Effects of a guided-field on particle diffusion in magnetohydrodynamic turbulence

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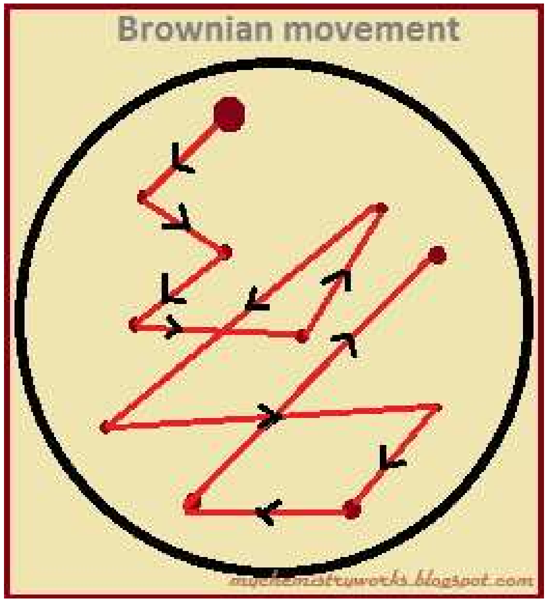
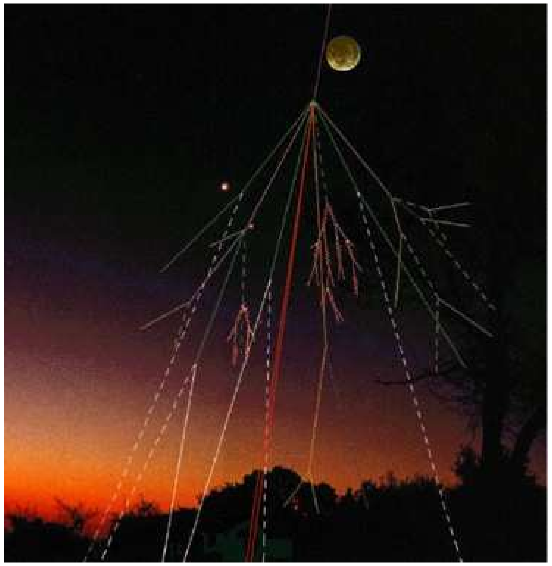
Particle transport in fluids





- Brownian motion observed under the microscope
- dispersion of pollutants in the atmosphere
- cosmic ray propagation through the interstellar medium

We consider passive tracer particles only —- the Lagrangian viewpoint provides an alternative view of the flow structure

Single-particle turbulent diffusion

- mean squared displacement:

$$\langle|\Delta\vec{X}(t)|^2\rangle\,, \quad \Delta\vec{X}(t)=\vec{X}(t)-\vec{X}(0)$$

- Taylor's formula (1920) for large t:

$$\vec{X}(t)=\vec{X}(0)+\int_0^t \mathrm{d}\tau\,\vec{V}(\tau)$$

$$\langle|\Delta\vec{X}(t)|^2]\rangle = 2\,t\int_0^\infty \mathrm{d}\tau\,\langle\vec{V}(\tau)\cdot\vec{V}(0)\rangle = 2tD$$

assume system is statistically homogeneous and stationary and the integral exists

- Lagrangian velocity correlation:

$$C_L(\tau)=\langle\vec{V}(\tau)\cdot\vec{V}(0)\rangle$$

- diffusion coefficient:

$$D=\int_0^\infty \mathrm{d}\tau\,\langle\vec{V}(\tau)\cdot\vec{V}(0)\rangle$$

Field-guided MHD turbulence + tracers

- Electrically conducting fluid in a 3D periodic domain:

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{1}{\rho_0}\nabla p + (\nabla \times \vec{B}) \times \vec{B} + \nu \nabla^2 \vec{u} + \vec{f}$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{u} \times \vec{B}) + \eta \nabla^2 \vec{B}$$

$$\nabla \cdot \vec{u} = \nabla \cdot \vec{B} = 0$$

$\vec{f}$: isotropic random forcing at the largest scales, Δt-correlated

- Field-guided MHD turbulence:

$$\vec{B}(\vec{x}, t) = B_0 \hat{z} + \vec{b}(\vec{x}, t)$$

- Evolution of passive tracer particles:

$$\frac{\mathrm{d}\vec{X}(t)}{\mathrm{d}t} = \vec{V}(t) = \vec{u}(\vec{X}(t), t)$$

$\vec{X}(0)$: uniformly distributed over the domain

Previous work: the 2D case

Turbulent transport (η_T) suppressed when $B_0 > B_0^* \sim Rm^{-1}$ $(Rm = UL/\eta)$

(Vainshtein & Rosner 1991, Cattaneo & Vainshtein 1991, Cattaneo 1994, Gruzinov & Diamond 1994, Kondić, Hughes & Tobias 2016)

ON THE EFFECTS OF A WEAK MAGNETIC FIELD ON TURBULENT TRANSPORT

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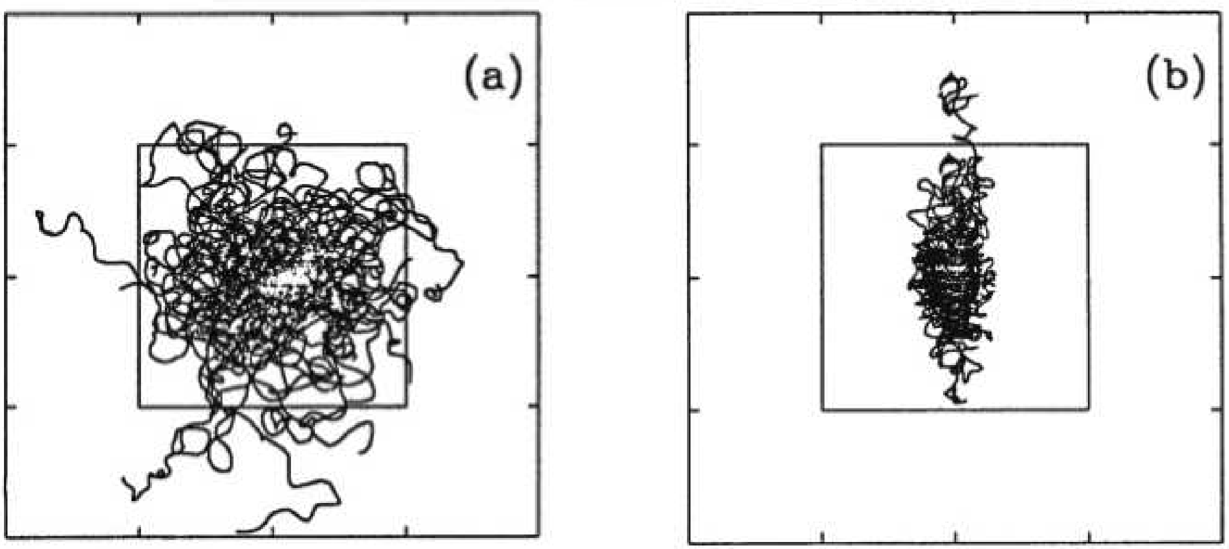
ABSTRACT

We discuss the effects of a weak large-scale magnetic field on turbulent transport. We show by means of a series of two-dimensional numerical experiments that turbulent diffusion can be effectively suppressed by a (large-scale) magnetic field whose energy is small compared to equipartition. The suppression mechanism is associated with a subtle modification of the Lagrangian energy spectrum, and it does not require any substantial reduction of the turbulent amplitude. We exploit the relation between diffusion and random walking to emphasize that the effect of a large-scale magnetic field is to induce a long-term memory in the field of turbulence. The implications for the general case of three-dimensional transport are briefly discussed.

Subject heading

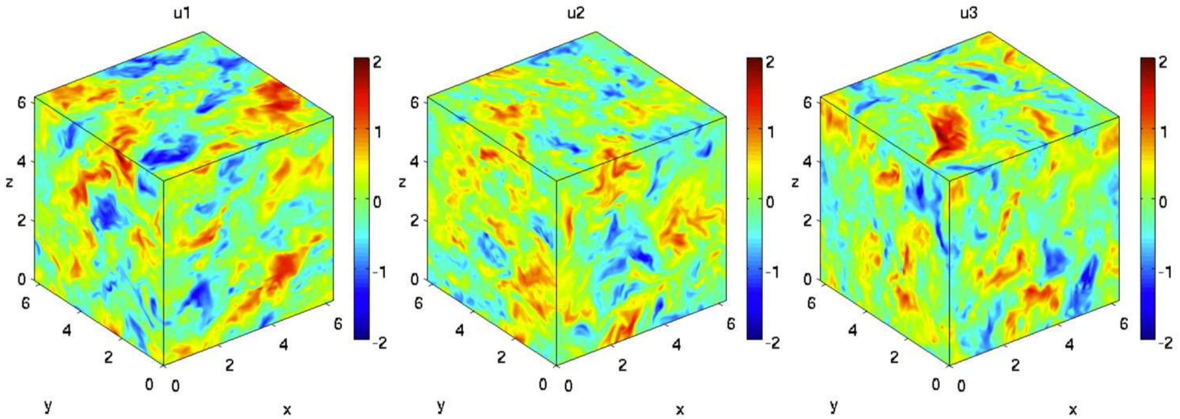


Q : Does suppression of turbulent diffusion occur in 3D ?

Eulerian fields

hydrodynamic

$\vec{u}$



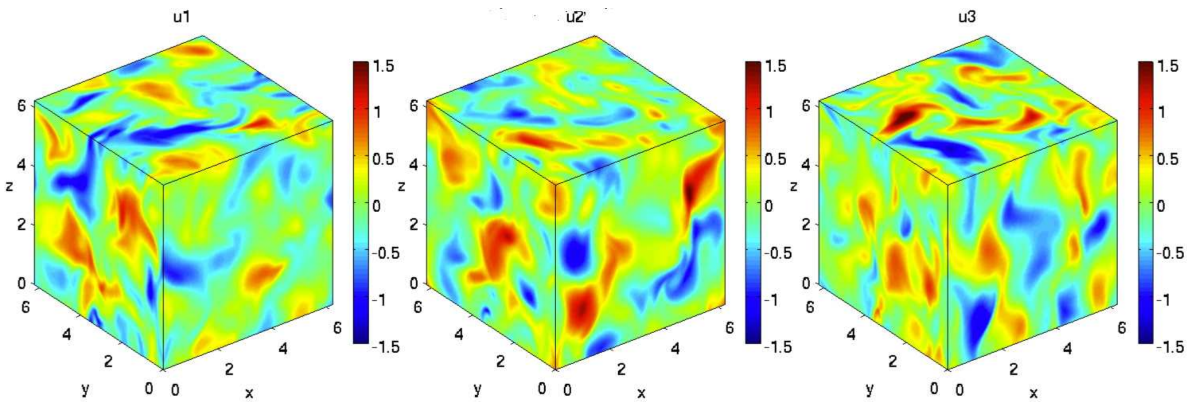
$\nu = \eta \sim 10^{-3}$

$Re = Rm \sim 10^{3}$

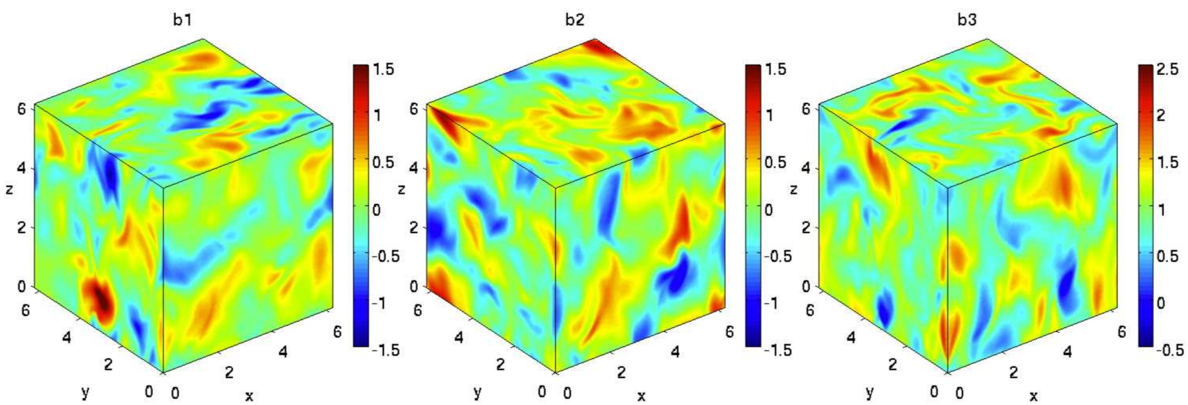
$256 \times 256 \times 256$

$B_0 = 1$

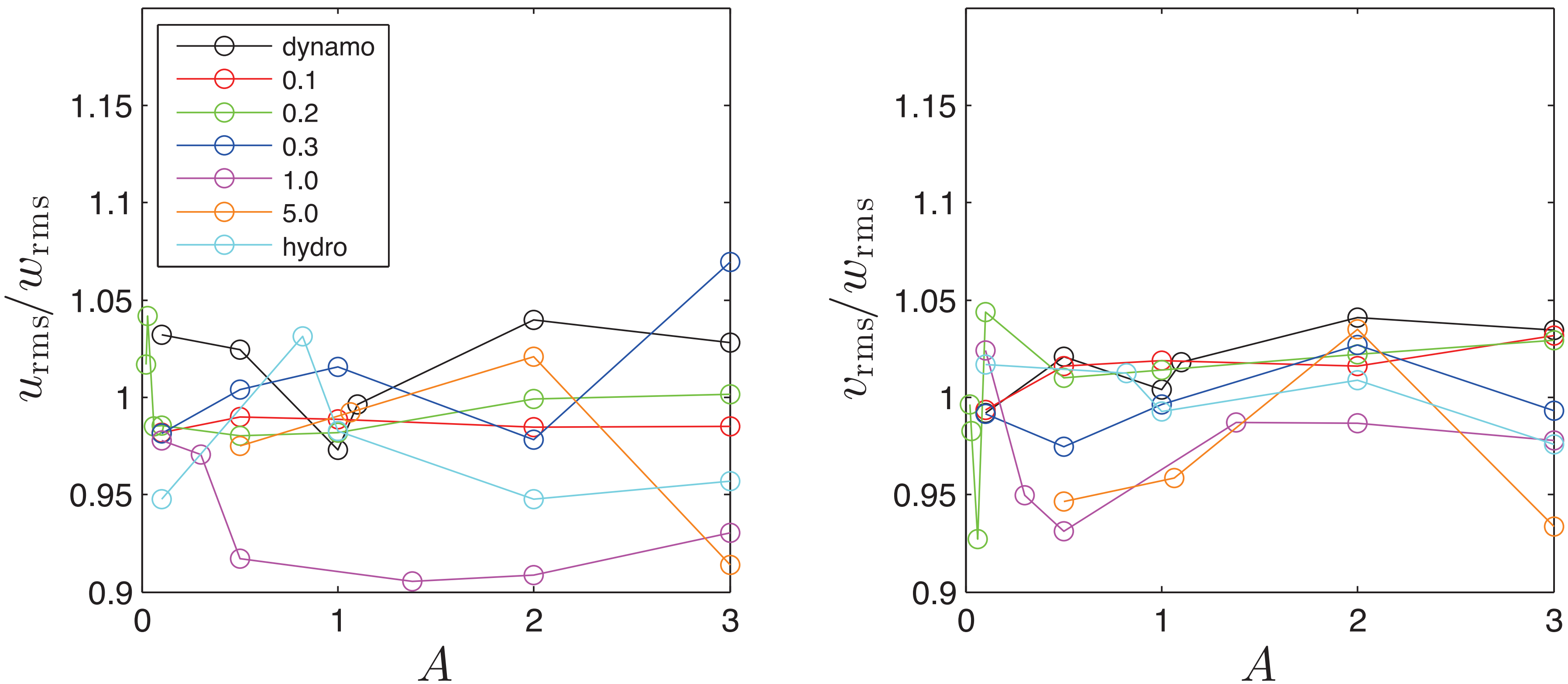
$\vec{u}$



$\vec{b}$



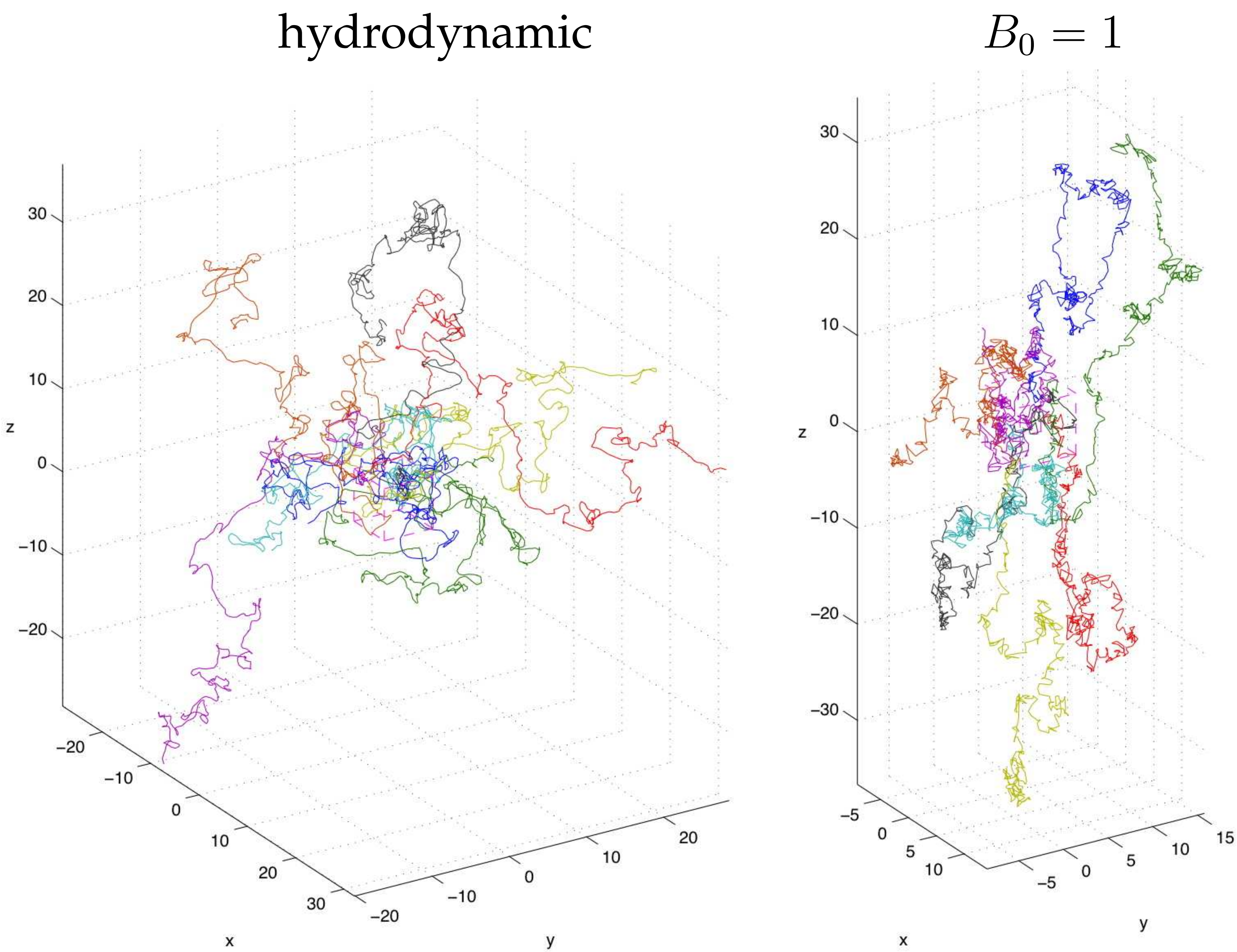
Ratios of Eulerian r.m.s. velocities



A = forcing amplitude

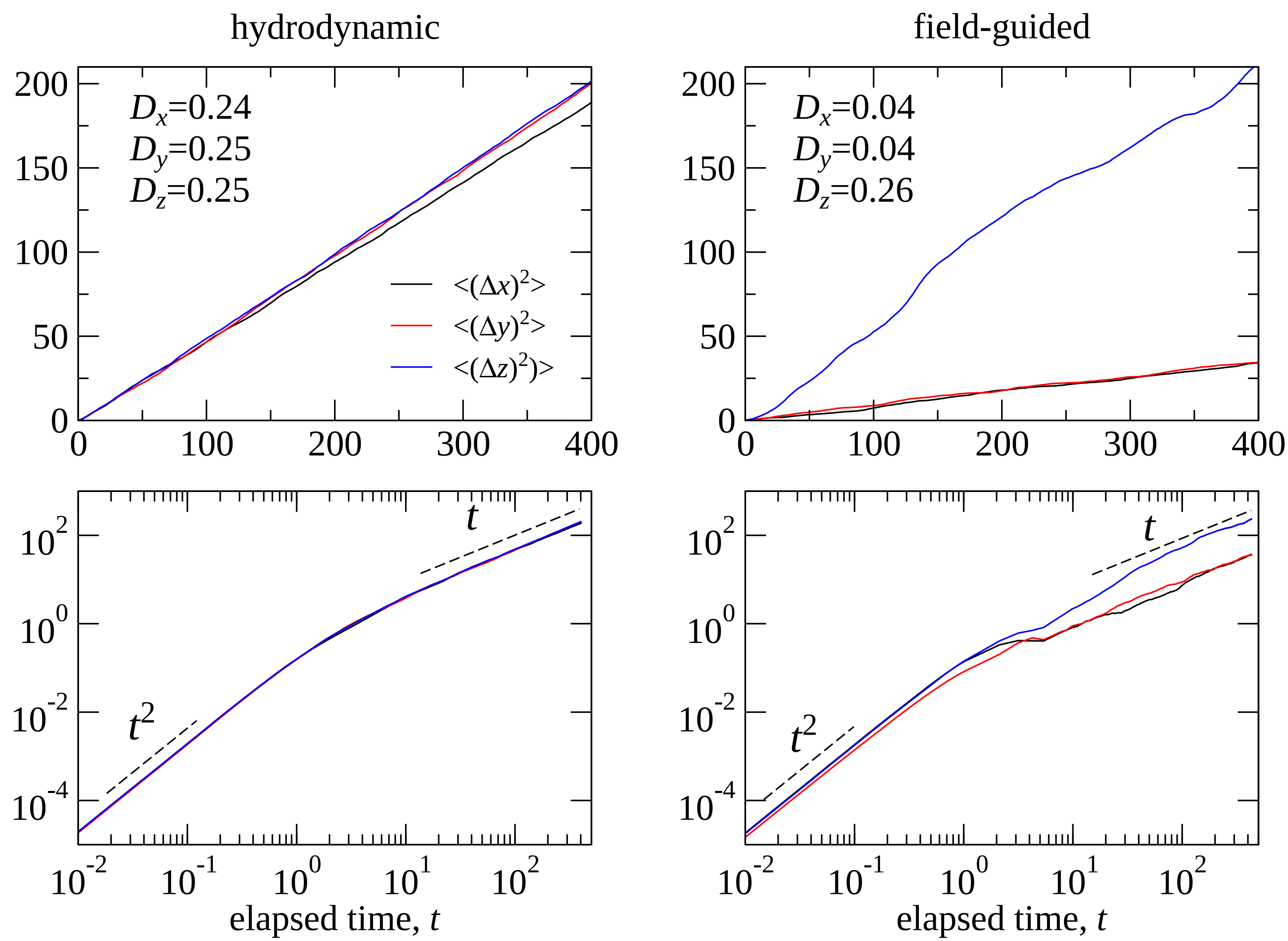
$$\frac{u_{\rm rms}}{w_{\rm rms}} \approx \frac{v_{\rm rms}}{w_{\rm rms}} \approx 1$$

Particle trajectories



- transport becomes anisotropic when $B_0 \neq 0$

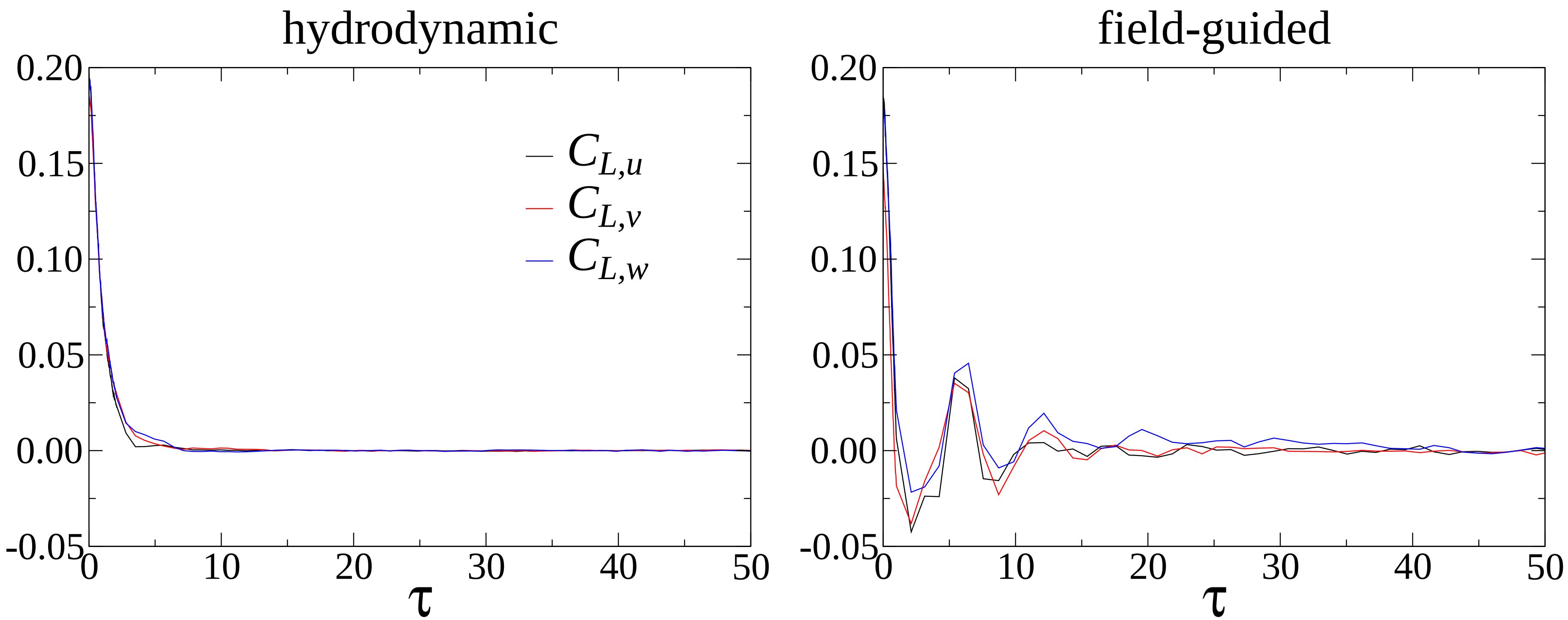
Scaling of mean squared displacement



- ballistic limit: $\sim t^2$ at small time
- diffusive scaling: $\sim t$ at large time, $\langle(\Delta x)^2\rangle \sim 2D_x t$, etc

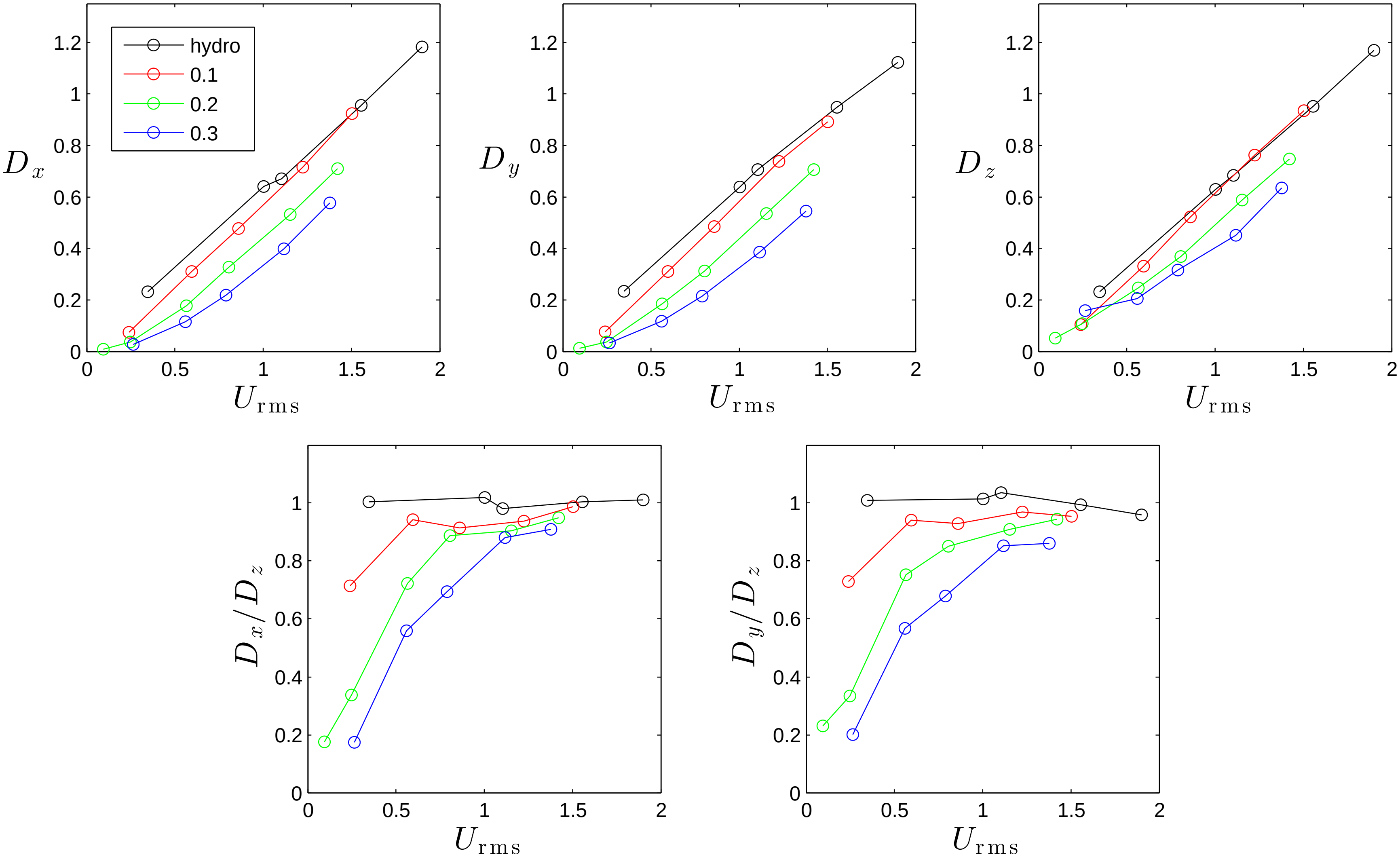
Lagrangian velocity correlation function

$$C_L(\tau) = \langle \vec{V}(\tau) \cdot \vec{V}(0) \rangle$$



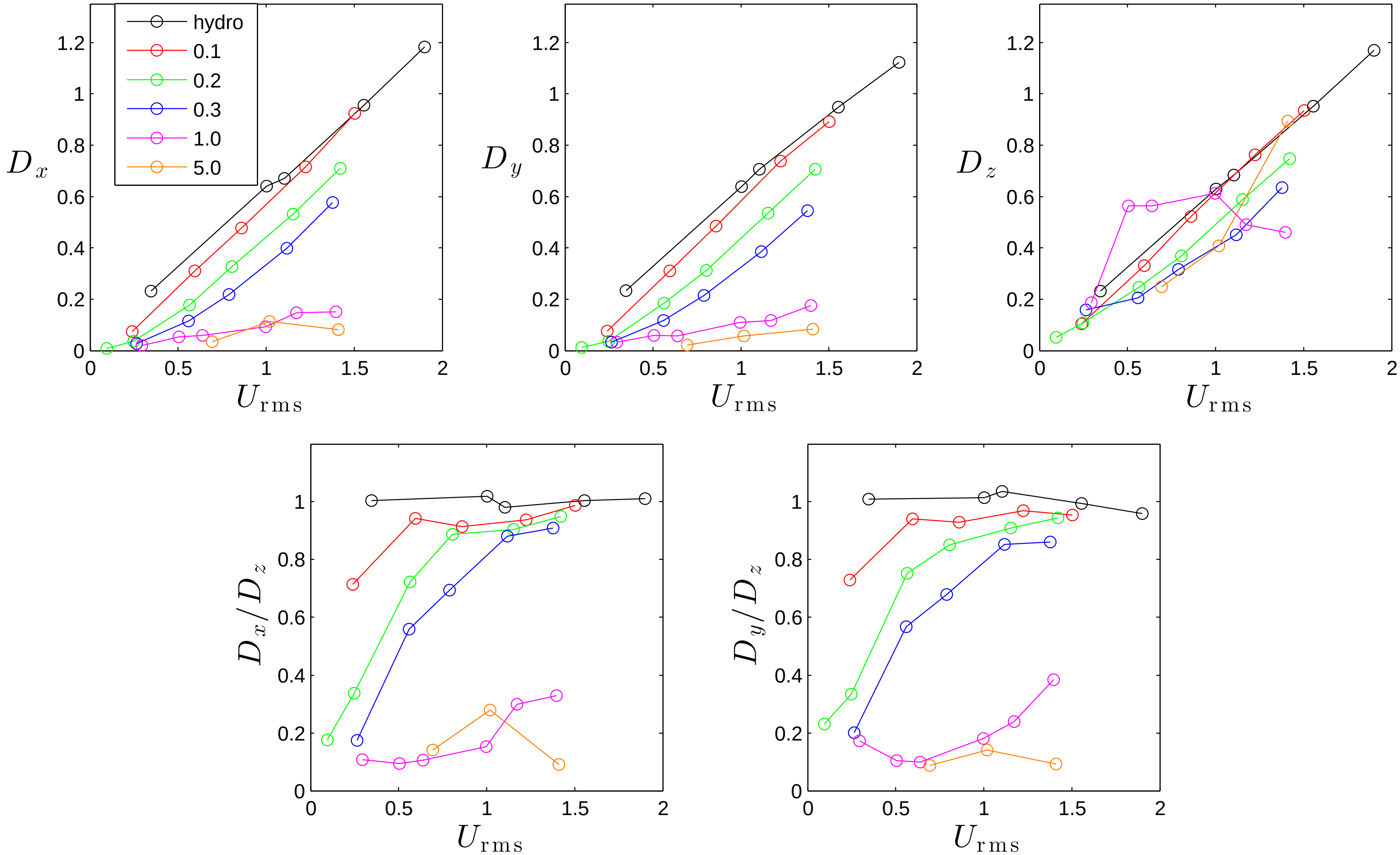
- hydrodynamic: $\sim \exp(-\tau)$, short correlation time
- field-guided: oscillatory, long correlation time
- how things depend on the guided-field strength B_0?

Diffusivity at different (weak) $B_0 \lesssim U_{\rm rms}$



- diffusion is reduced by B_0, including the z-direction
- anisotropic suppression: $D_x, D_y \lesssim D_z$
- strong $U_{\rm rms}(\gtrsim B_0)$ reduces the anisotropy in D's

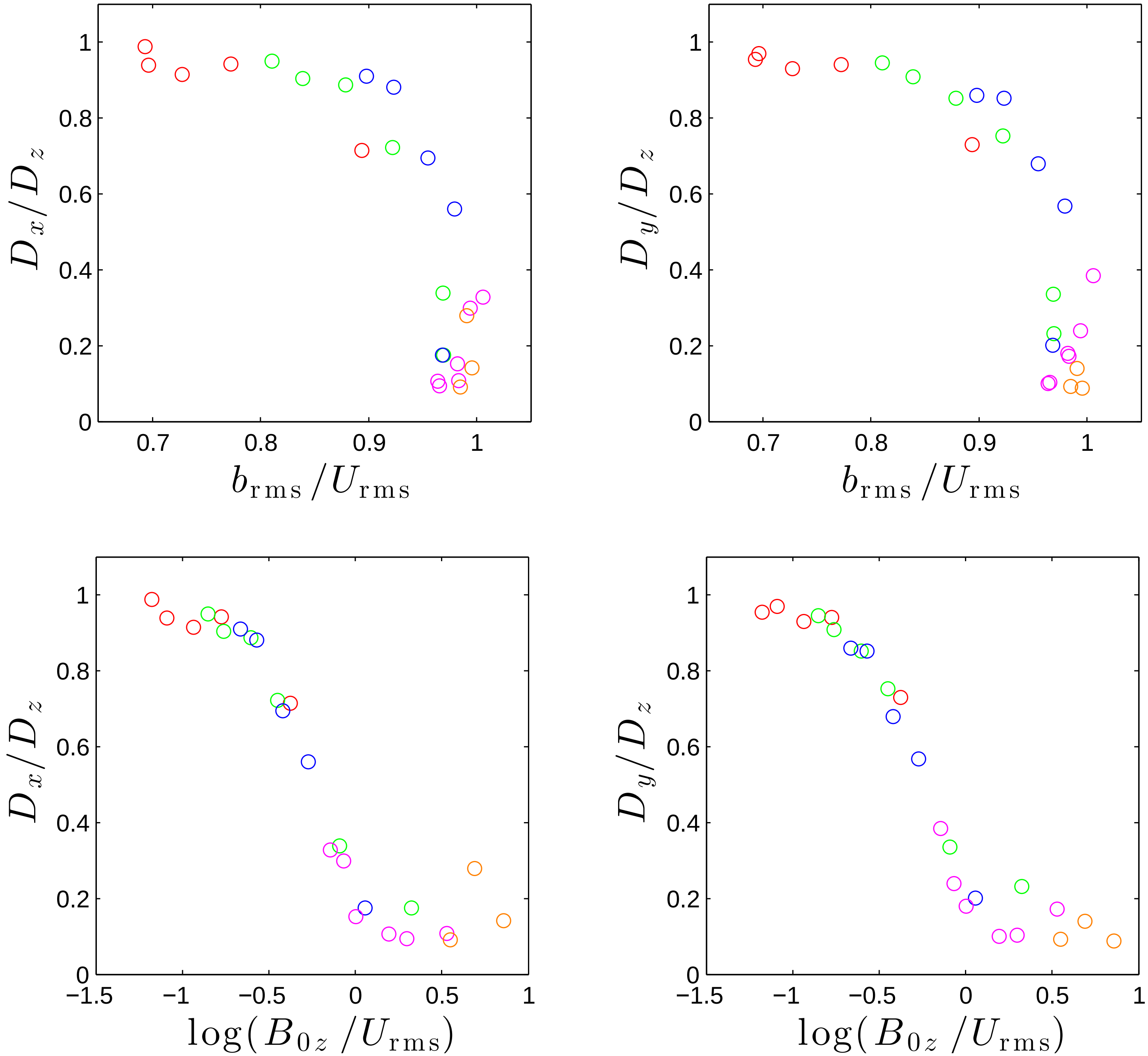
Diffusivity at different B_0



At strong guided-field strength, $B_0 \gtrsim U_{\rm rms}$

- D_x, D_y are strong suppressed, anomalous behavior of D_z
- D_x/D_z , $D_y/D_z \ll 1$ for the values of $U_{\rm rms}$ studied

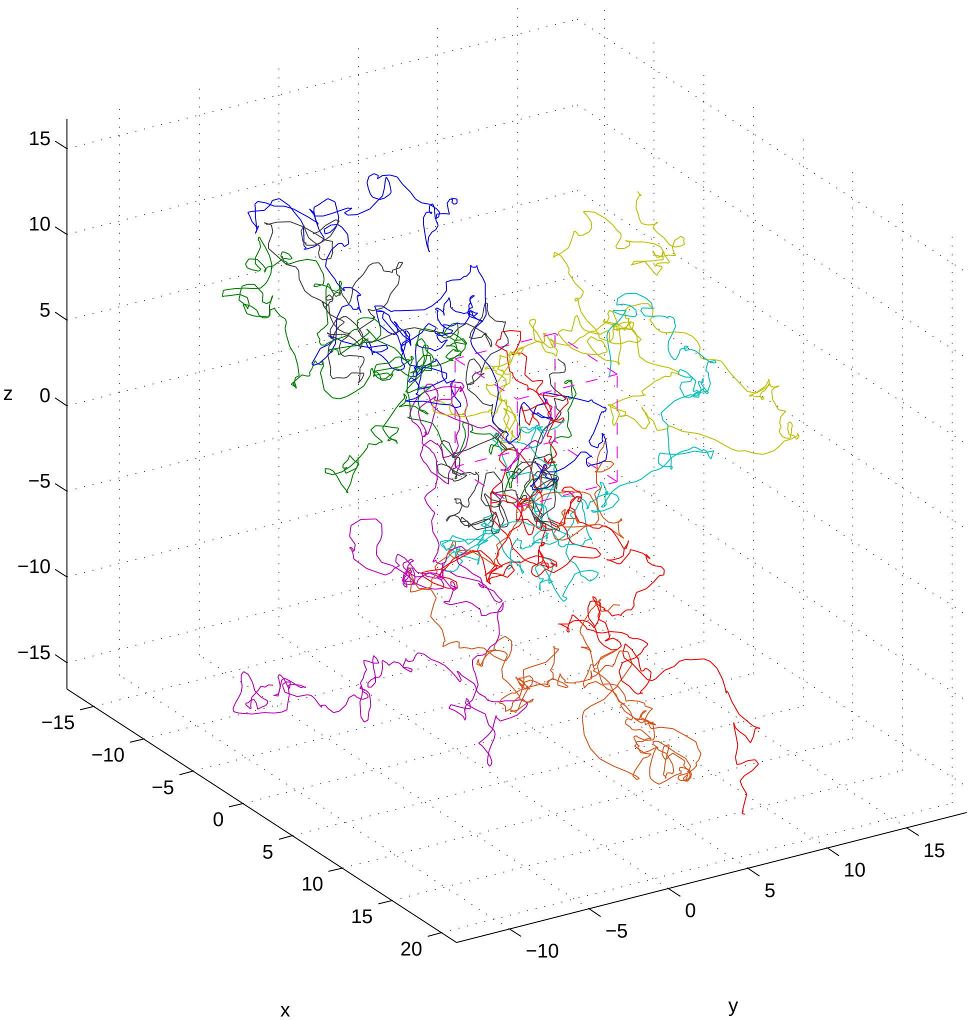
Anisotropic turbulent diffusion



Particle trajectories

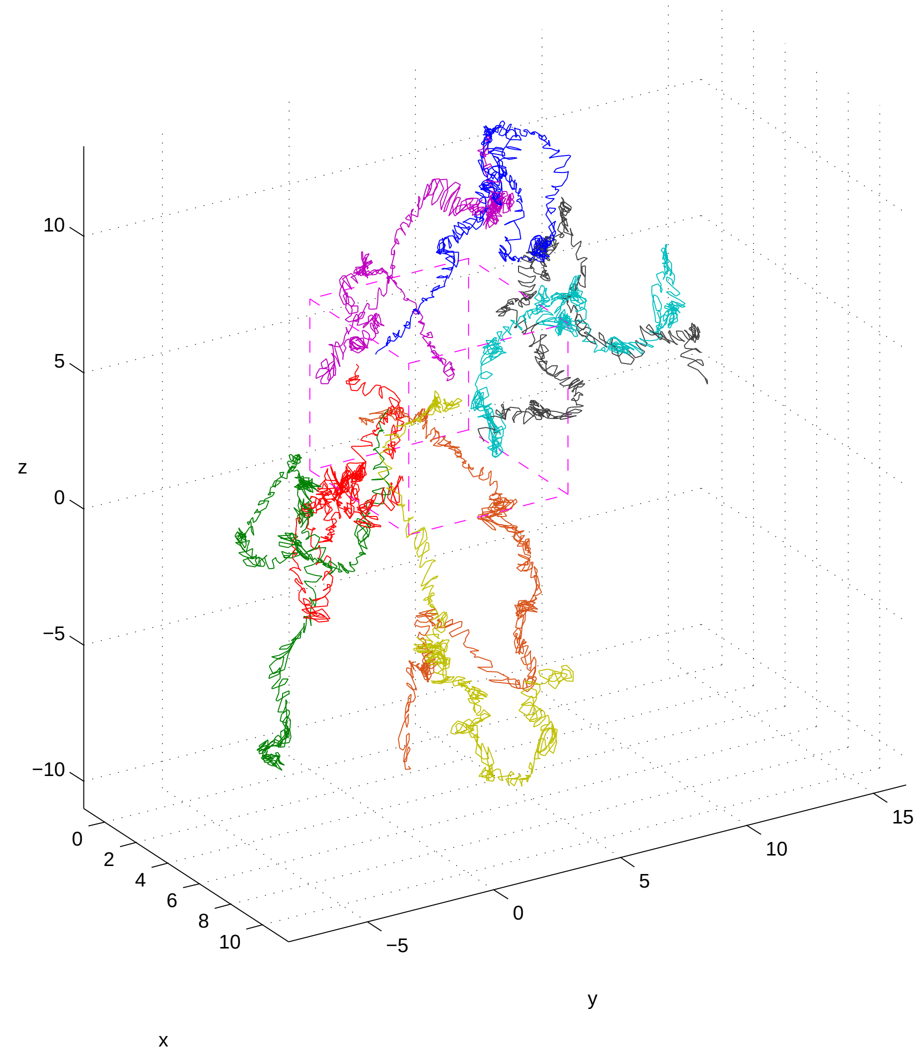
$$B_0 = 0.2,\ U_{\rm rms} = 1.42$$
$$D_x/D_z = 0.95$$

amp=3 , ν=1.25e−03 , η=1.25e−03 , $B0_z$=0.2 , L_z=1 , nx=256 , ny=256 , nz=256



$$B_0 = 1.0,\ U_{\rm rms} = 0.29$$
$$D_x/D_z = 0.24$$

amp=0.1 , ν=1.25e−03 , η=1.25e−03 , $B0_z$=1 , L_z=1 , nx=256 , ny=256 , nz=256

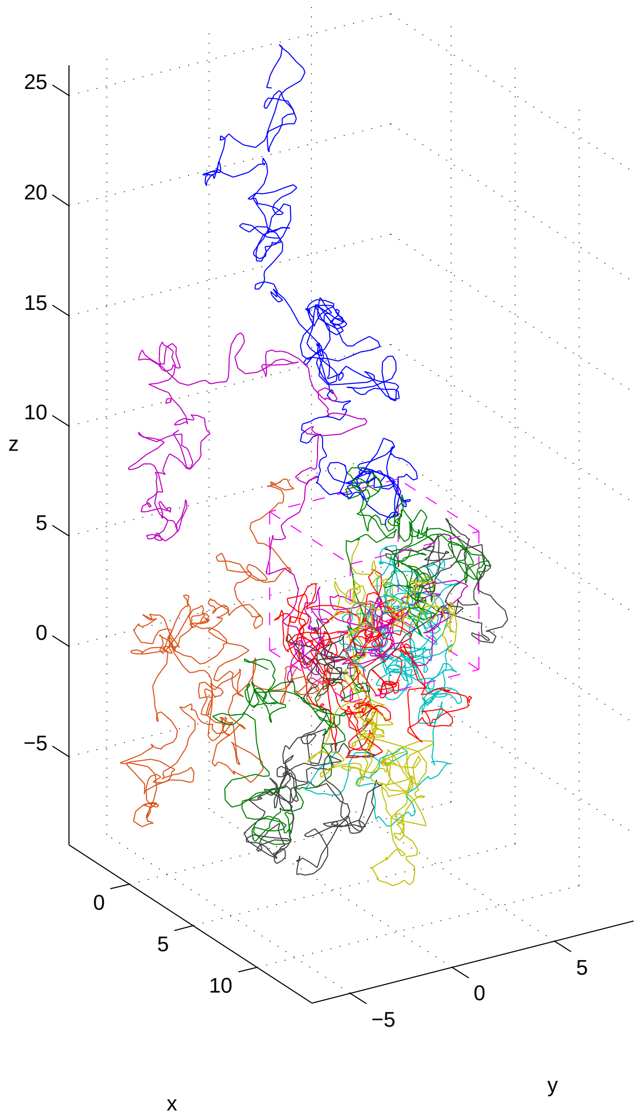


Particle trajectories

$B_0 = 0.2,\ U_{\rm rms} = 0.25$

$D_x/D_z = 0.34$

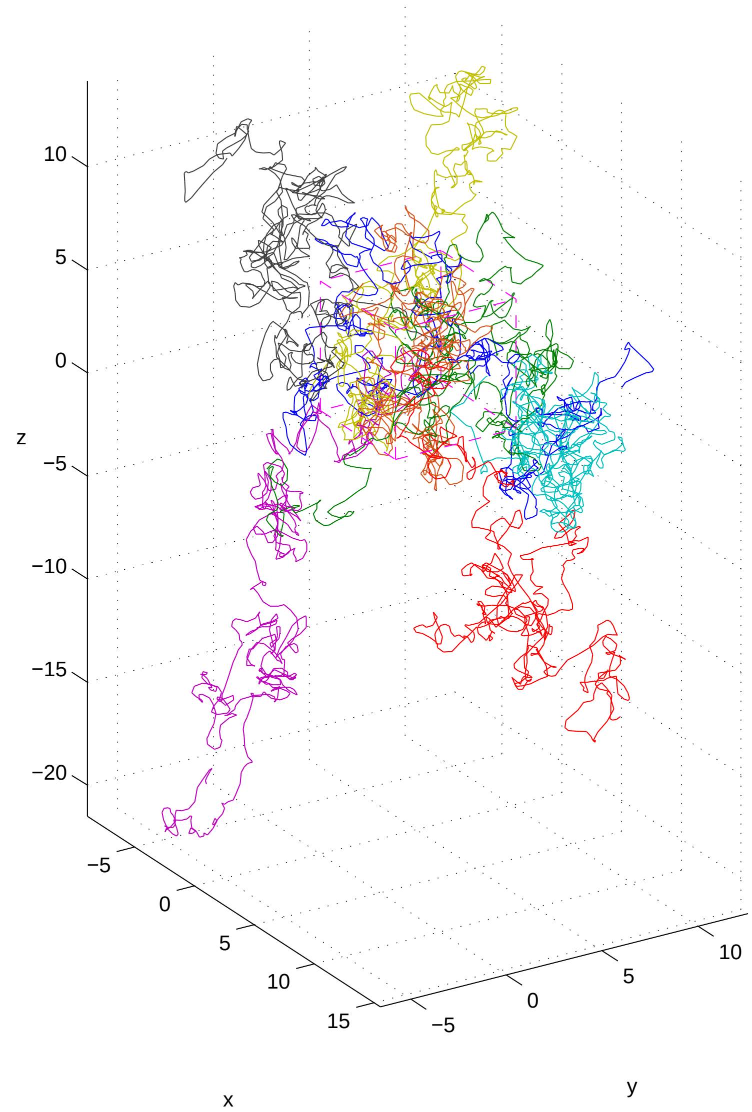
amp=0.1 , ν=1.25e−03 , η=1.25e−03 , $B0_z$=0.2 , L_z=1 , nx=256 , ny=256 , nz=256



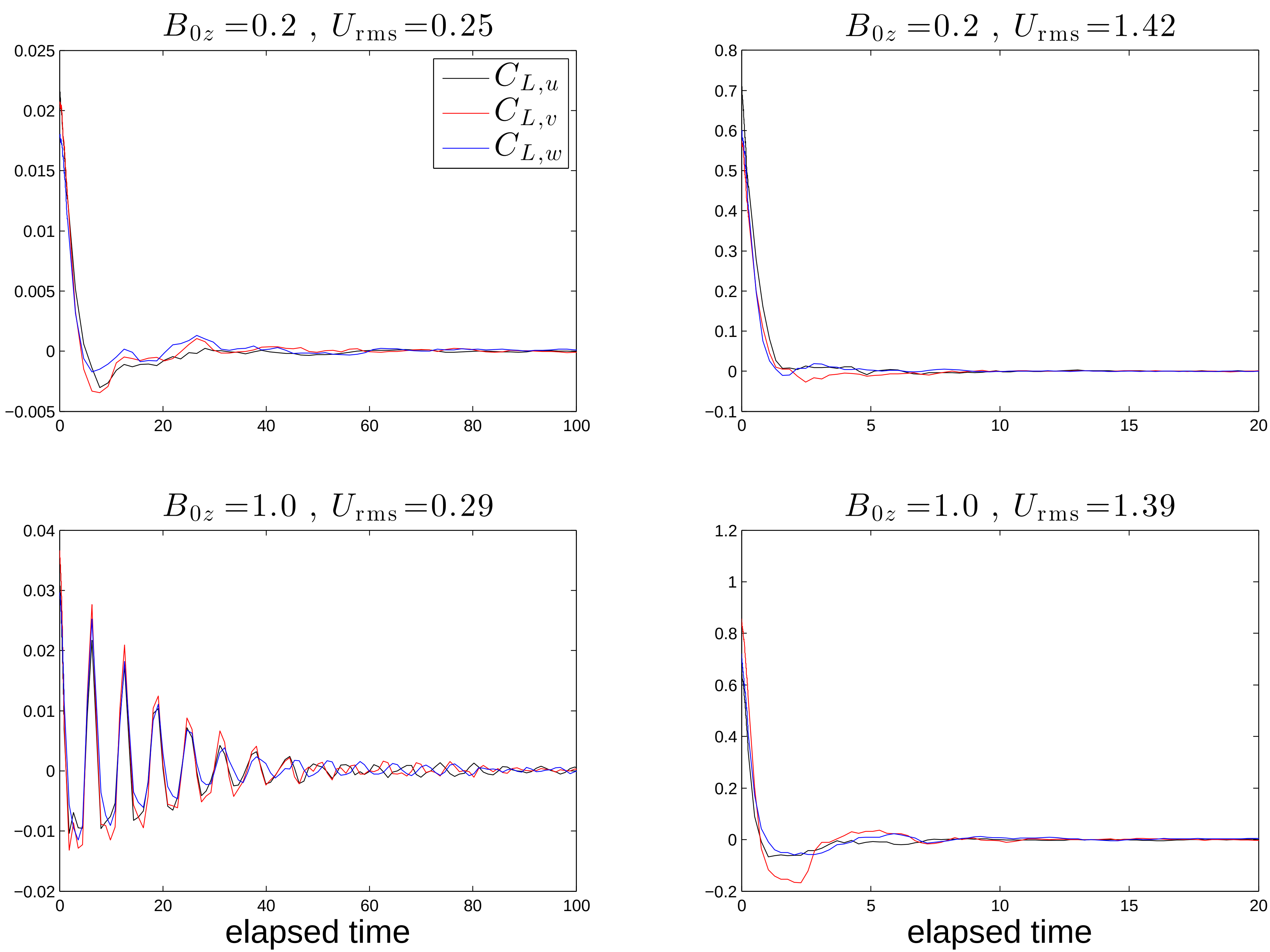
$B_0 = 1.0,\ U_{\rm rms} = 1.39$

$D_x/D_z = 0.34$

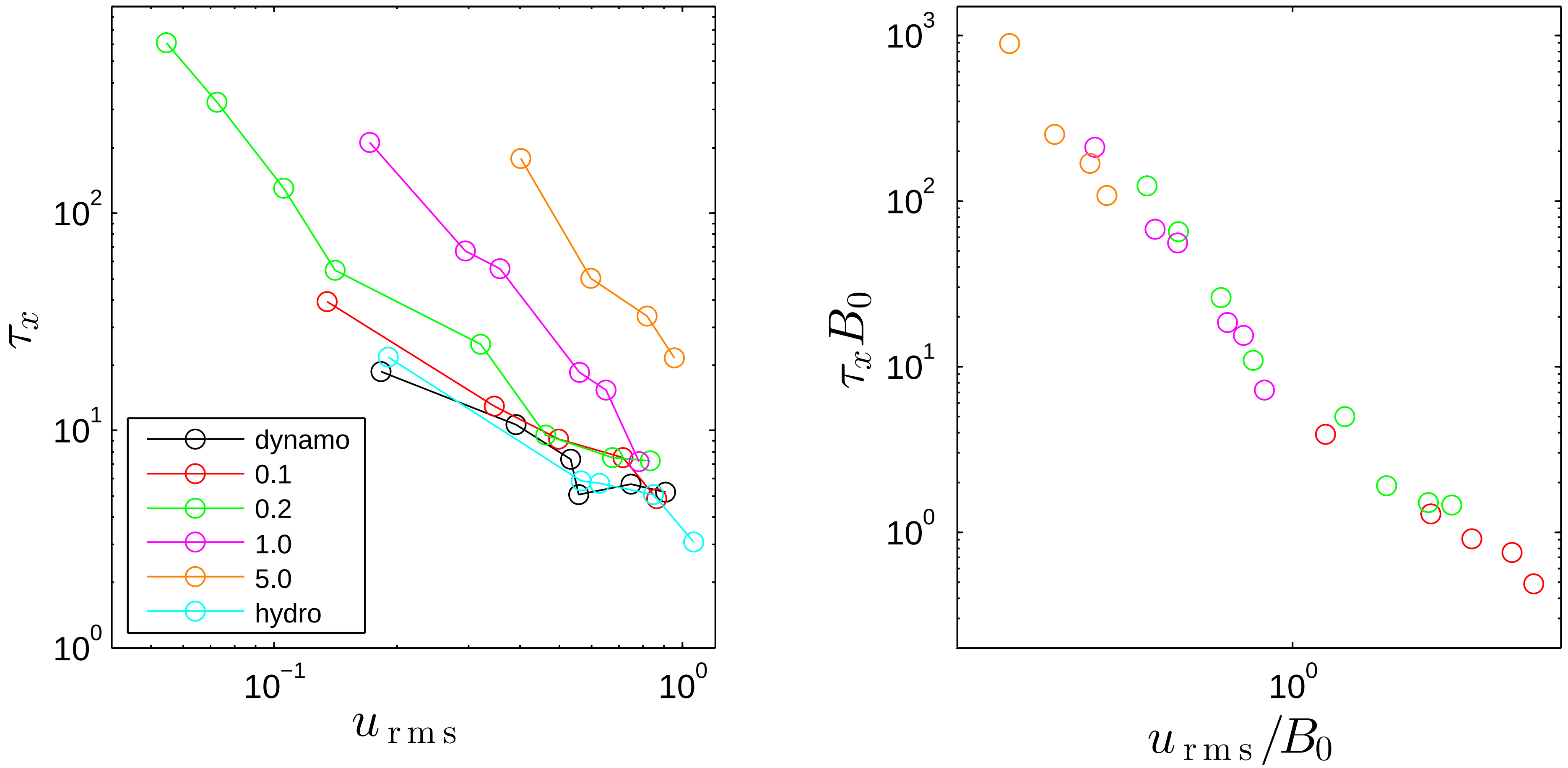
amp=3 , ν=1.25e−03 , η=1.25e−03 , $B0_z$=1 , L_z=1 , nx=256 , ny=256 , nz=256



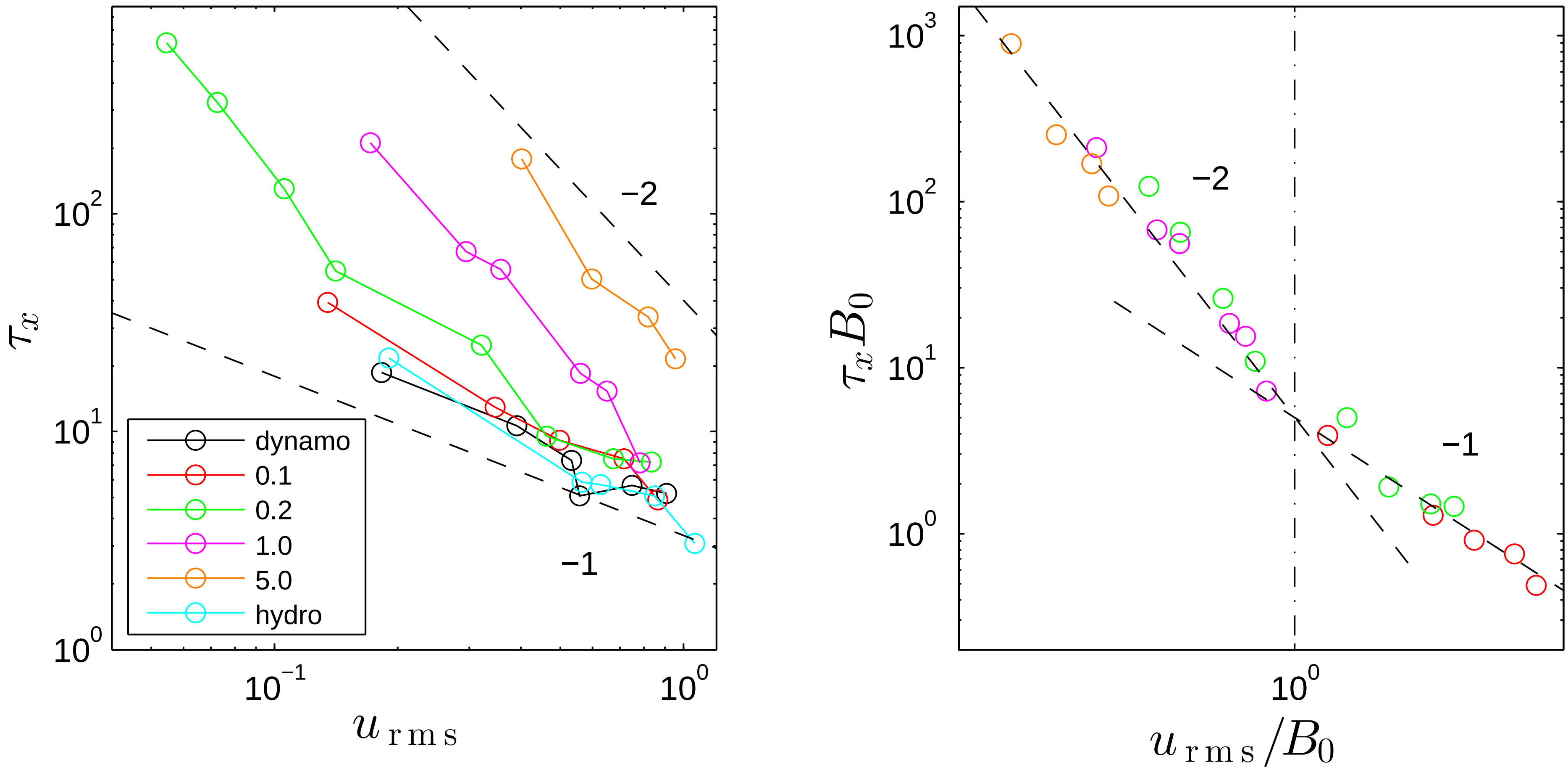
Lagrangian velocity correlation



Velocity decorrelation time



Velocity decorrelation time



- $u_{\rm rms}/B_0 > 1 : \tau_x \sim (u_{\rm rms})^{-1}$ $\quad [\tau_x B_0 \sim (u_{\rm rms}/B_0)^{-1}]$
- $u_{\rm rms}/B_0 < 1 : \tau_x \sim B_0 (u_{\rm rms})^{-2}$ $\quad [\tau_x B_0 \sim (u_{\rm rms}/B_0)^{-2}]$

ongoing work: ensemble averaging to get better statistics

A tentative physical picture ...

- wave induces memory into the system

  wave time scale: $\tau_A \sim B_0^{-1}$

- background turbulence removes memory

  turbulent decorrelation time: $\tau_u \sim (U_{\rm rms})^{-1}$

- a competition between τ_A and τ_u

- anisotropic diffusion:
  - $B_0/U_{\rm rms} \gtrsim 1$
  - $\tau_A \lesssim \tau_u$

A tentative physical picture . . .

Iroshnikov–Kraichnan picture of weak MHD turbulence

$$V_A \sim B_0$$ Alfven wave speed

$$\tau_A \sim \frac{\ell}{V_A}$$ wave packet interaction time

$$\frac{\Delta u}{\tau_A} \sim \frac{u^2}{\ell}$$ distortion each interaction

$$u \sim \sqrt{N}\Delta u$$ distortion after N interactions

$$\Rightarrow \quad \tau_{\mathrm{cas}} \sim N\tau_A \sim \frac{\ell B_0}{u^2}$$ cascade time

Summary

- study single-particle diffusion in 3D MHD turbulence
- transport mostly shows diffusive scaling at large time
- anisotropic suppression of turbulent diffusion by a guided-field $(D_x\,, D_y \lesssim D_z)$
- competition between waves and background turbulence