Particle diffusion in magnetohydrodynamic turbulence

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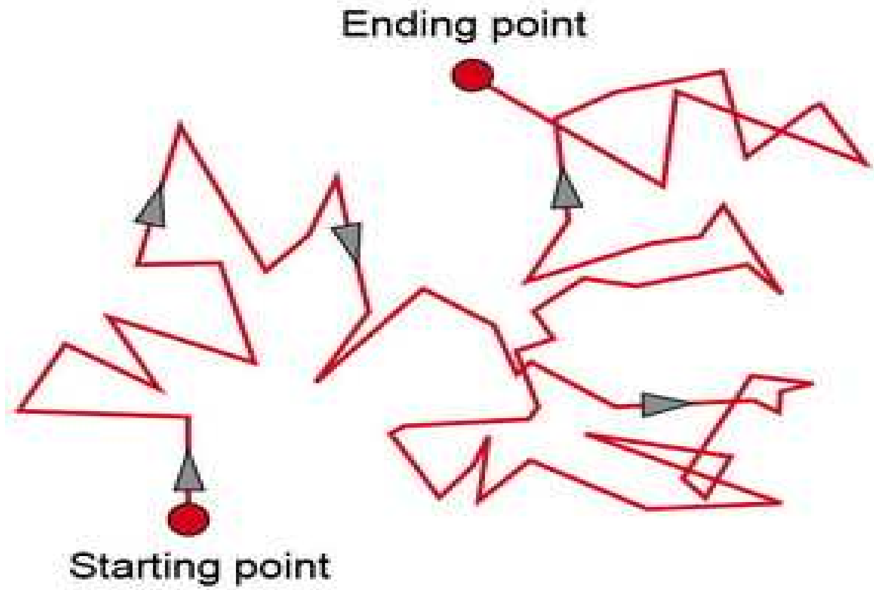
Single-particle diffusion



- transport properties in fusion experiments
- astrophysical pheonomena:
  - cosmic ray propagation
  - thermal conductivity in galaxy-cluster plasma
- mean scalar ϕ evolution:

$$\langle \phi(\vec{x}, t) \rangle = \int \mathrm{d}\vec{\alpha} \, \langle \phi_0(\vec{\alpha}) \rangle \, P(\vec{x}, t | \vec{\alpha})$$

Diffusive turbulent transport

- mean squared displacement:

$$\langle |\Delta \vec{X}(t)|^2 \rangle \,, \quad \Delta \vec{X}(t) = \vec{X}(t) - \vec{X}(0)$$

- Taylor's formula (1921) for large t:

$$\vec{X}(t) = \vec{X}(0) + \int_0^t \mathrm{d}\tau \, \vec{V}(\tau)$$

$$\langle |\Delta \vec{X}(t)|^2] \rangle = 2\,t \int_0^\infty \mathrm{d}\tau \, \langle \vec{V}(\tau) \cdot \vec{V}(0) \rangle = 2tD$$

- Lagrangian velocity correlation:

$$C_L(\tau) = \langle \vec{V}(\tau) \cdot \vec{V}(0) \rangle$$

- diffusion coefficient:

$$D = \int_0^\infty \mathrm{d}\tau \, \langle \vec{V}(\tau) \cdot \vec{V}(0) \rangle$$

MHD turbulence

- The governing equations:

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\nabla p + (\nabla \times \vec{B}) \times \vec{B} + \nu \nabla^2 \vec{u} + \vec{f}$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{u} \times \vec{B}) + \eta \nabla^2 \vec{B}$$

$$\nabla \cdot \vec{u} = \nabla \cdot \vec{B} = 0$$

$\vec{f}$: random forcing at the largest scales

- Evolution of passive tracer particles:

$$\frac{\mathrm{d}\vec{X}(t)}{\mathrm{d}t} = \vec{V}(X(t), t)$$

$$\vec{X}(0) = \vec{\alpha}$$

- Field-guided MHD turbulence:

$$\vec{B}(\vec{x}, t) = B_0 \hat{z} + \vec{b}(\vec{x}, t)$$

Previous work: the 2D case

ON THE EFFECTS OF A WEAK MAGNETIC FIELD ON TURBULENT TRANSPORT

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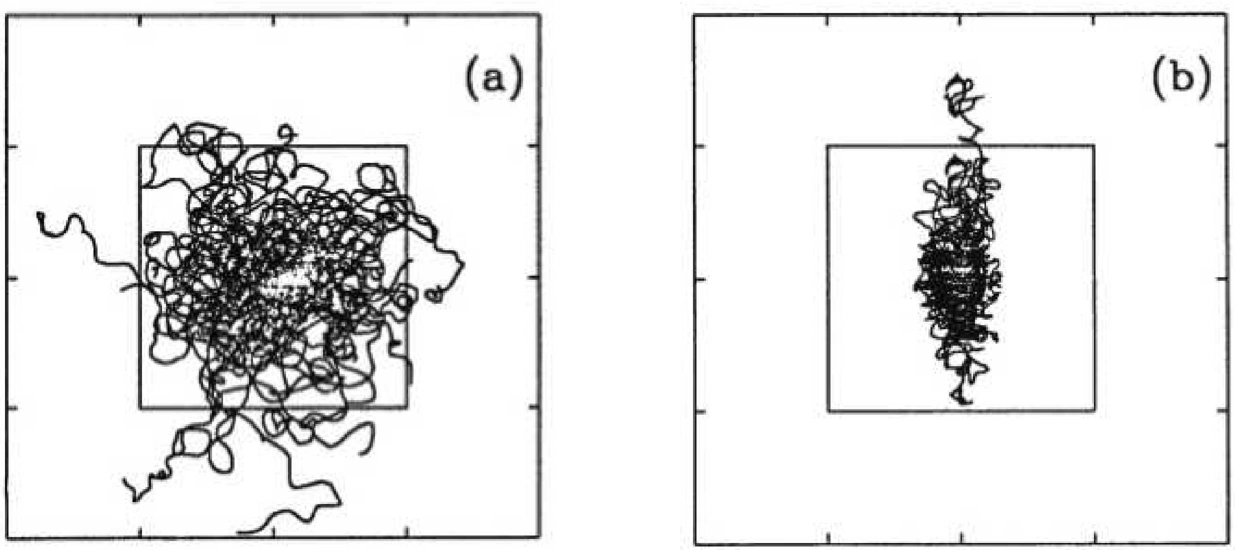
ABSTRACT

We discuss the effects of a weak large-scale magnetic field on turbulent transport. We show by means of a series of two-dimensional numerical experiments that turbulent diffusion can be effectively suppressed by a (large-scale) magnetic field whose energy is small compared to equipartition. The suppression mechanism is associated with a subtle modification of the Lagrangian energy spectrum, and it does not require any substantial reduction of the turbulent amplitude. We exploit the relation between diffusion and random walking to emphasize that the effect of a large-scale magnetic field is to induce a long-term memory in the field of turbulence. The implications for the general case of three-dimensional transport are briefly discussed.

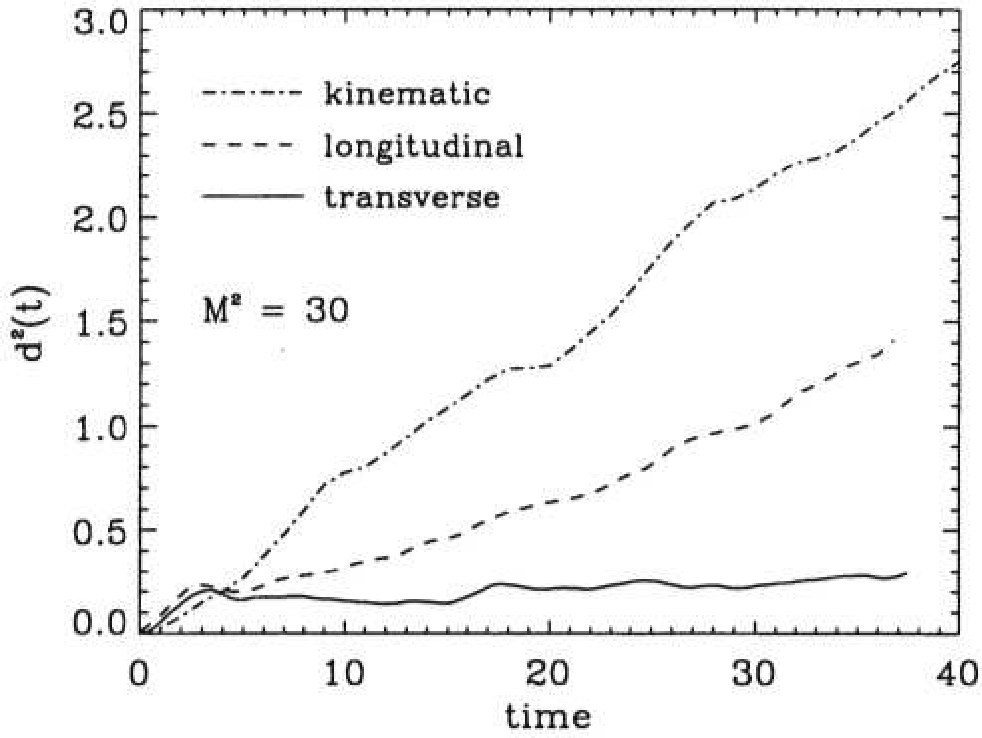
Subject headings: diffusion — MHD — turbulence

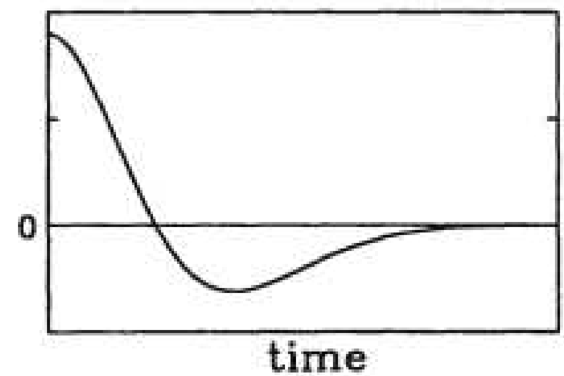
1. transport suppressed in direction $\perp$ to $B_0\hat{y}$



Previous work: the 2D case

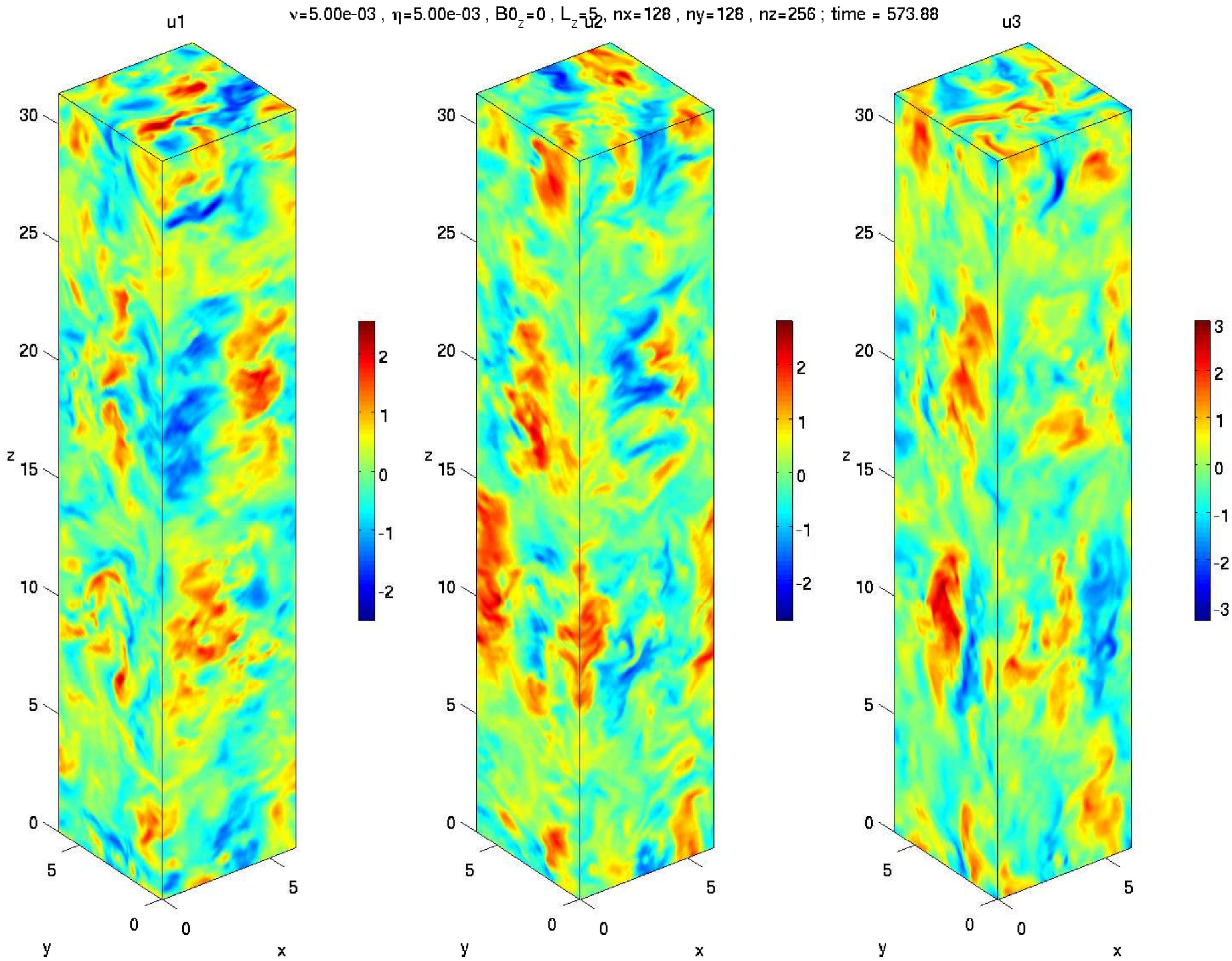
2. field-perpendicular transport is not diffusive
3. the system has long-term memory: slow decay of $C_L(\tau)$





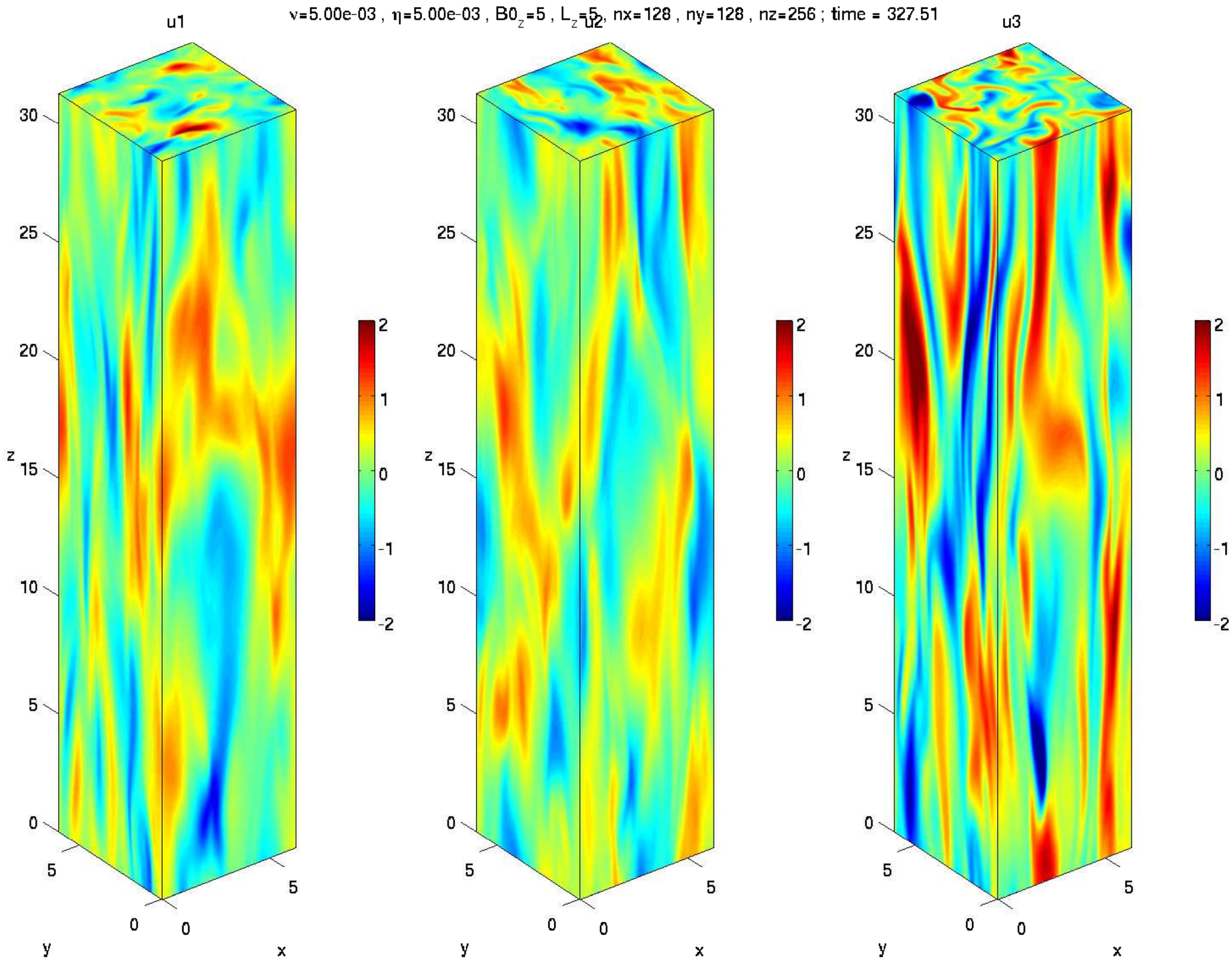
- *"...it is unlikely that in three dimensions the turbulent diffusivity becomes suppressed ... in three dimensions, motions that interchange field lines can bring together oppositely directed field lines without bending them."*

The hydrodynamic case, $\vec{B} = 0$



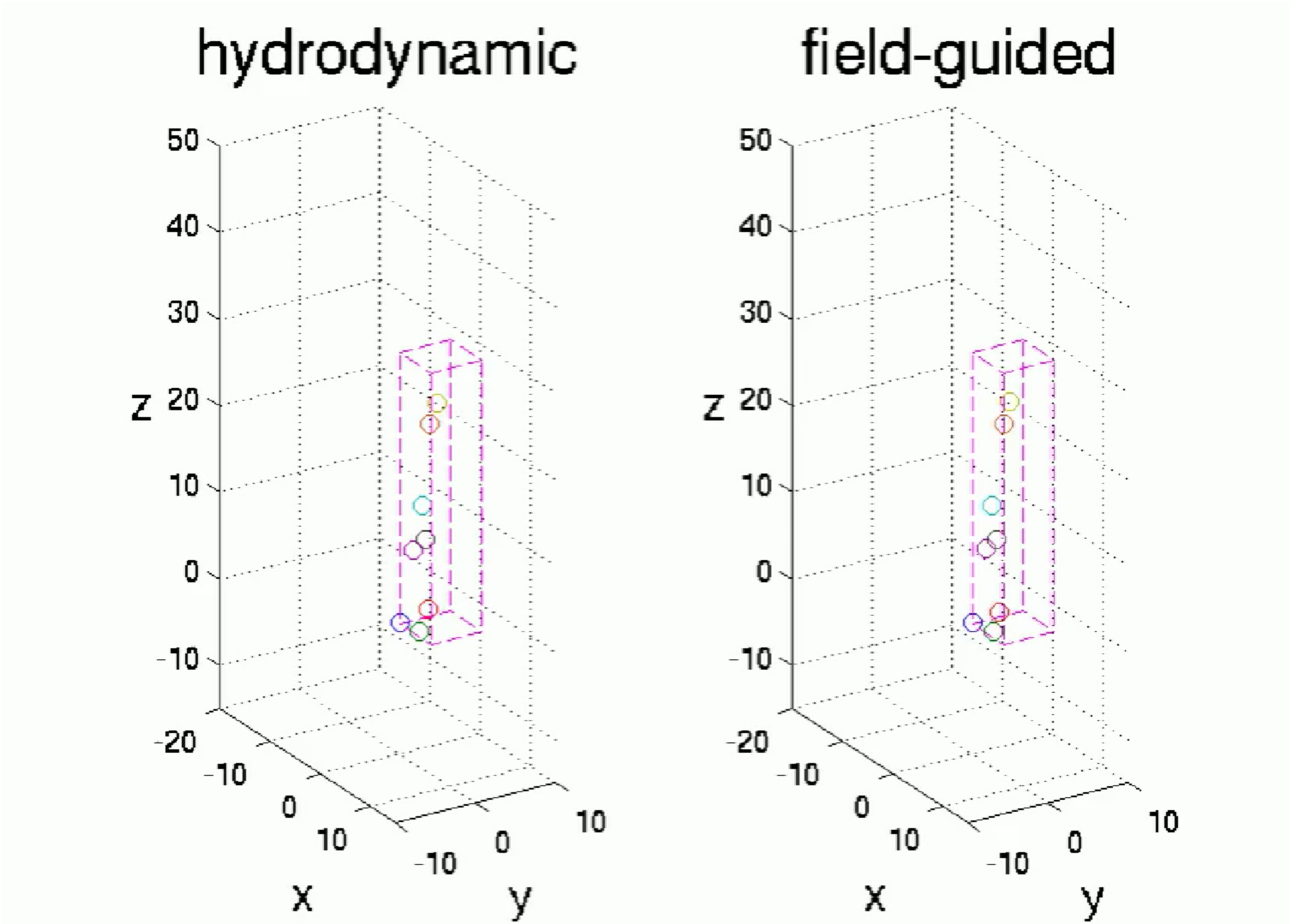
- system is homogeneous and isotropic

The field-guided case, $\vec{B} = B_0\hat{z}$

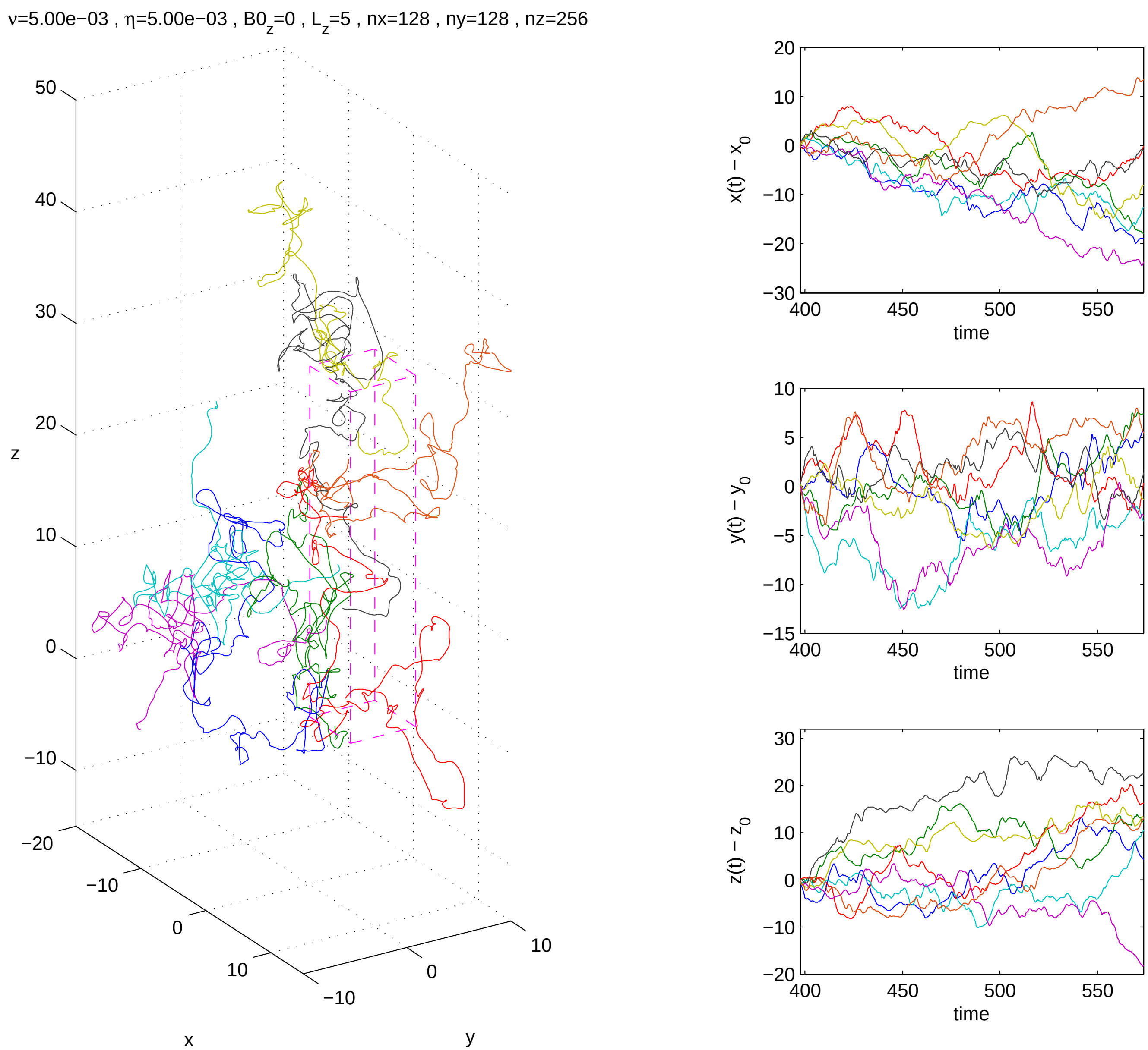


- anisotropic: elongation in the along-field direction

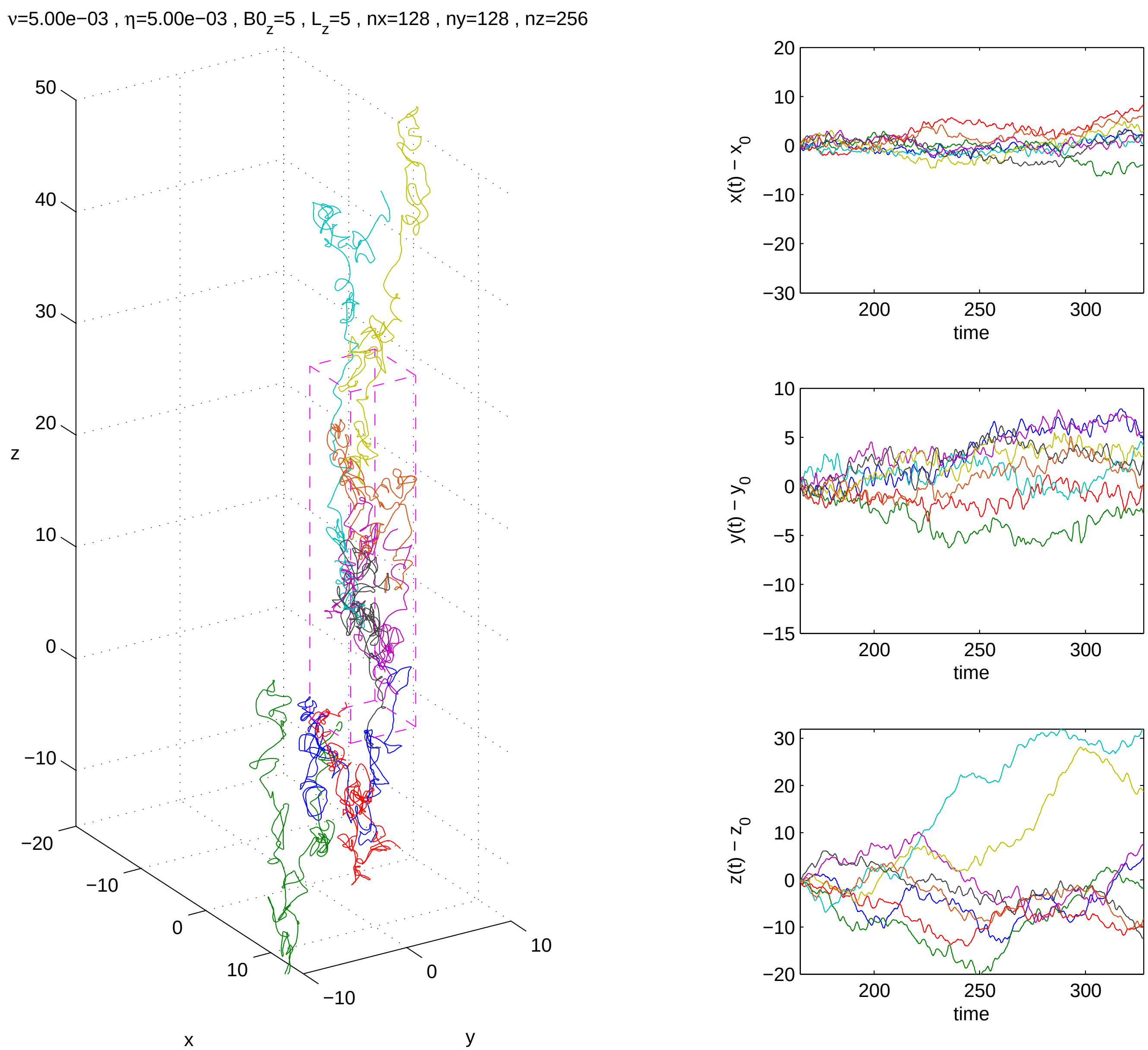
Particle tracking



The hydrodynamic case, $\vec{B} = 0$

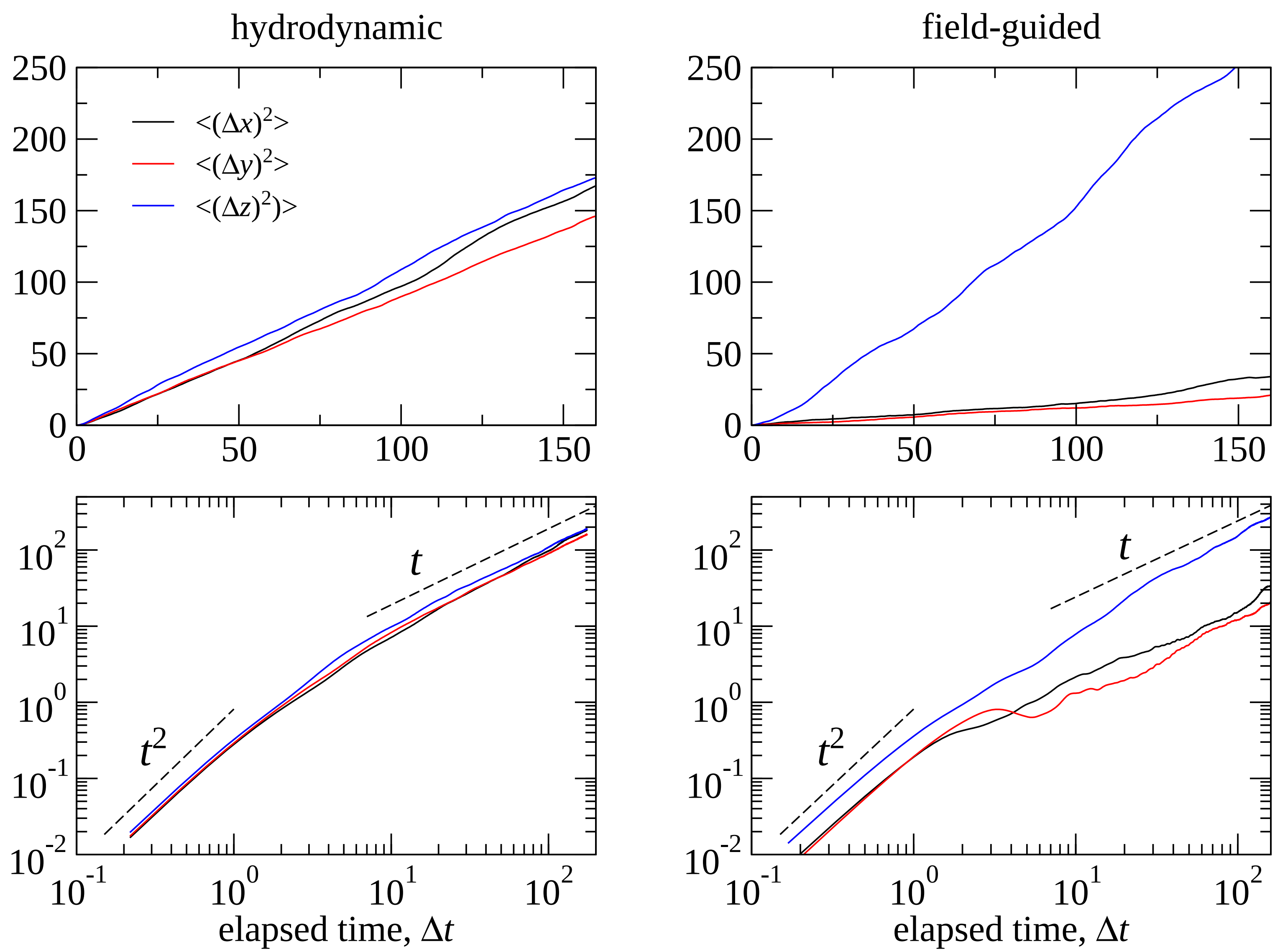


The field-guided case, $\vec{B} = B_0 \hat{z}$



- transport suppressed in the field-perpendicular direction!

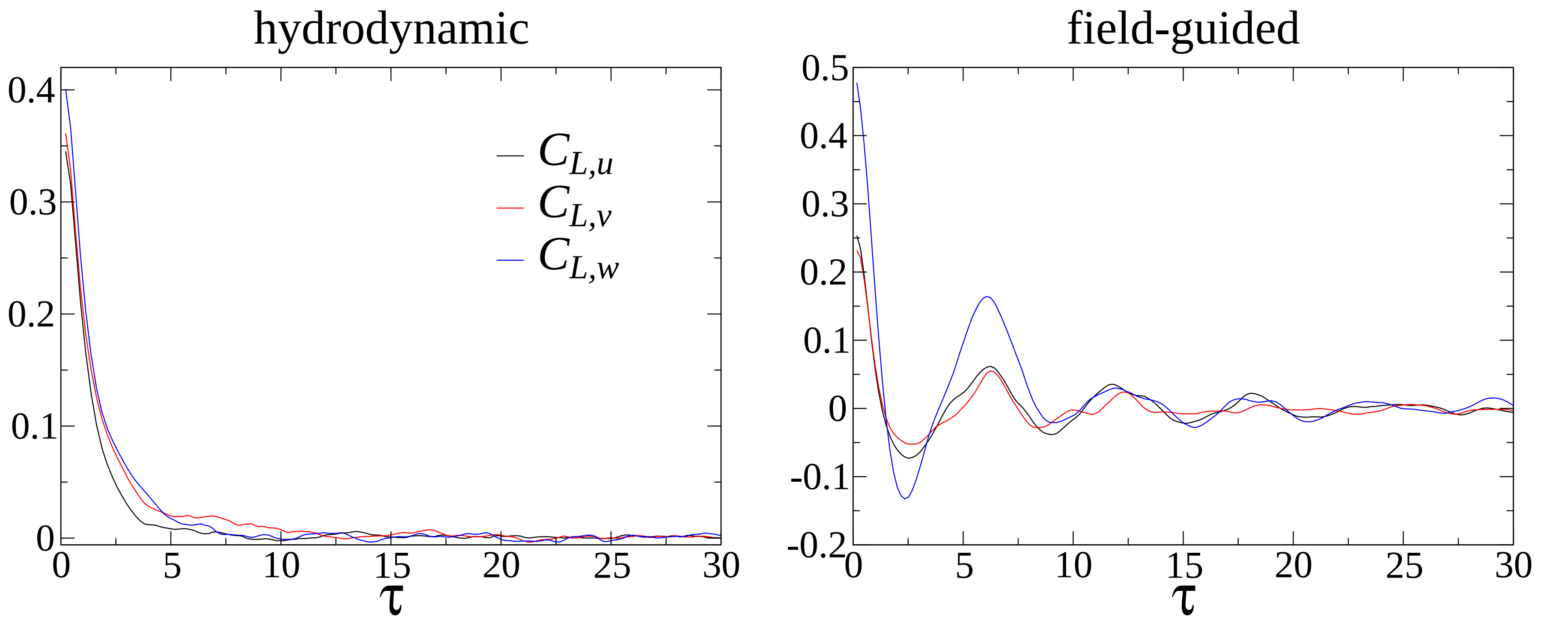
Scaling of mean-squared displacement



- ballistic limit: $\sim t^2$ at small time
- diffusive scaling: $\sim t$ at large time

Lagrangian velocity correlation function

$$C_L(\tau) = \langle \vec{V}(\tau) \cdot \vec{V}(0) \rangle$$



- hydrodynamic: $\sim \exp(-\tau)$, short correlation time
- field-guided: oscillatory, long correlation time

Summary

- study single-particle diffusion in 3D MHD turbulence
- strong field-guided case versus the hydrodynamics case
- suppression of turbulent transport in the field-perpendicular direction
- transport shows diffusive scaling at large time
- Is the mechanism of transport suppression the same or different in 2D and 3D?