Revealing small-scale structures in turbulent Rayleigh-Bénard convection

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Thermal convection

- Free convection
  - imposed temperature gradient leads to density difference in a fluid
  - hot fluid tends to rise, cold fluid tends to fall
  - flow is driven by buoyancy
- Applications
  - kitchen:
    - boiling water in a kettle
    - air flow in an oven
  - atmosphere and ocean:
    - formation of cloud and thunderstorms
    - oceanic *deep convection* → moderate winter climate in northern Europe
  - Earth’s interior: mantle convection

Rayleigh-Bénard convection

Fluid in a box heated from below and cooled from above

- Rayleigh number

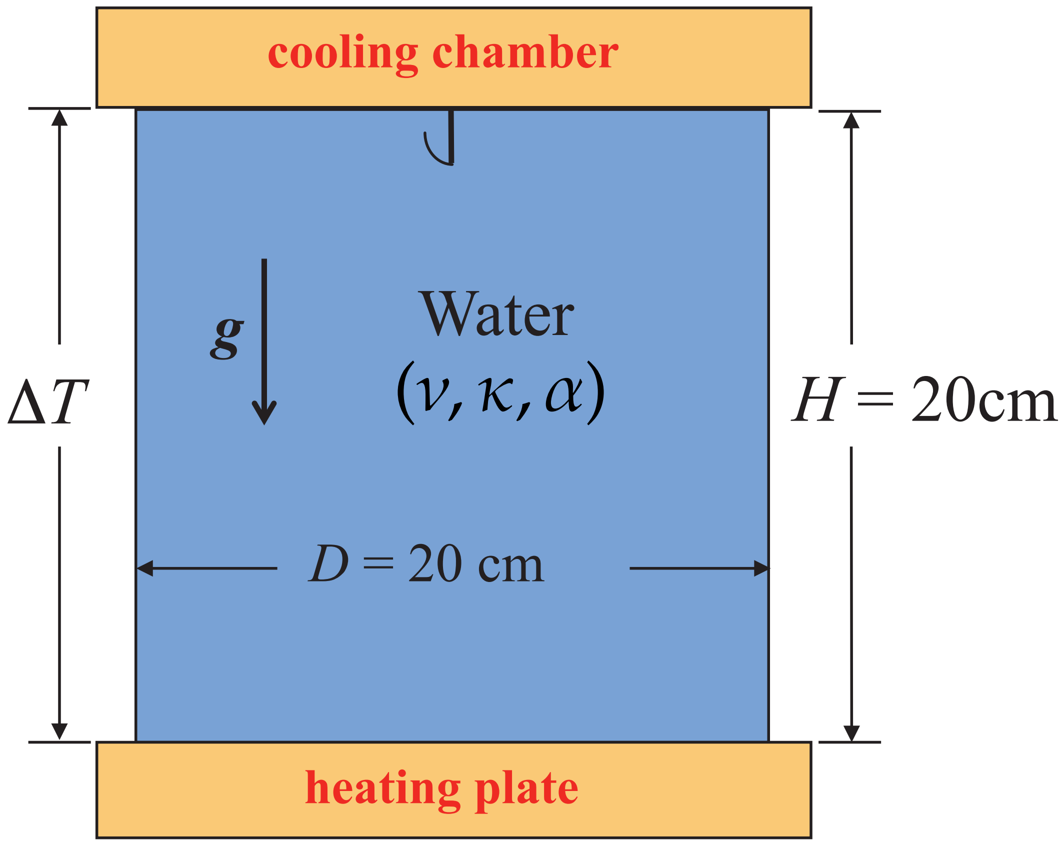
$$\mathrm{Ra} = \frac{\alpha g H^3 \Delta T}{\nu\kappa}$$

- Prandtl number

$$\mathrm{Pr} = \frac{\nu}{\kappa}$$

- Aspect ratio

$$\Gamma = \frac{D}{H}$$

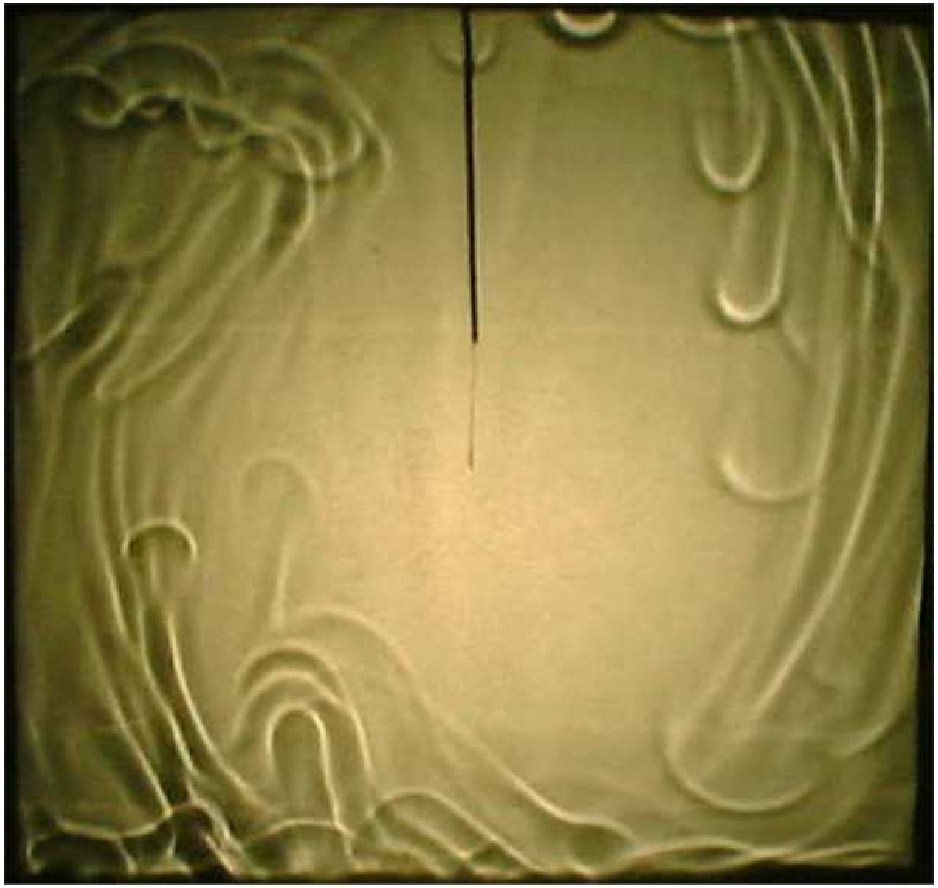


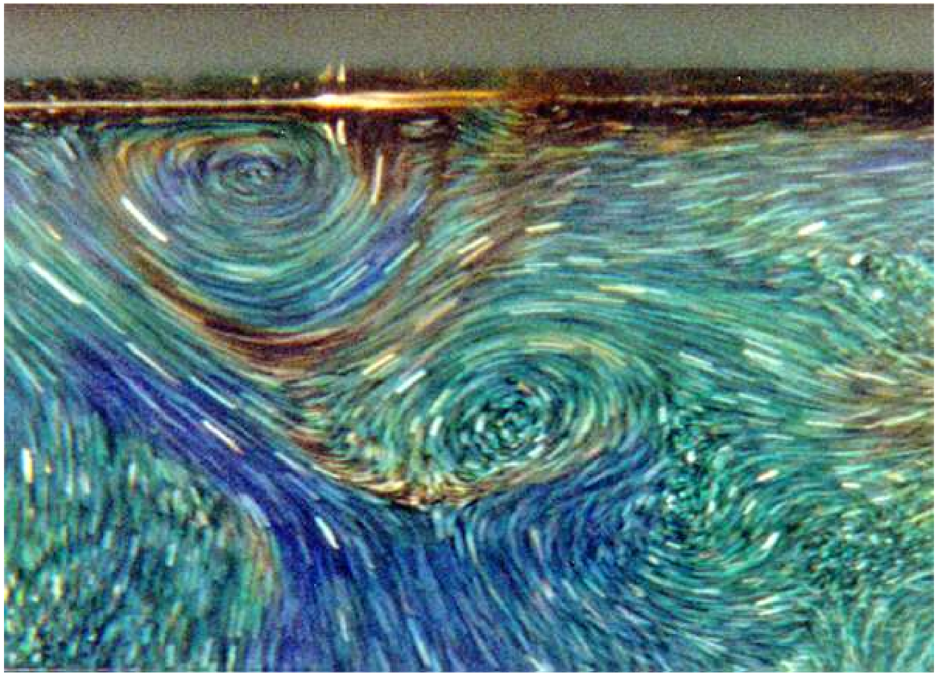
ν: viscosity

κ: thermal diffusivity

α: volume expansion coefficient

Rayleigh-Bénard convection





Left: Ra $= 6.8 \times 10^8$, Pr $= 596$ (dipropylene glycol) , $\Gamma = 1$

X. D. Shang, X. L. Qiu, P. Tong, and K.-Q. Xia, Phys. Rev. Lett. 90, 074501 (2003)

Right: Ra $= 2.6 \times 10^9$, Pr $= 5.4$ (water) , $\Gamma = 1$

Y. B. Du and P. Tong, J. Fluid Mech. 407, 57 (2000)

Global and local properties

- Large-scale (global) quantites, e.g. total heat transfer across the system

- Small-scale (local) quantities
  - structure of velocity and temperature fields
  - effects of thermal plums
  - Tool: structure functions, e.g.

$$S_u^{(p)}(r) = \langle\, |u(\vec{x} + \vec{r}) - u(\vec{x})|^p \,\rangle_{\vec{x}}$$

$$S_T^{(p)}(r) = \langle\, |T(\vec{x} + \vec{r}) - T(\vec{x})|^p \,\rangle_{\vec{x}}$$

  - expect different behaviour in the bulk and near the boundaries

Temperature structure functions

$$S_T^{(p)}(r) = \langle\, |T(\vec{x} + \vec{r}) - T(\vec{x})|^p \,\rangle_{\vec{x}}$$

- probing activities at scale r
- larger p emphasizes more extreme events
- motivations from Kolmogorov-type phenomenology
- scaling behavior:

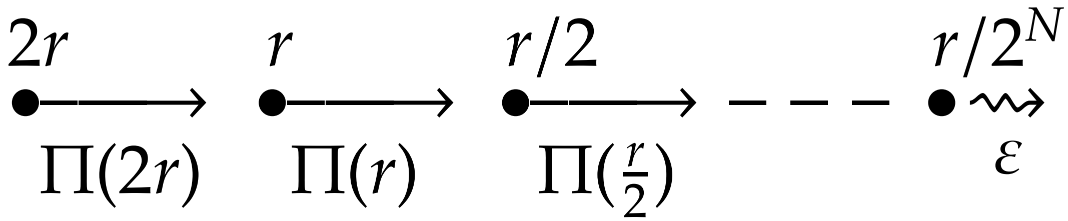
$$S_T^{(p)}(r) \sim r^{\zeta_T}$$

Given a time-series of measurement $T(t)$ at a fixed location, one can define a time domain structure function:

$$S_T^{(p)}(\tau) = \langle\, |T(t + \tau) - T(t)|^p \,\rangle_t$$

- Taylor's frozen flow hypothesis $\Rightarrow$ $S_T^{(p)}(\tau) \sim \tau^{\zeta_T}$

Cascade picture: passive scalar



Energy and temperature variance transferred from large scales to small scales, eventually being dissipated at the smallest scales

$$\varepsilon = \text{mean energy dissipation rate}$$

$$\chi = \text{mean thermal dissipation rate}$$

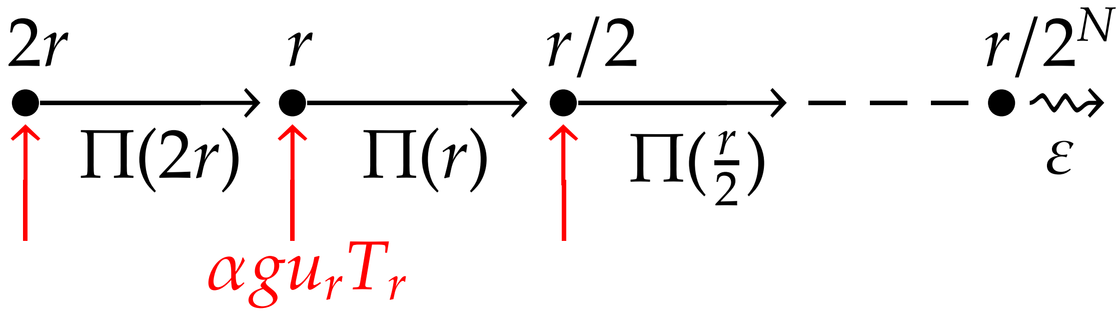
- no buoyancy, energy transfer rate Π is scale independent

$$\Pi = \varepsilon \quad \text{in the inertial range}$$

- relevant parameters are: ε, χ, r
- Obukhov-Corrsin scaling:

$$S_T^{(p)}(r) \sim \varepsilon^{-p/6}\, \chi^{p/2}\, r^{p/3}$$

Cascade picture: active scalar



$$\partial_t \vec{u} + (\vec{u} \cdot \nabla)\vec{u} = -\nabla p + \nu \nabla^2 \vec{u} + \alpha g T \hat{z}$$

$$\partial_t T + (\vec{u} \cdot \nabla) T = \kappa \nabla^2 T$$

- buoyancy is important, $\Pi(2r)$ is negligble at r

$$\Pi(r) = \alpha g u_r T_r \quad \text{in the inertial range}$$

- relevant parameters are: $\alpha g, \chi, r$
- Bolgiano-Obukhov scaling:

$$S_T^{(p)}(r) \sim (\alpha g)^{-p/5} \chi^{2p/5} r^{p/5}$$

Intermittency correction

- ε and χ varies significantly in space
- Refined similarity hypothesis: replace ε and χ by their local average over a ball of radius r about $\vec{x}$, $\mathcal{B}(\vec{x}, r)$

$$\varepsilon_r(\vec{x}) = \langle \varepsilon(\vec{x}') \rangle_{\vec{x}' \in \mathcal{B}}$$

$$\chi_r(\vec{x}) = \langle \chi(\vec{x}') \rangle_{\vec{x}' \in \mathcal{B}}$$

- The scaling predictions become

$$\text{OC (passive)}: \; S_T^{(p)}(r) \sim \langle \varepsilon_r^{-p/6} \rangle_{\vec{x}} \, \langle \chi_r^{p/2} \rangle_{\vec{x}} \, r^{p/3}$$

$$\text{BO (active)}: \; S_T^{(p)}(r) \sim (\alpha g)^{-p/5} \, \langle \chi_r^{2p/5} \rangle_{\vec{x}} \, r^{p/5}$$

- $\langle \varepsilon_r^{-p/6} \rangle_{\vec{x}}$ and $\langle \chi_r^{p/2} \rangle_{\vec{x}}$ are r-dependent, hence modifying the scaling exponents of $S_T^{(p)}(r)$

Some previous experimental work

Early time-domain measurements

- Wu et al.(PRL 1990) reported BO scaling at the convection cell center (using helium gas)
- Niemela et al. (Nature 2000) found BO scaling at large τ and OC scaling at small τ (using similar Ra, Pr and $\Gamma = 0.5$ as in Wu et al. 1990)
- Skrbet et al. (PRE 2002) found no scaling range at all (using the same setup as Niemela et al. 2000 but with $\Gamma = 1$)
- Zhou & Xia et al. (PRL 2001) observed BO scaling at the cell center and an apparent OC scaling in the mixing zone (using water)

Recent space-domain measurements

- Sun et al. (PRL 2006) demonstrated that behaviour at the cell center does not obey BO scaling and is closer to OC scaling
- Kunnen et al. (PRE 2008) reported a possible BO scaling at larger scales

Difficulties in comparing experimental results to theory:

- limited scaling range
- validity of the frozen flow hypothesis
- anisotropy and inhomogeneity, ...

Conditional structure functions

Recall in the space-domain, $\chi_r(\vec{x}) = \langle \chi(\vec{x}\,') \rangle_{\vec{x}\,' \in \mathcal{B}(\vec{x},r)}$

$$\text{OC (passive)}: \quad S_T^{(p)}(r) \sim \langle \chi_r^{p/2} \rangle_{\vec{x}}\, r^{p/3}$$

$$\text{BO (active)}: \quad S_T^{(p)}(r) \sim \langle \chi_r^{2p/5} \rangle_{\vec{x}}\, r^{p/5}$$

In the *time-domain*, given the time-series $T(t)$ and $\chi(t)$

$$\text{Define:} \quad \chi_\tau(t) = \langle \chi(t') \rangle_{t' \in \mathcal{B}(t,\tau)}$$

$$\text{OC (passive)}: \quad S_T^{(p)}(\tau) \sim \langle \chi_\tau^{p/2} \rangle_t\, \tau^{p/3}$$

$$\text{BO (active)}: \quad S_T^{(p)}(\tau) \sim \langle \chi_\tau^{2p/5} \rangle_t\, \tau^{p/5}$$

Define the conditional structure functions:

$$\hat{S}_T^{(p)}(\tau, X) = \langle\, |T(t+\tau) - T(t)|^p \,\big|\, \chi_\tau(t) = X \,\rangle_t$$

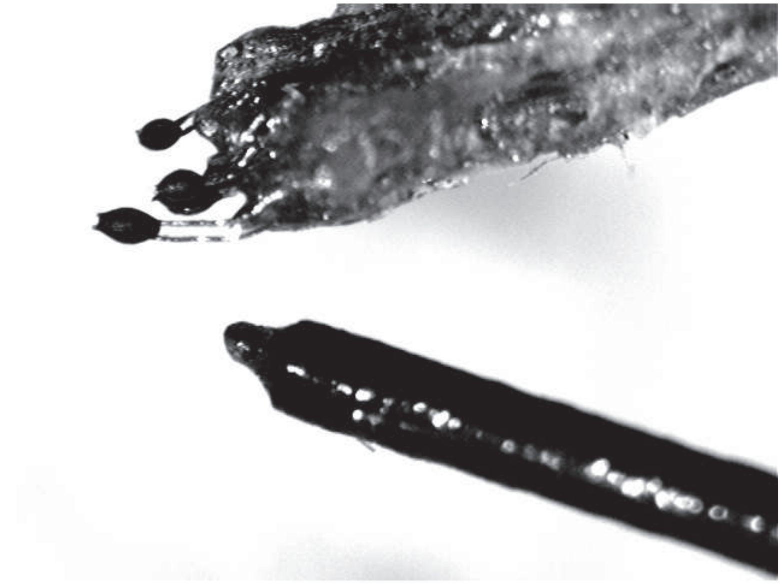
$$\text{OC (passive)}: \quad \hat{S}_T^{(p)}(\tau, X) \sim X^{p/2}\, \tau^{p/3}$$

$$\text{BO (active)}: \quad \hat{S}_T^{(p)}(\tau, X) \sim X^{2p/5}\, \tau^{p/5}$$

Measuring local thermal dissipation rate

$$\chi_\tau(\vec{x}, t) = \frac{1}{\tau}\int_t^{t+\tau} \kappa|\nabla T_f(\vec{x}, t')|^2 \, \mathrm{d}t'$$

where T_f = temperature fluctuation



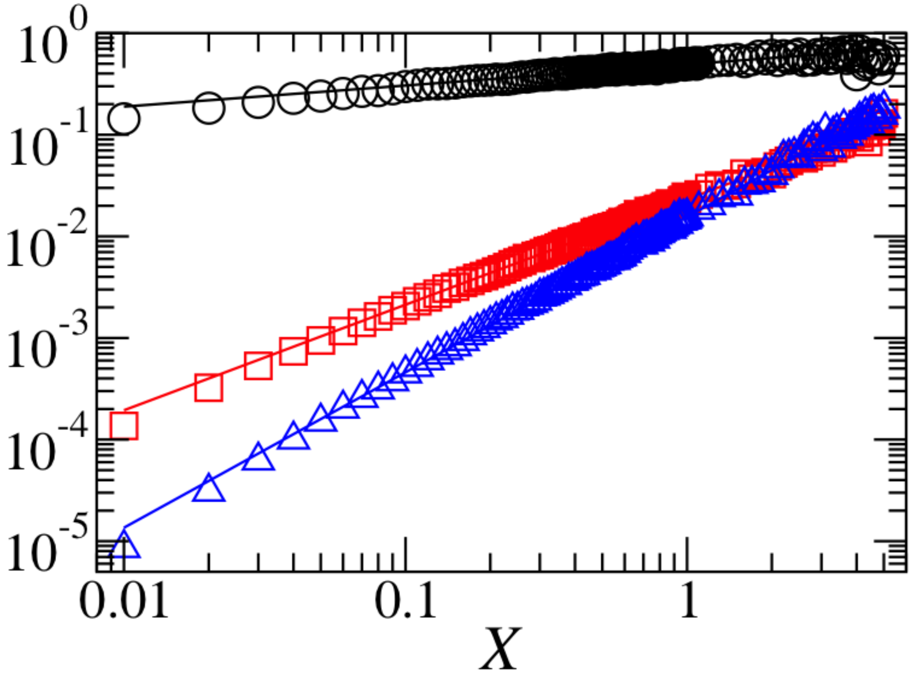
Home-made temperature gradient probe

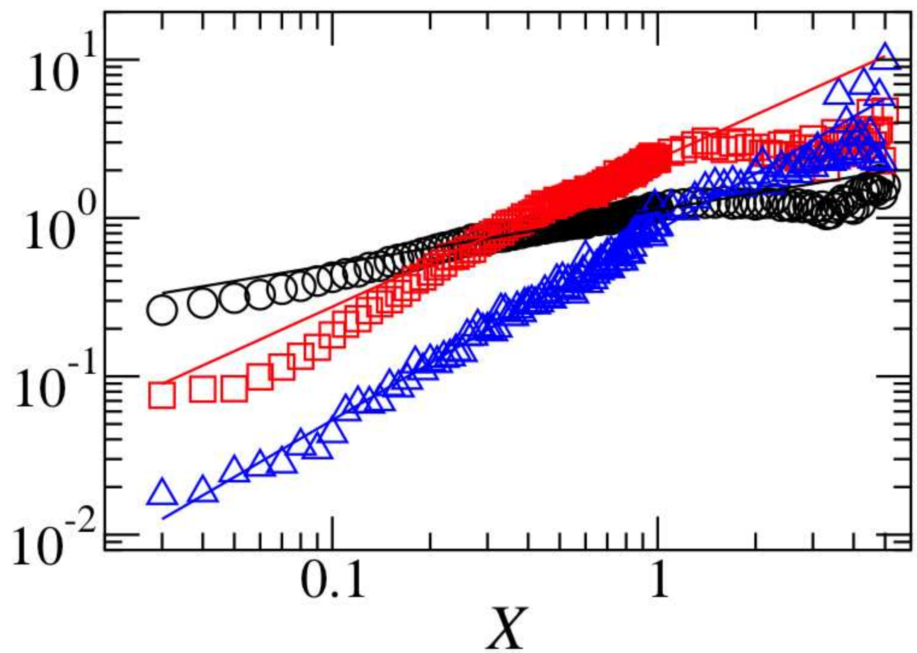
- four temperature sensors of diameter 0.11mm
- separation between sensors = 0.25mm
- temperature resolution ~ 5mK

He & Tong, Phys. Rev. E **79**, 026306 (2009)

Results: conditional structure functions

$$\hat{S}_T^{(p)}(\tau, X) = \langle\, |T(t+\tau) - T(t)|^p \,\big|\, \chi_\tau(t) = X \,\rangle_t \sim X^{\beta(p)}$$





cell center **bottom plate**

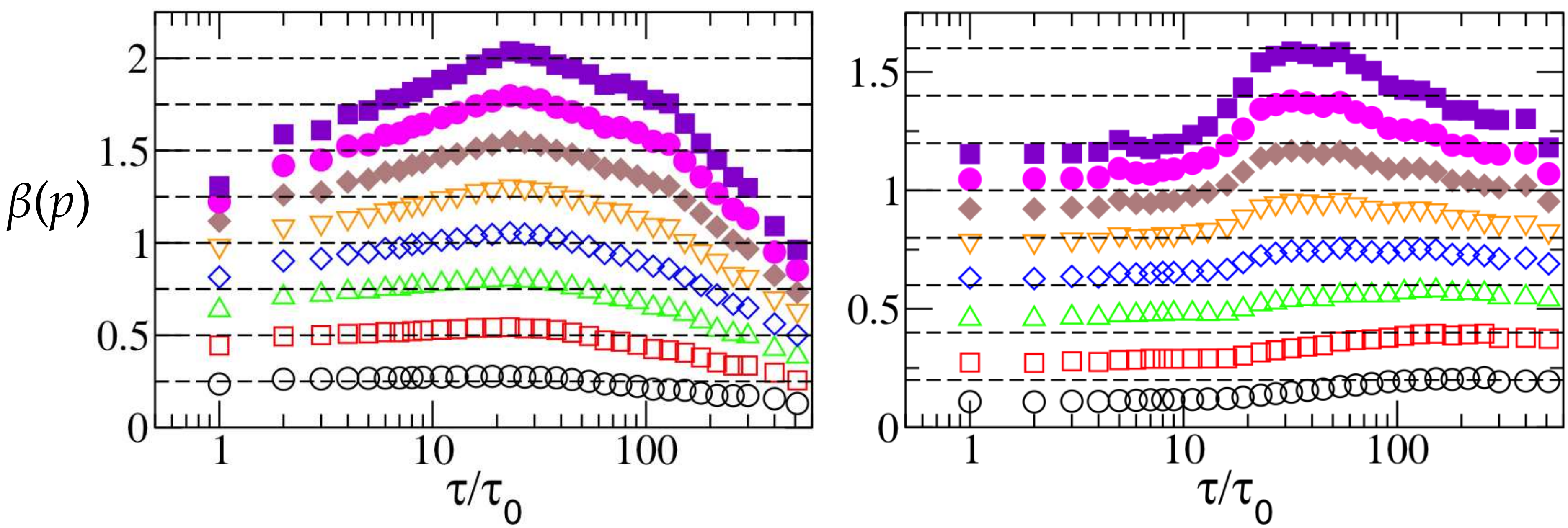
(top to bottom: decreasing τ and increasing p)

We have found significant scaling ranges in both cases.

$$\mathrm{Ra} = 8.3 \times 10^9 \;,\; \mathrm{Pr} = 5.5 \;,\; \Gamma = 1$$

Results: the scaling exponents

$$\hat{S}_T^{(p)}(\tau, X) = \langle\, |T(t+\tau) - T(t)|^p \,\big|\, \chi_\tau(t) = X \,\rangle_t \sim X^{\beta(p)}$$

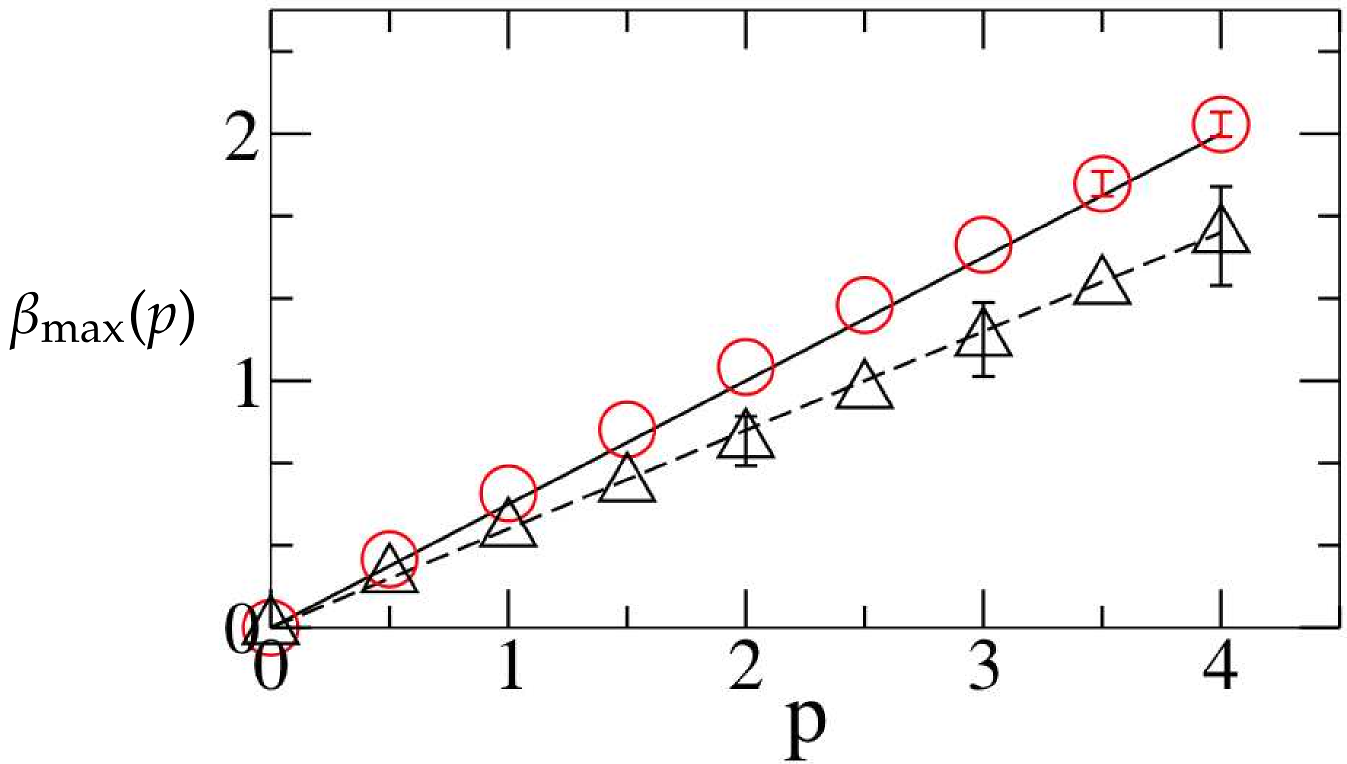


cell center **bottom plate**

p=0.5 to 4 from bottom to top, τ_0 is the data sampling interval

- $\beta(p)$ depends on τ
- for each p, $\beta(p)$ attains a maximum $\beta_{\max}(p)$

Results: passive vs. active



Experimental data: cell center (circles)

bottom plate (triangle)

Theory: $p/2$ passive OC scaling (solid)

$2p/5$ active BO scaling (dashed)

Summary

- introduce the conditional structure functions

$$\hat{S}_T^{(p)}(\tau, X) = \langle\, |T(t+\tau) - T(t)|^p \,\big|\, \chi_\tau(t) = X \,\rangle_t$$

  χ_τ = local time-averaged thermal dissipation rate

- investigate the scaling with X (rather than τ) and found significant scaling ranges,

$$\hat{S}_T^{(p)}(\tau, X) \sim X^{\beta(p)}$$

- results using experimental data at $\mathrm{Ra} = 8.3 \times 10^9$ suggest that temperature obeys the
  - the Obukhov-Corrsin scaling for a passive scalar at the convection cell center
  - the Bolgiano-Obukhov scaling for an active scalar near the bottom plate