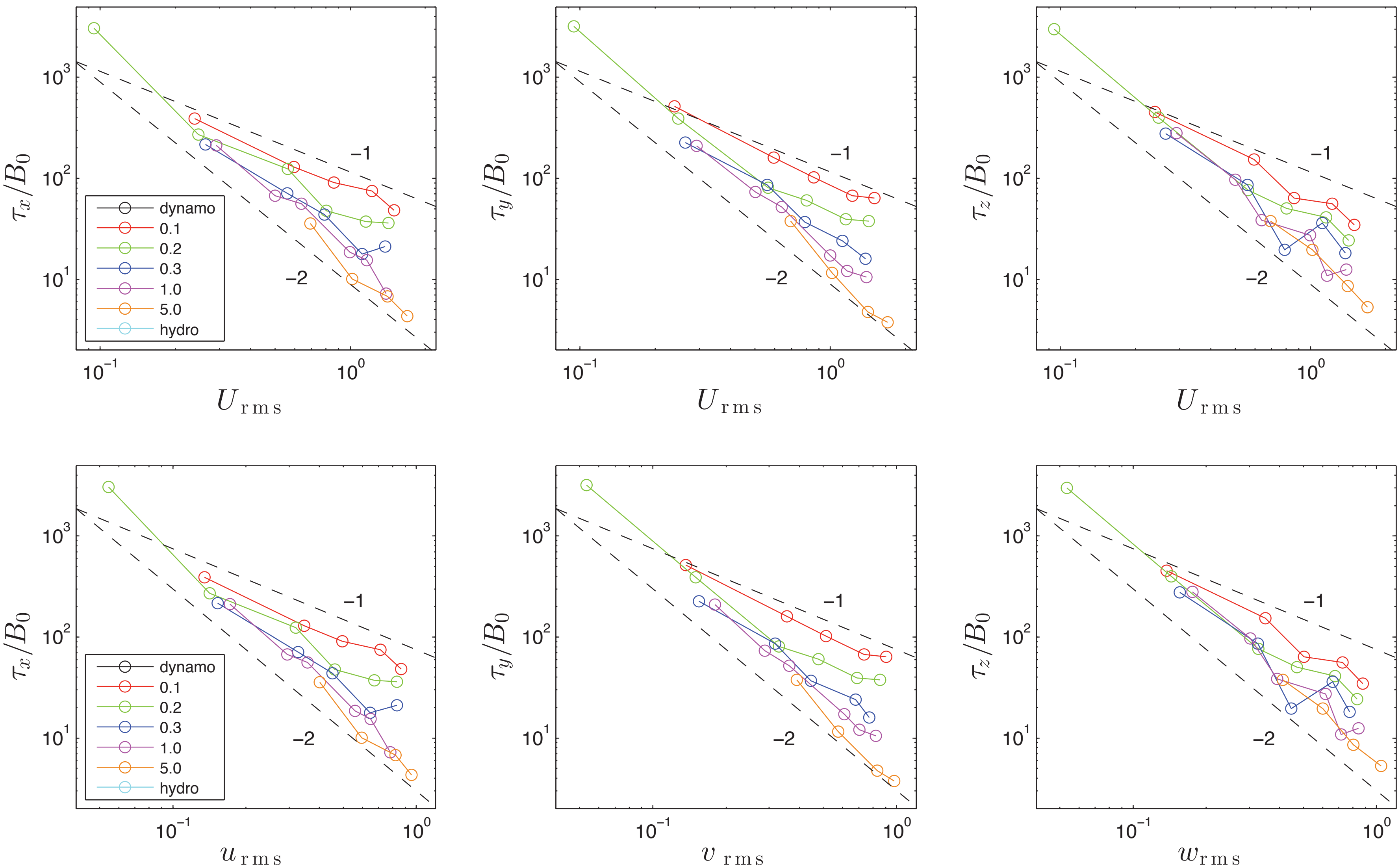
$B_0 = 1.0, U_{\rm rms} = 1.39$

$D_x/D_z = 0.34$

Lagrangian velocity correlation



Velocity decorrelation time



A physical picture ...

- wave induces memory into the system

  wave frequency: $\tau_A^{-1} \sim B_0$

- background turbulence removes memory

  decorrelation time: τ_u

- a competition between τ_A and τ_u

- anisotropic diffusion:
  - $b_{\rm rms}/U_{\rm rms} \approx 1$
  - $\tau_A \ll \tau_u$
  - $E_u(k) \approx E_b(k)$

- quantitative theory in development

Summary

- study single-particle diffusion in 3D MHD turbulence
- transport mostly shows diffusive scaling at large time
- anisotropic suppression of turbulent diffusion by a guided-field $(D_x \, , D_y \lesssim D_z)$
- competition between waves and background turbulence