How deep is Jupiter's metallic hydrogen region?

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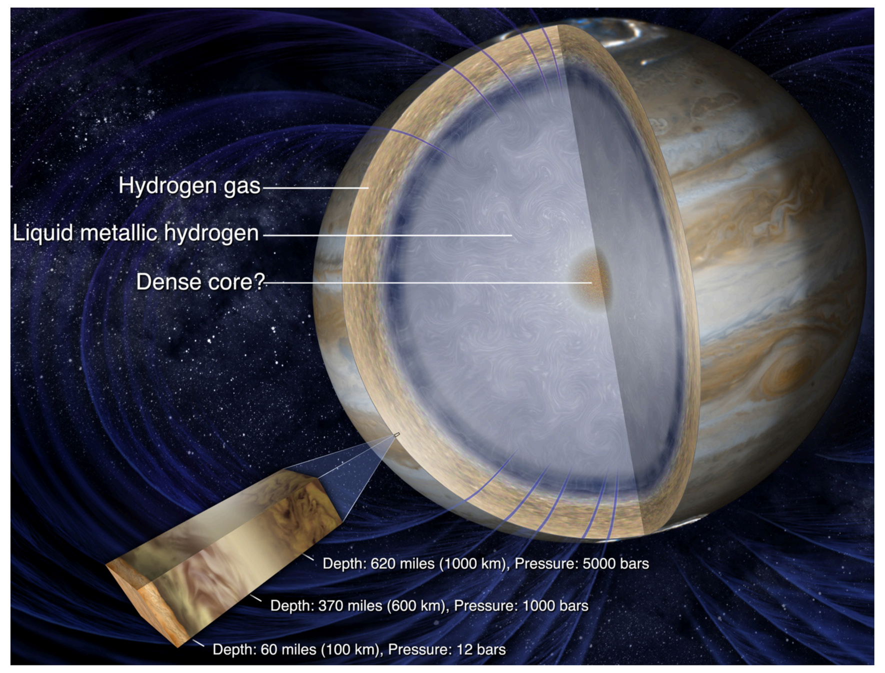
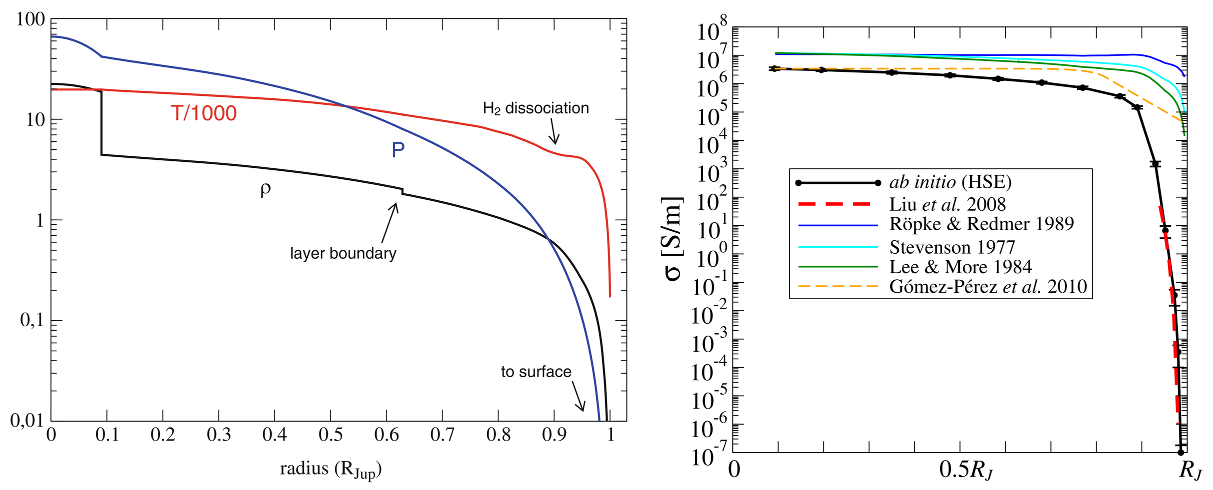
Interior structure of Jupiter



(JPL)

- mainly composed of hydrogen and helium
- surface: $T \sim 170\,\mathrm{K}$ and $P \sim 1\,\mathrm{bar}$
  centre: $T \sim 20000\,\mathrm{K}$ and $P \sim 70\,\mathrm{Mbar}$
- hydrogen molecules ionise to form metallic hydrogen at $T \sim 4300\,\mathrm{K}$ and $p \sim 0.5\,\mathrm{Mbar}$ ($\sim 0.9\,r_J$)
- continuous transition from the outer insulating molecular layer to the inner conducting region

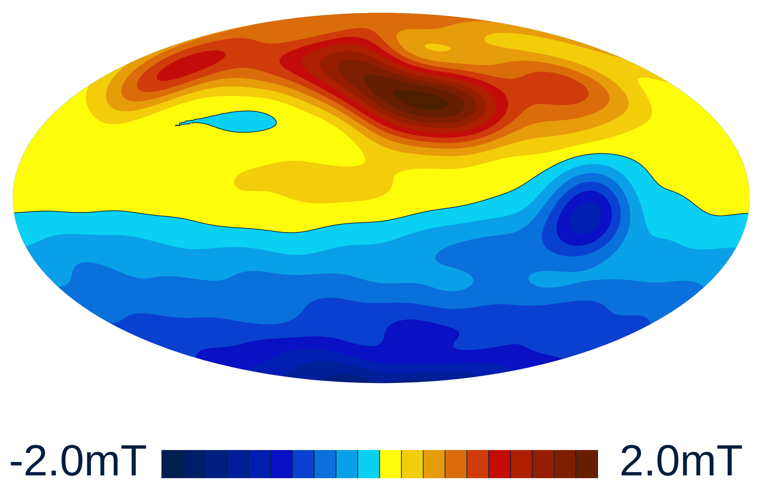
Theoretical calculation of EOS and conductivity $\sigma(r)$



(French, Becker, Lorenzen and Nettelmann 2012)

Jupiter's dynamo

- secular cooling of the planet drives convective motion of the liquid metallic hydrogen
- current is being generated $\Longrightarrow$ magnetic field
- recent observation of the radial component of the magnetic field from the Juno mission



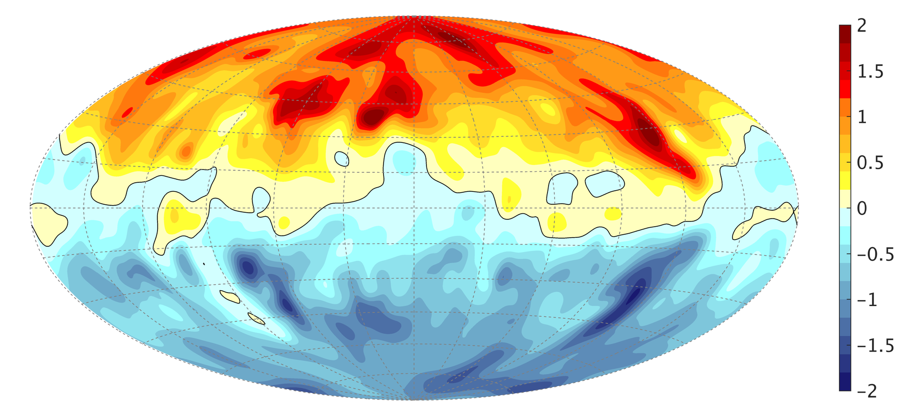
How do we characterise the depth of the dynamo region for a continuously varying $\sigma(r)$?

A numerical model of Jupiter

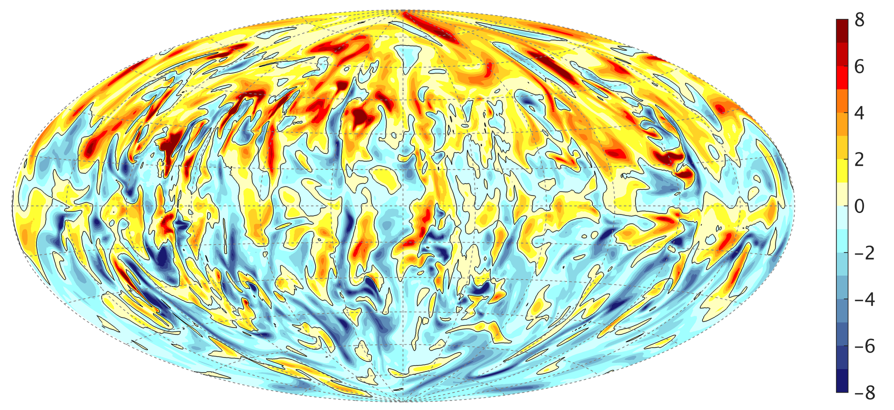
- Anelastic rotating MHD spherical shell with radius ratio $r_{\rm in}/r_{\rm out} = 0.0963$ (small core, $r_{\rm out} \approx 0.96\,r_J$)
- convection driven by secular cooling of the planet
- boundary conditions: no-slip at $r_{\rm in}$ and stress-free at $r_{\rm out}$, electrically insulating outside $r_{\rm in} < r < r_{\rm out}$
- dimensionless numbers: Ra, Pm, Ek, Pr
- Large Pm to produce strong field dynamo (Dormy 2016): Jupiter's field strongly influences its flow

$Ra = 2\times10^7, Pm = 10, Ek = 1.5\times10^{-5}, Pr = 0.1$

radial magnetic field

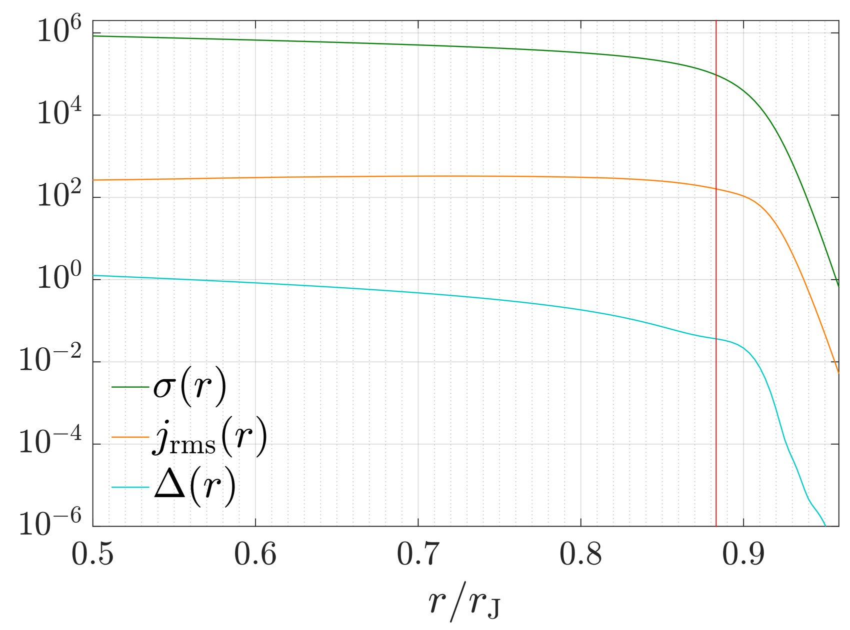


$r = r_{\rm out}$



$r = 0.75 r_{\rm out}$

Where does the current start flowing?



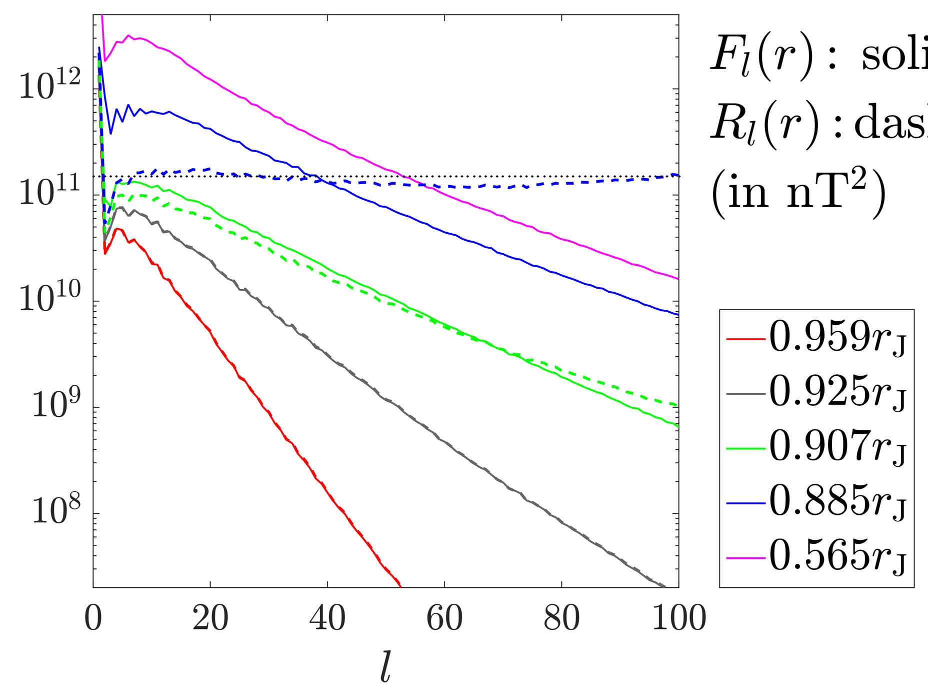
- average current over a spherical surface of radius r

$$\mu_0 \boldsymbol{j} = \nabla\times\boldsymbol{B}$$

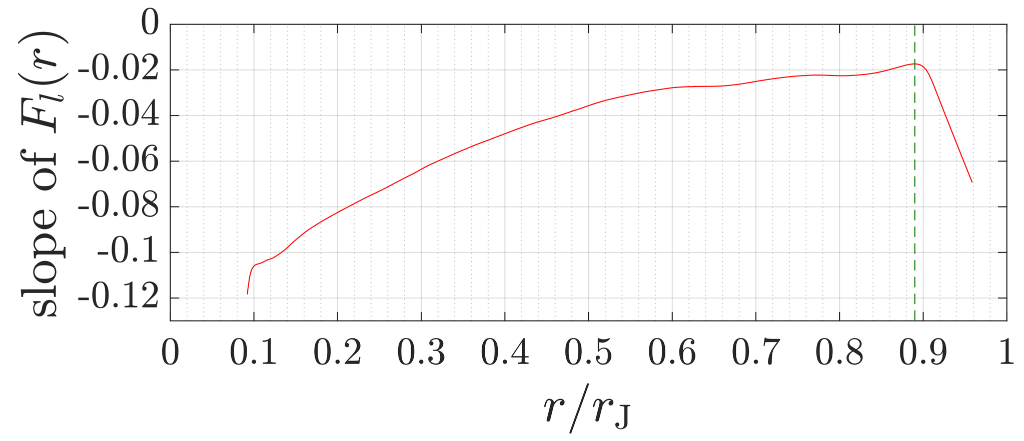
$$j^2_{\rm rms}(r,t) \equiv \frac{1}{4\pi}\int_0^{2\pi}\!\!\int_0^{\pi} |\boldsymbol{j}|^2 \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

- $j_{\rm rms}$ drops quickly but smoothly in transition region

Magnetic energy spectrum



$F_l(r)$: solid lines
$R_l(r)$: dashed lines
(in nT^2)



- average magnetic energy over a spherical surface

$$E_B(r) = \frac{1}{4\pi}\int_0^{2\pi}\!\!\int_0^{\pi} \frac{|\boldsymbol{B}|^2}{2\mu_0}\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

- write $\boldsymbol{B}$ as sum of spherical harmonics modes (l,m)

$$2\mu_0 E_B(r) = \sum_{l=1}^{\infty} F_l(r)$$

$F_l(r) \sim$ magnetic energy in the lth mode

- near the surface $r \approx r_{\rm out}$, $F_l(r)$ decays steeply with l
- in the interior (away from the core), $F_l(r)$ becomes shallow and maintains roughly the same shape
- indicates a qualitative change in the dynamics at

$$r_{\rm dyn} \approx 0.889\,r_J$$

Downward continuation in a $\boldsymbol{j}=0$ region

- in an interior region $r_1 < r < r_{\rm out}$ where $\boldsymbol{j} = \boldsymbol{0}$, the spectrum for $r < r_{\rm out}$ can be related to $F_l(r_{\rm out})$ at the surface
- denote such "source-free" spectrum by $R_l(r)$:

$$R_l(r) = \left(\frac{r_{\rm out}}{r}\right)^{2l+4} F_l(r_{\rm out}) \qquad (\star)$$

- if $F_l(r) \approx R_l(r)$, it implies $\boldsymbol{j} \approx \boldsymbol{0}$ at depth $r < r_{\rm out}$
- the deviation of $R_l(r)$ from $F_l(r)$ estimated by

$$\Delta(r) = \sqrt{\frac{\Sigma_l(\ln R_l - \ln F_l)^2}{\Sigma_l(\ln F_l)^2}}$$

is a measure of the significance of $\boldsymbol{j}$ at r

Characterisation using surface observation

- direct measurement of Jupiter's interior magnetic field is not possible, hence $F_l(r)$ is not available from observation
- can we estimate the depth of the metallic hydrogen region from $\boldsymbol{B}$ observed near the surface?
- **white source hypothesis:** *at the depth where $\boldsymbol{j}$ becomes significant, R_l becomes independent of l*
- a characteristic radius $r_{\rm flat}$ can be obtained from the spectrum $F_l(r_{\rm out})$ at the surface by assuming $R_l(r)$ at $r = r_{\rm flat}$ is independent of l, then by $(\star)$

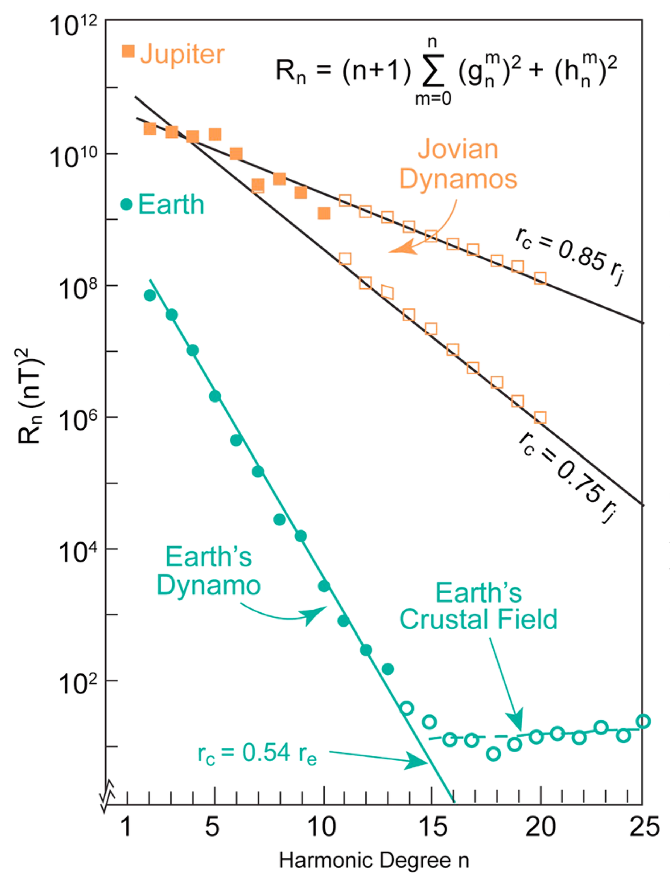
$$\ln F_l(r_{\rm out}) = 2\ln\left(\frac{r_{\rm flat}}{r_{\rm out}}\right) l + \ln R_l(r_{\rm flat}) + 4\ln\left(\frac{r}{r_{\rm out}}\right)$$

is linear in l, a least-squares linear fit gives the slope of $\ln F_l(r_{\rm out})$ from which we obtain

$$r_{\rm flat} = 0.885\,r_J$$

- we have also calculated $R_l(r_{\rm flat})$ using $(\star)$ and verified that it is indeed flat

Comparison with Juno data



The latest data from the Juno mission produces a magnetic spectrum (Connerney et al. 2018) that is steeper than that from our numerical model.

This spectrum is consistent with having a white noise source at a depth of $r = 0.85\,r_J$ as compared to our $r_{\rm flat} = 0.885\,r_J$.

The discrepancy between our numerical model and the Juno data suggests that

- theoretical calculation may have overestimated the conductivity in the Jupiter interior
- the metallic hydrogen layer, thus the transition region in the conductivity profile $\sigma(r)$, could be deeper than predicted
- our numerical model has more small-scale forcing than Jupiter does