Effective diffusivities in a two-layer, isopycnal, wind-driven basin model

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Question(s)

- To what degree can baroclinic eddy transport be described in terms of a local eddy closure?

- Is there a scale separation between eddy and mean?

- Is the small-scale, local eddy flux isotropic or anisotropic?

Extremely preliminary results on these questions....

Define eddy relative to low-pass mean, but approximate this via running average in time and space:

$$\bar{f}(x,y,t) = \frac{1}{\tau\ell^2}\int_{t-\tau/2}^{t+\tau/2} dt' \int_{x-\ell/2}^{x+\ell/2} dx' \int_{y-\ell/2}^{y+\ell/2} dy' f(x',y',t')$$

Large-scale equation

Assume advection of passive tracer in one isopycnal layer by quasi-oceanic velocity field:

$$c_t + \nabla\cdot(\mathbf{u}c) = \kappa\nabla^2 c + S$$

Apply our average:

$$\boxed{\bar{c}_t + \nabla\cdot(\mathbf{F} + \bar{\mathbf{u}}\bar{c}) = \kappa\nabla^2\bar{c} + \bar{S}}$$

where

$$\mathbf{F} = \overline{\mathbf{u}'c'}$$

and primed quantities are deviations from the mean,

$$c' = c - \bar{c}, \quad \mathbf{u}' = \mathbf{u} - \bar{\mathbf{u}}, \quad \text{such that} \quad \overline{c'} \simeq \overline{\mathbf{u}'} \simeq 0.$$

A perfect numerical model would yield $\bar{c}$ subsampled on discrete points separated by distances ℓ and times τ.

The effective diffusivity: A review

Assuming that the eddy flux is a linear function of the large-scale mean gradient, we write it as

$$\mathbf{F} = \overline{\mathbf{u}'c'} = -\mathsf{K}\nabla\bar{c}, \quad \text{where} \quad \mathsf{K} = \begin{pmatrix} K^{\mathsf{xx}} & K^{\mathsf{xy}} \\ K^{\mathsf{yx}} & K^{\mathsf{yy}} \end{pmatrix} = \mathsf{K}(\mathbf{x}, t)$$

Can decompose the effective diffusivity into symmetric and antisymmetric parts as $\mathsf{K} = \mathsf{K}^s + \mathsf{K}^a$, where

$$\mathsf{K}^s = \begin{pmatrix} K^{\mathsf{xx}} & \alpha \\ \alpha & K^{\mathsf{yy}} \end{pmatrix} - \kappa\mathsf{I} \quad \text{and} \quad \mathsf{K}^a = \begin{pmatrix} 0 & -\gamma \\ \gamma & 0 \end{pmatrix}$$

with $\alpha = (K^{\mathsf{xy}} + K^{\mathsf{yx}})/2$ and $\gamma = (K^{\mathsf{yx}} - K^{\mathsf{xy}})/2$. Eddy transport velocity is

$$\mathbf{u}^\star = \nabla \times (-\hat{\boldsymbol{z}}\gamma) = \hat{\boldsymbol{z}} \times \nabla\gamma$$

and so averaged flow is

$$\boxed{\bar{c}_t + \nabla\cdot(\mathbf{u}^\dagger\bar{c}) = \nabla\cdot(\mathsf{K}^s\nabla\bar{c}) + \bar{S}}$$

where

$$\mathbf{u}^\dagger = \mathbf{u}^\star + \bar{\mathbf{u}}$$

Objectives

K is a property of the flow

1. Measure **K** directly in an high-resolution ocean model

2. Develop theory to predict **K** from local ocean parameters

Measuring K

On cell: $\overline{\mathbf{u}'c'} = -\mathsf{K}\boldsymbol{\Gamma}$ (where **K** $\approx$ constant on cell), or

$$\overline{u'c'} = \mathsf{K}^{\mathsf{xx}}\Gamma_x + \mathsf{K}^{\mathsf{xy}}\Gamma_y$$

$$\overline{v'c'} = \mathsf{K}^{\mathsf{yx}}\Gamma_x + \mathsf{K}^{\mathsf{yy}}\Gamma_y$$

$\Rightarrow$ four unknowns and two equations.

Since **K** is property of flow, use two independent tracers a and b (forced by different large-scale gradients), with

$$\boldsymbol{\Gamma} \approx \nabla\bar{a}, \quad \boldsymbol{\Lambda} \approx \nabla\bar{b}$$

So that:

$$\overline{u'a'} = \mathsf{K}^{\mathsf{xx}}\Gamma_x + \mathsf{K}^{\mathsf{xy}}\Gamma_y$$

$$\overline{v'a'} = \mathsf{K}^{\mathsf{yx}}\Gamma_x + \mathsf{K}^{\mathsf{yy}}\Gamma_y$$

$$\overline{u'b'} = \mathsf{K}^{\mathsf{xx}}\Lambda_x + \mathsf{K}^{\mathsf{xy}}\Lambda_y$$

$$\overline{v'b'} = \mathsf{K}^{\mathsf{yx}}\Lambda_x + \mathsf{K}^{\mathsf{yy}}\Lambda_y$$

$\Rightarrow$ four unknowns and four equations.

Model/Simulation

Two-layer, adiabatic isopycnal PE model (HIM) in $22^\circ \times 20^\circ$ basin

- Sinusoidal wind-forcing (double-gyre) and small linear bottom drag

- Biharmonic Smagorinsky viscosity plus low-level biharmonic constant background.

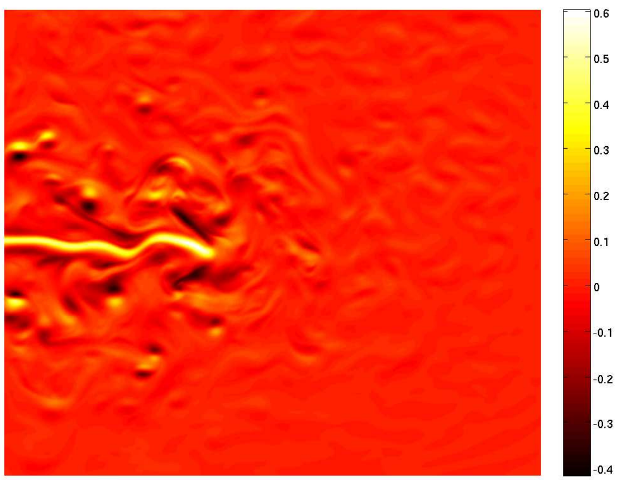
- 2 passive scalars in top layer, one forced with meridional gradient the other with zonal gradient (** need improved method for gradients)

- Current resolution: 1/20 degree, but will go higher.

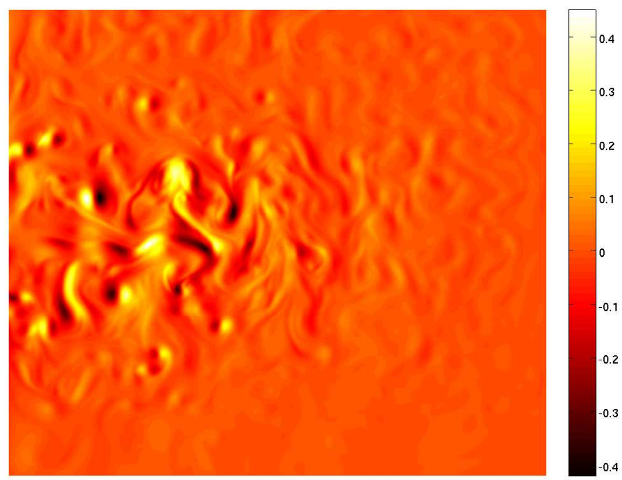
- Running mean in space and time calculated using 100km $\times$ 100km $\times$ 10 day cells, and eddy fluxes of each tracer calculated on each cell (** need bigger cells in ℓ and t)

Snapshot of the flow

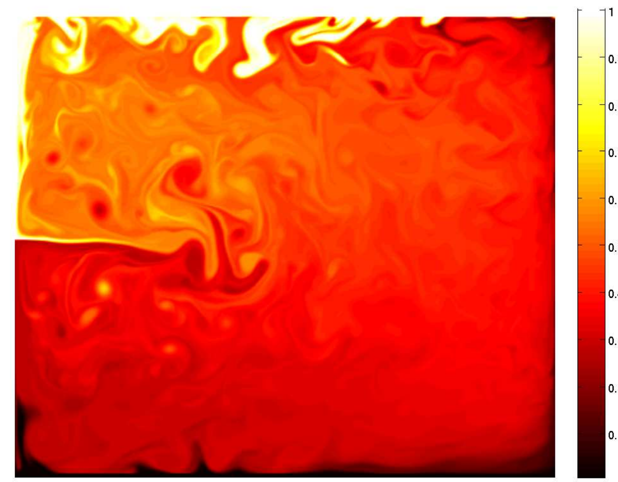
$\mathbf{u}'$



$\mathbf{v}'$

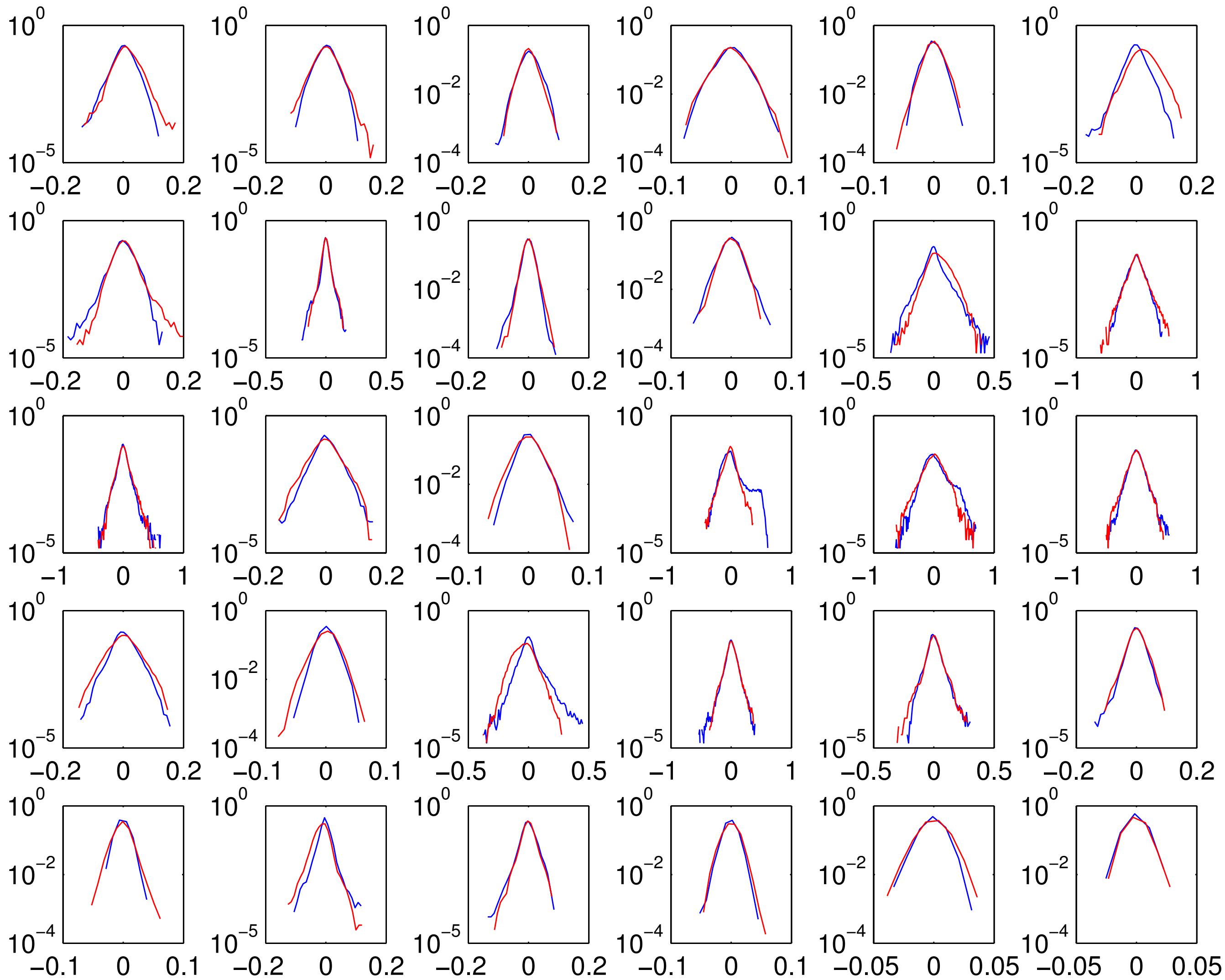


Tracer b



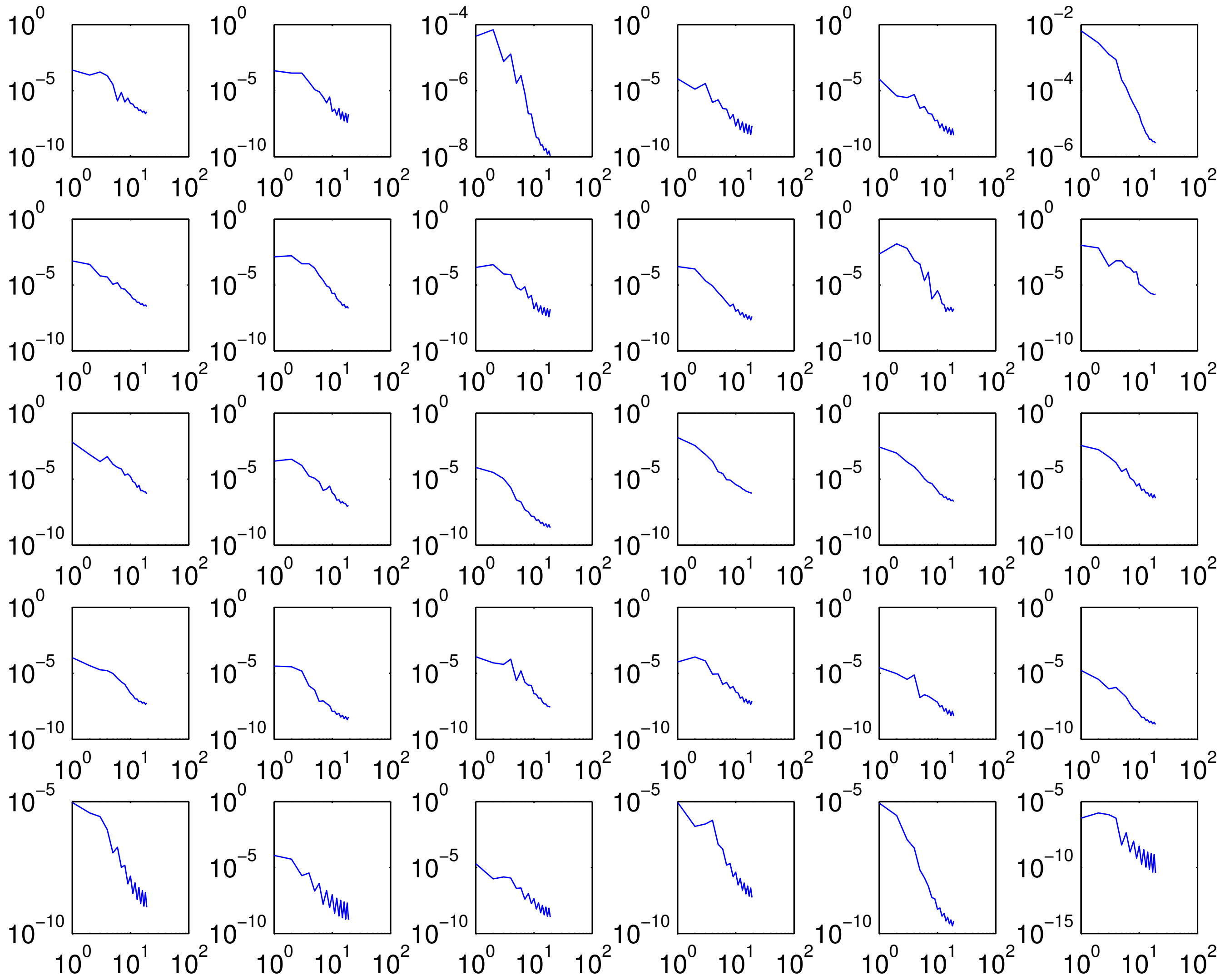
PDF of u' and v' in cells

(Regions far from the jet are nearly isotropic)



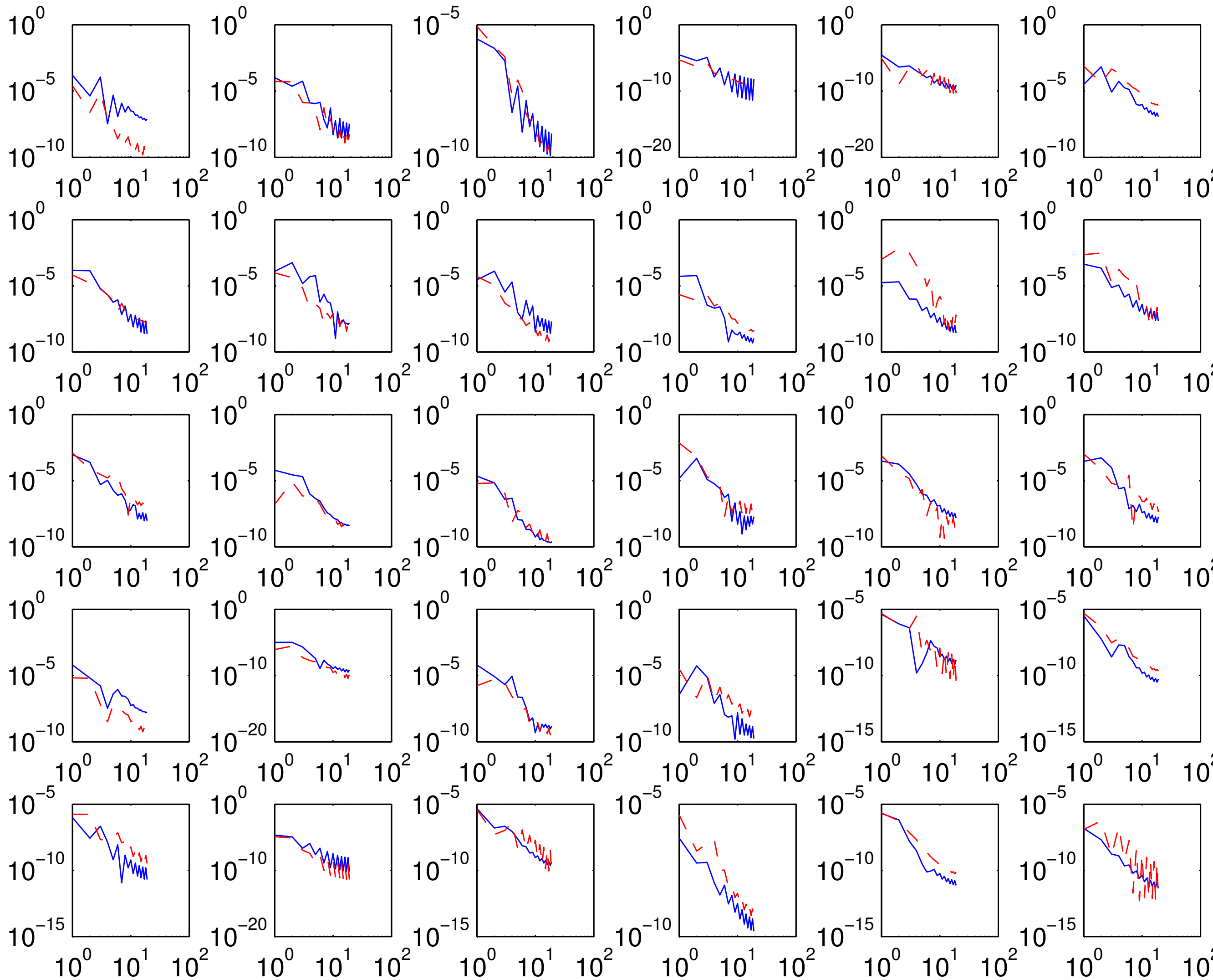
Energy spectrum in cells

(gives info about eddy energy and scale in each cell: mixing length and mixing velocity)



Zonal and meridional spectra in each cell

(second measure of anisotropy on each cell)



Predicting K: Small-scale 'cell' problem

Consider a region (cell) with linear dimension ℓ centered at $\mathbf{x}_i$ and a period of time τ centered at t_i such that

1. ℓ is much smaller than the length scale over which $\nabla \bar{c}$ varies, i.e.

$$\ell \ll \left. \frac{|\nabla \bar{c}|}{|\nabla^2 \bar{c}|} \right|_{\mathbf{x}_i}$$

2. τ is much smaller than the time scale over which $\bar{c}$ varies

Then in such a cell we have

$$c(\mathbf{x}, t) \approx \mathbf{\Gamma} \cdot \mathbf{x} + c'(\mathbf{x}, t), \quad \mathbf{\Gamma} \approx \nabla \bar{c} \quad \text{(linear approximation)}$$

and

$$\frac{\partial c'}{\partial t} + \nabla \cdot (\mathbf{u} c') = \kappa \nabla^2 c' - \mathbf{u} \cdot \mathbf{\Gamma} - \bar{c} \nabla \cdot \mathbf{u}$$

Use analytical solutions to simplified cell problem, based on general set of flow types from model, to predict **K**...

Plans

- Increase horizontal resolution

- Change cell size

- Calculate diffusivity and compare to local theories

- Develop suite of cell problem solutions to test

- Increase vertical resolution and diagnose eddy PV flux directly...