Stochastic modelling and parametrization of atmospheric moisture transport

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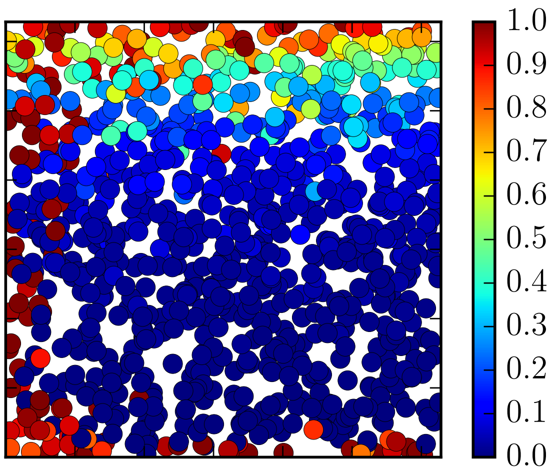
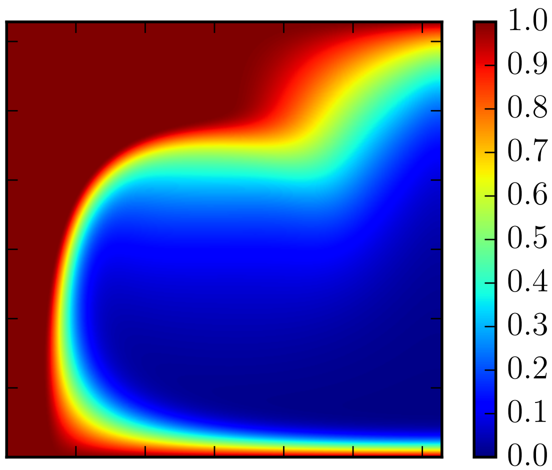


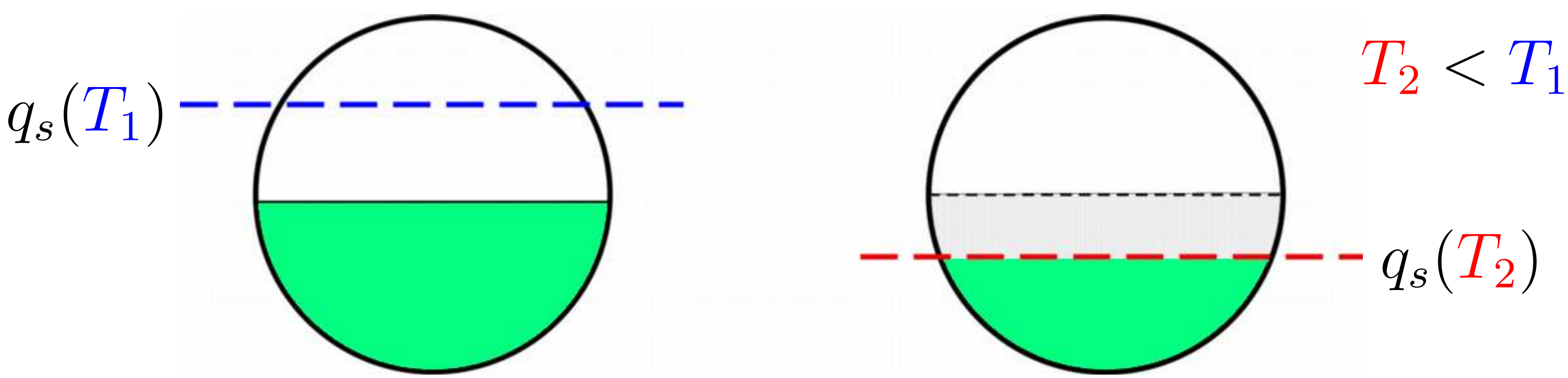


Condensation of water vapour

- specific humidity of an air parcel:

$$q = \frac{\text{mass of water vapor}}{\text{total air mass}}$$

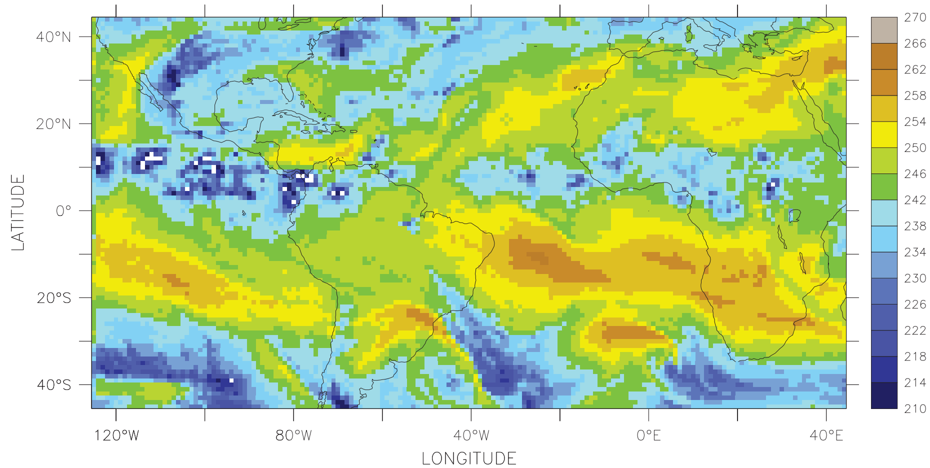
- saturation specific humidity, $q_s(T)$
  - when $q > q_s$, condensation occurs
  - excessive moisture precipitates out, $q \to q_s$
  - $q_s(T)$ decreases with temperature T
- probability distribution of water vapor in the atmosphere?



Advection–condensation paradigm

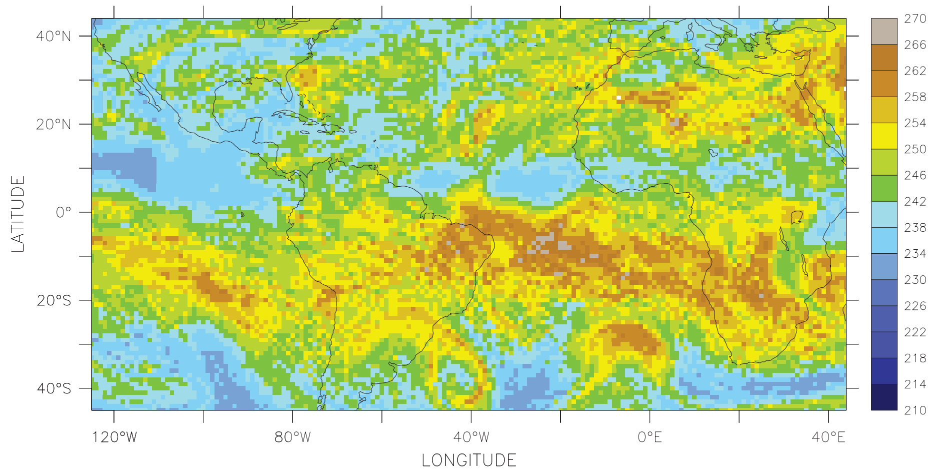
Large-scale advection + condensation

$\rightarrow$ reproduce (leading-order) observed humidity distribution



Observed WVEBBT

Observation



Simulated WVEBBT

Simulation

– velocity and q_s field from observation
– trace parcel trajectories backward to the lower boundary layer (source)
– track condensation along the way

ignore: cloud-scale microphysics, molecular diffusion, ...

(Pierrehumbert & Roca, GRL, 1998)

Advection–condensation model

- Particle formulation:

$$\mathrm{d}\vec{X}(t) = \vec{u}\,\mathrm{d}t\,, \quad \mathrm{d}Q(t) = (S - C)\mathrm{d}t$$

  air parcel at location $\vec{X}(t)$ carrying specific humidity $Q(t)$

  - $S =$ moisture source (evaporation)
  - $C =$ condensation sink, in the rapid condensation limit

$$C : Q \mapsto \min\left[\,Q\,,\; q_s(\vec{X})\,\right]$$

  - saturation profile: $q_s(y) = q_0 \exp(-\alpha y)$
  - $y =$ latitude (advection on a midlatitude isentropic surface) or altitude (vertical convection in troposphere)

- Mean-field formualtion:

$$\frac{\partial q}{\partial t} + \vec{u} \cdot \nabla q = S - C$$

  $q(\vec{x}, t)$ is treated as a passive scalar field advected by $\vec{u}$

Particle models: previous analytical results

1D stochastic models: $u \sim$ spatially uncorrelated random process

- **Pierrehumbert, Brogniez & Roca 2007**: white noise, $S = 0$
- **O'Gorman & Schneider 2006**: Ornstein–Uhlenbeck process, $S = 0$

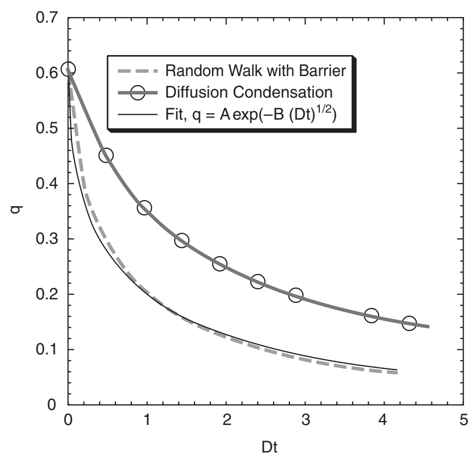


FIGURE 6.8. Decay of ensemble mean specific humidity at $y = 0.5$ for the bounded random walk with a barrier at $y = 0$. The thin

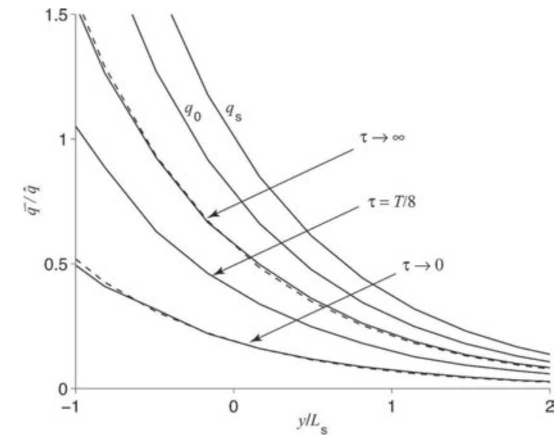
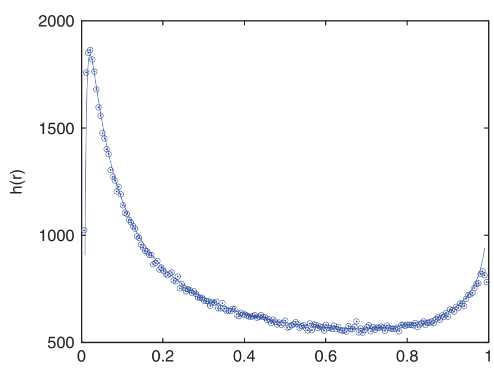
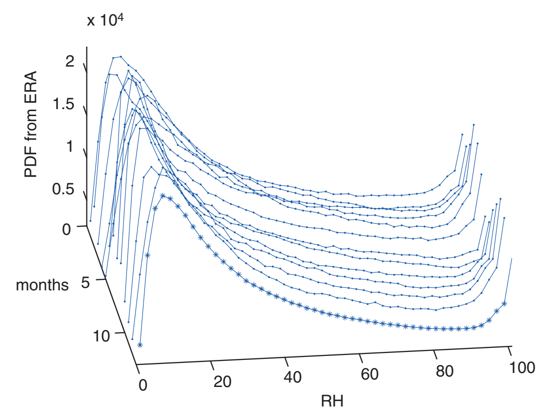


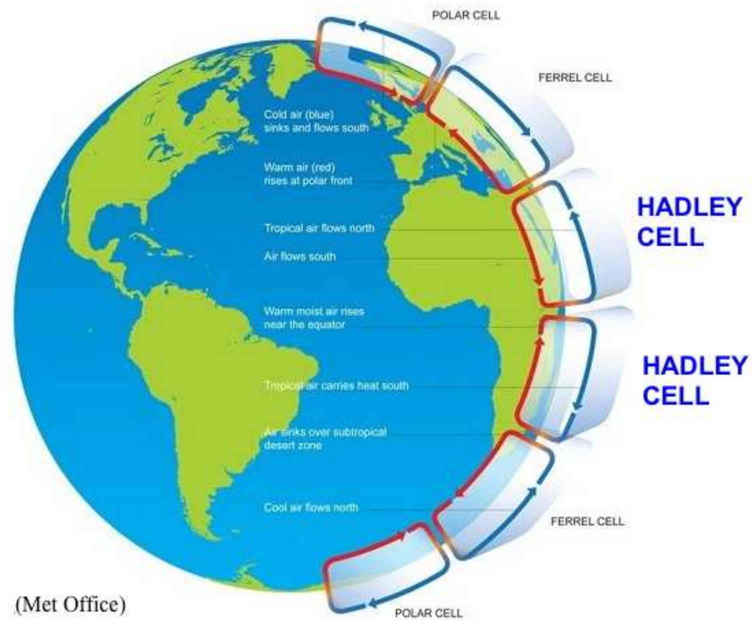
FIG. 2. Mean specific humidity vs meridional distance for initial value problem. Moisture distributions are shown after the evolution times T at which $L(T) = 4L_s$ in each case. Solid lines are

- **Sukhatme & Young 2011**: white noise with a boundary source





Coherent circulation in the atmosphere



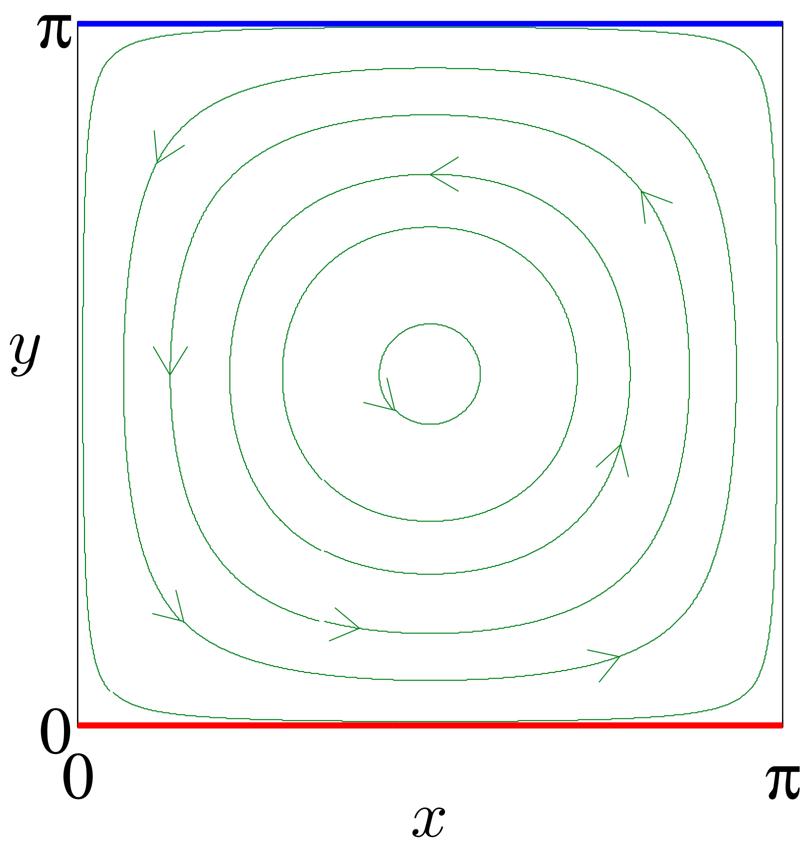
(Met Office)

- moist, warm air rises near the equator
- poleward transport in the upper troposphere
- subsidence in the subtropics ($\sim 30^\circ$N and 30°S)
- transport towards the equator in the lower troposphere

Q: response of rainfall patterns to changes in the Hadley cells?

Steady-state problem

- bounded domain: $[0, \pi] \times [0, \pi]$, reflective B.C.
- $q_s(y) = q_{\max} \exp(-\alpha y)$: $q_s(0) = q_{\max}$ and $q_s(\pi) = q_{\min}$
- resetting source: $Q = q_{\max}$ if particle hits $y = 0$



cellular flow: $\psi = -U \sin(x) \sin(y)$; $\quad (u, v) = (-\psi_y, \psi_x)$

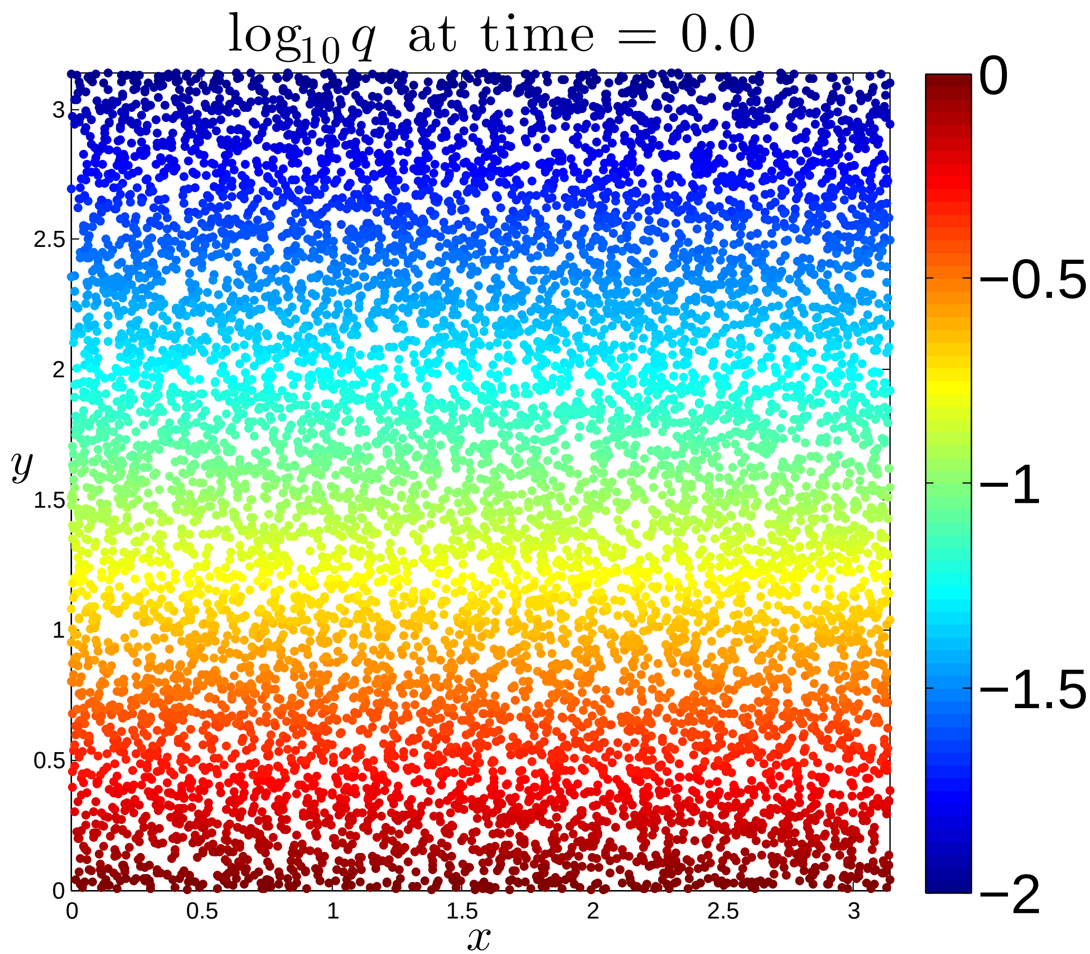
Stochastic system with source

$$\mathrm{d}X(t) = u(X,Y)\,\mathrm{d}t + \sqrt{2\kappa}\,\mathrm{d}W_1(t)$$

$$\mathrm{d}Y(t) = v(X,Y)\,\mathrm{d}t + \sqrt{2\kappa}\,\mathrm{d}W_2(t)$$

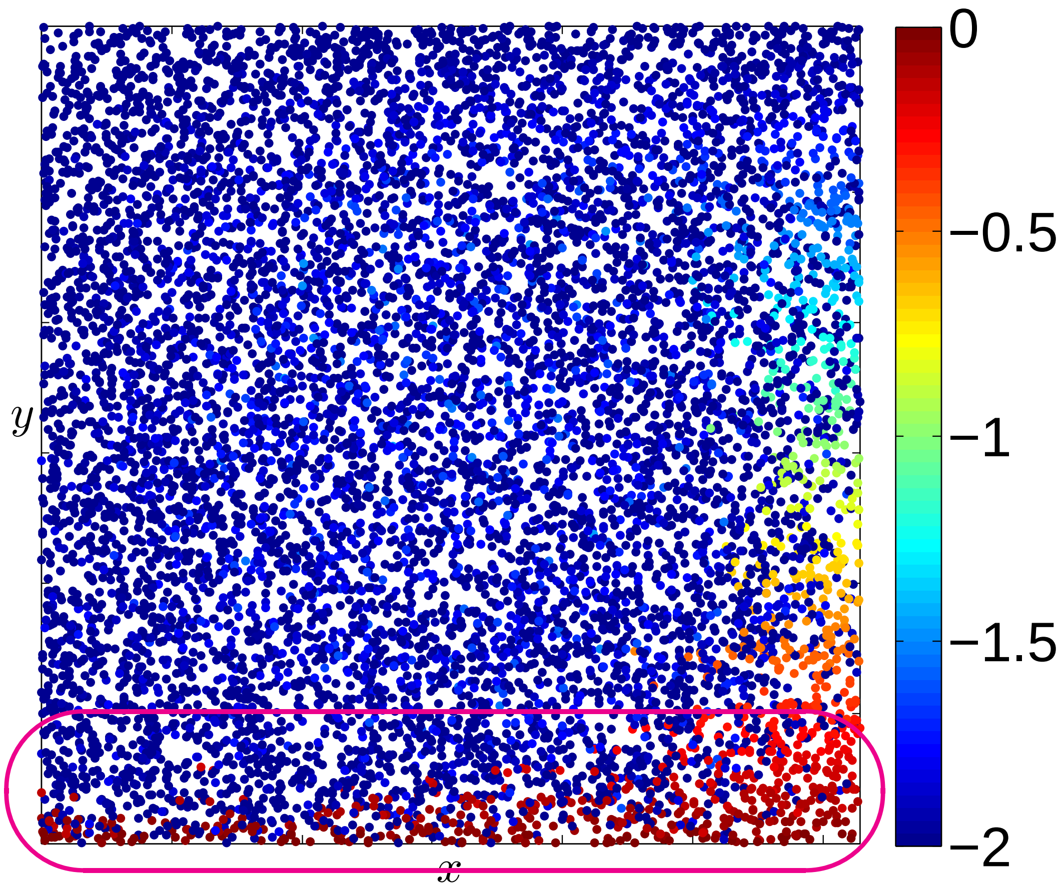
$$\mathrm{d}Q(t) = [S(Y) - C(Q,Y)]\mathrm{d}t$$

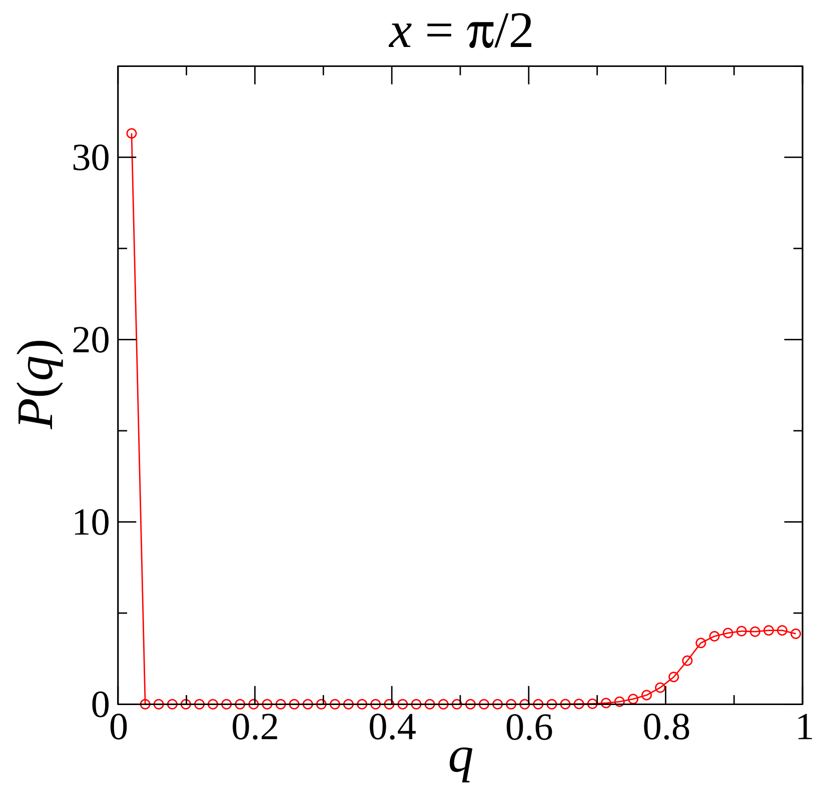
$$\psi = -U \sin x \sin y$$
$$u = -\psi_y$$
$$v = \psi_x$$



$$U = 1$$
$$\kappa = 10^{-2}$$

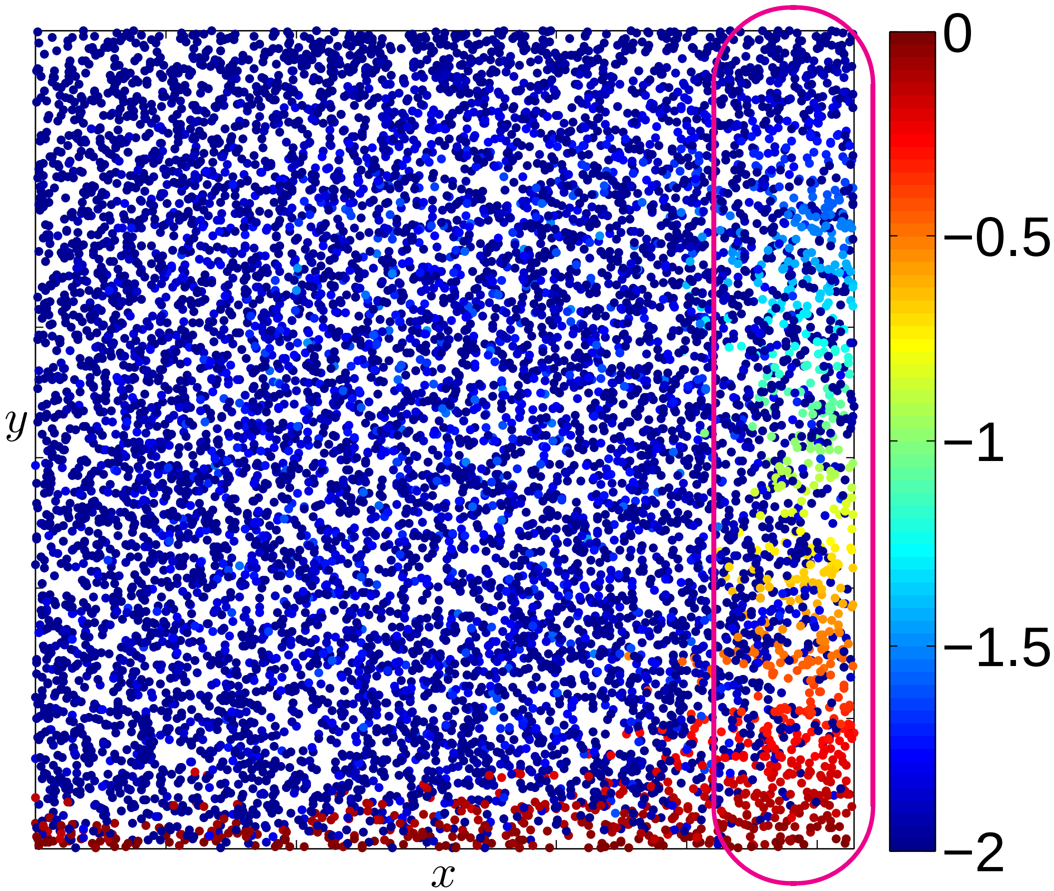
Source boundary layer

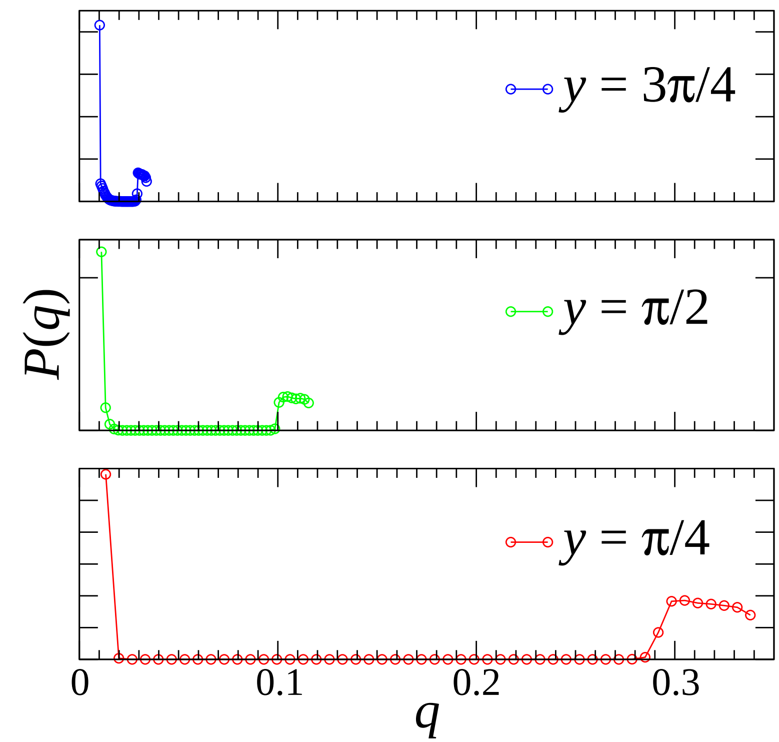




- **Bimodal distribution**: layer consists mainly of either:
  - $Q = q_{\min}$ from upstream of the flow and diffuse in from the domain interior
  - $Q \approx q_{\max}$ from the resetting source
- particles with $Q \approx q_{\max}$ spreading into the domain as x increases

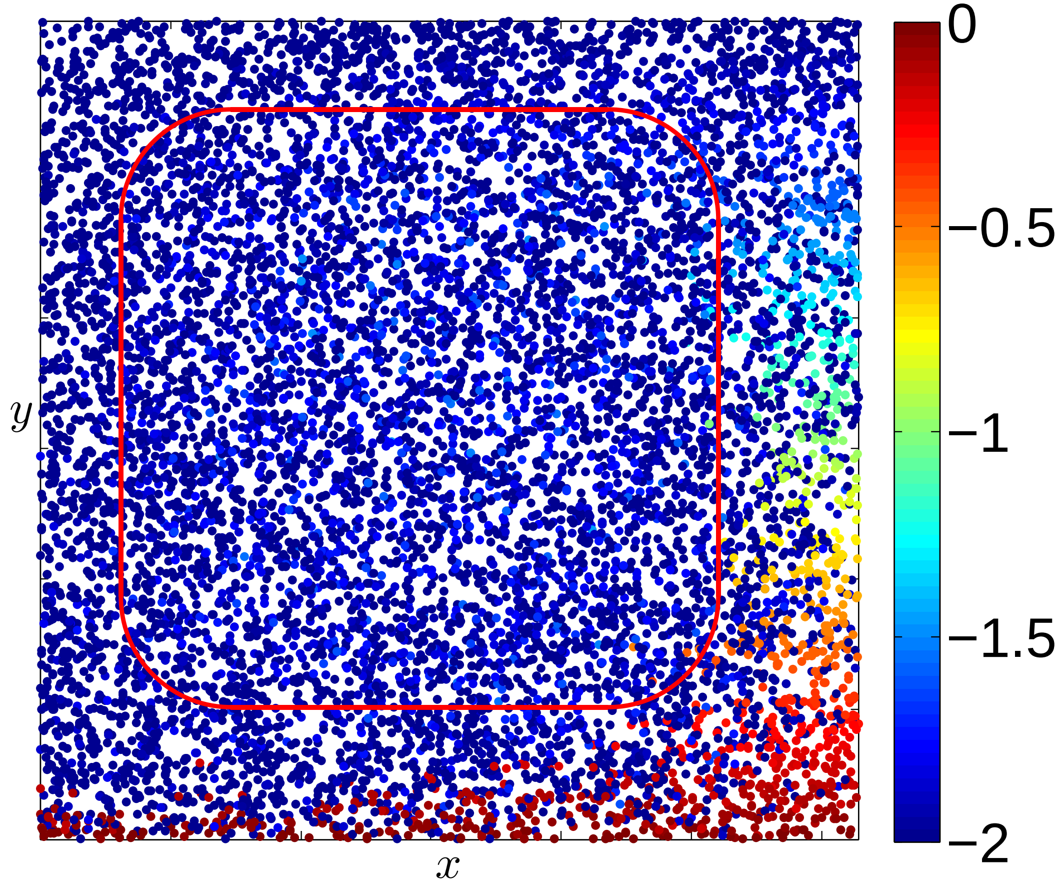
Condensation boundary layer

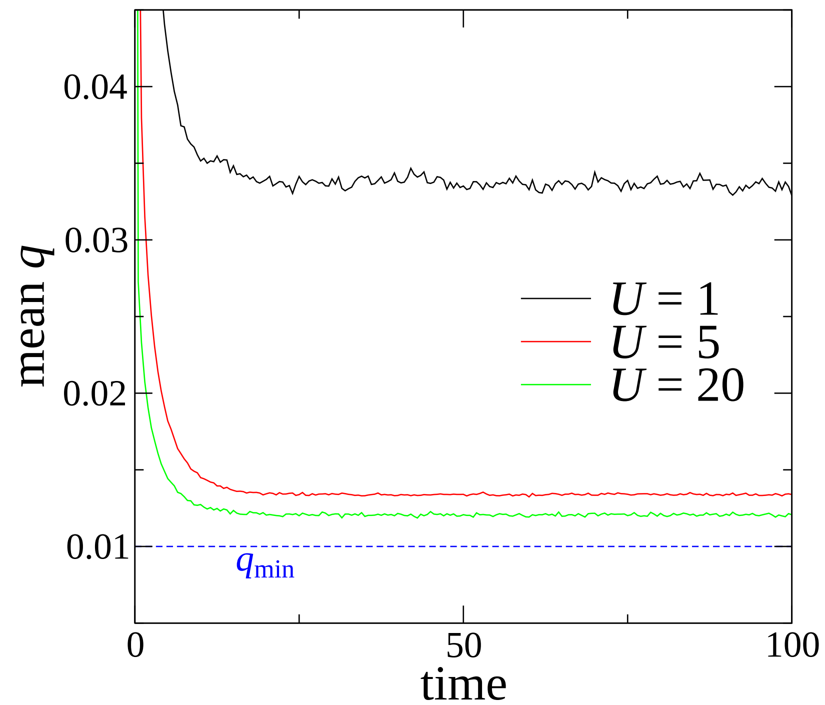




- moist particles move up into region of low $q_s(y)$
- at some fixed height y_1: mainly consists of $Q = q_{\min}$ (diffuse in from the interior) and $Q = q_s(y_1)$ — **Bimodal distribution**
- condensation $\Rightarrow$ localized rainfall over a narrow $O(\epsilon^{1/2})$ region

Interior region





- a homogeneous region of very dry air $Q \approx q_{\min}$ is created in the domain interior
- the vortex "shields" the source from the interior
- interior effectively undergoing stochastic drying

Steady-state problem

Steady-state Fokker-Planck equation for $P(x, y, q)$:

$$\epsilon^{-1}\vec{u} \cdot \nabla P + \partial_q[(S - C)P] = \nabla^2 P\,, \qquad \epsilon = \kappa/(UL) \ll 1$$

Rapid condensation limit:

$$\left.\begin{array}{r} P(x, y, q) \neq 0 \\ C = 0 \end{array}\right\} \text{ for } x, y \in [0, \pi] \text{ and } q \in [q_{\min}, q_s(y)]$$

Resetting source at bottom boundary:

$$P(x, y = 0, q) = \pi^{-1}\delta(q - q_{\max})$$

At the top boundary: $P(x, y = \pi, q) = \pi^{-1}\delta(q - q_{\min})$

Hence,

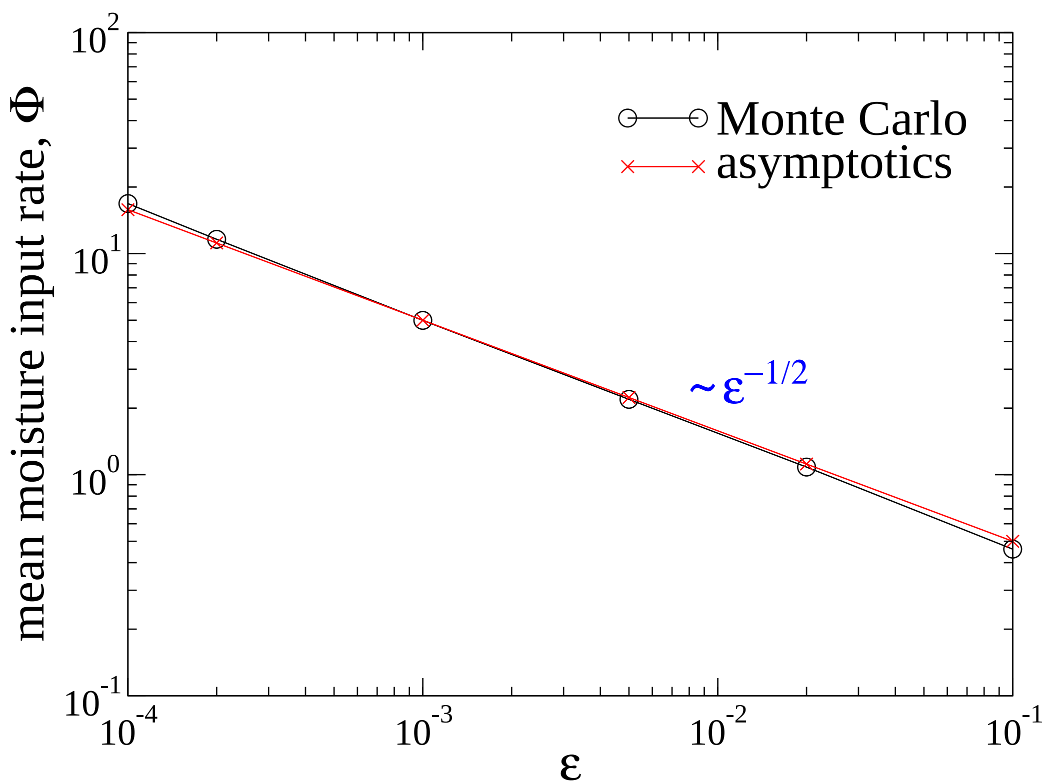
$$\epsilon^{-1}\vec{u} \cdot \nabla P = \nabla^2 P$$

which predicts a boundary layer of thickness $O(\epsilon^{1/2})$

Solution and diagnostics

- solve $P(x, y, q)$ by matched asymptotics as $\epsilon \to 0$
- dry peak: $P(x, y, q) = \delta(q - q_{\min})\beta(x, y)/\pi^2 + F(x, y, q)$
- mean moisture input rate:

$$\Phi = \epsilon^{-1/2}\sqrt{8\kappa/\pi}(q_{\max} - q_{\min})\,, \quad \epsilon = \kappa/(UL)$$



Other diagnostics: horizontal rainfall profile, vertical moisture flux, …etc

Mean-field PDE model

- Weather/climate models represent atmospheric moisture as a coarse-grained field $\bar{q}(\vec{x}, t)$ governed by deterministic PDE
- Advection–condensation–diffusion:

$$\frac{\partial \bar{q}}{\partial t} + \vec{u} \cdot \nabla \bar{q} = \kappa_q \nabla^2 \bar{q} - C + S$$

  - κ_q: eddy diffusivity representing un-resolved processes

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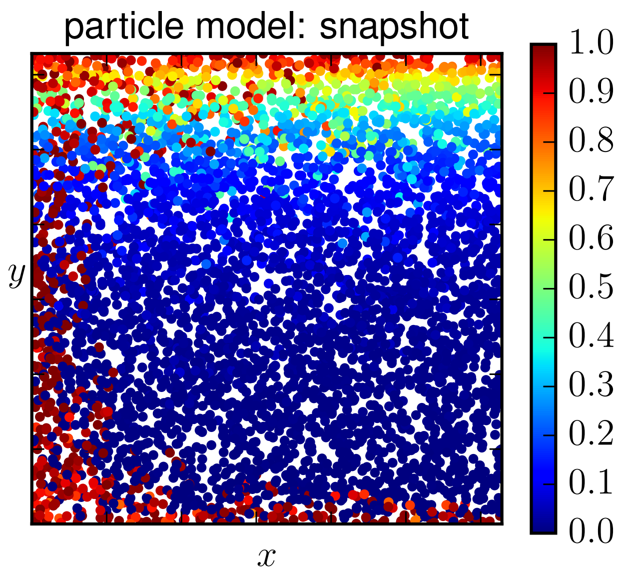

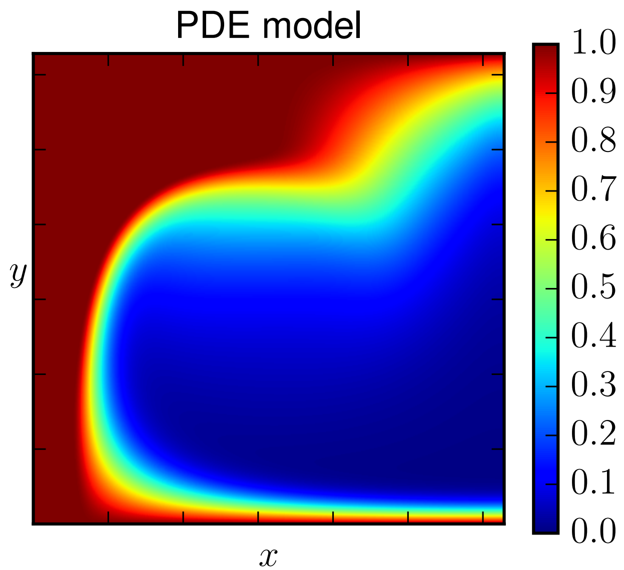

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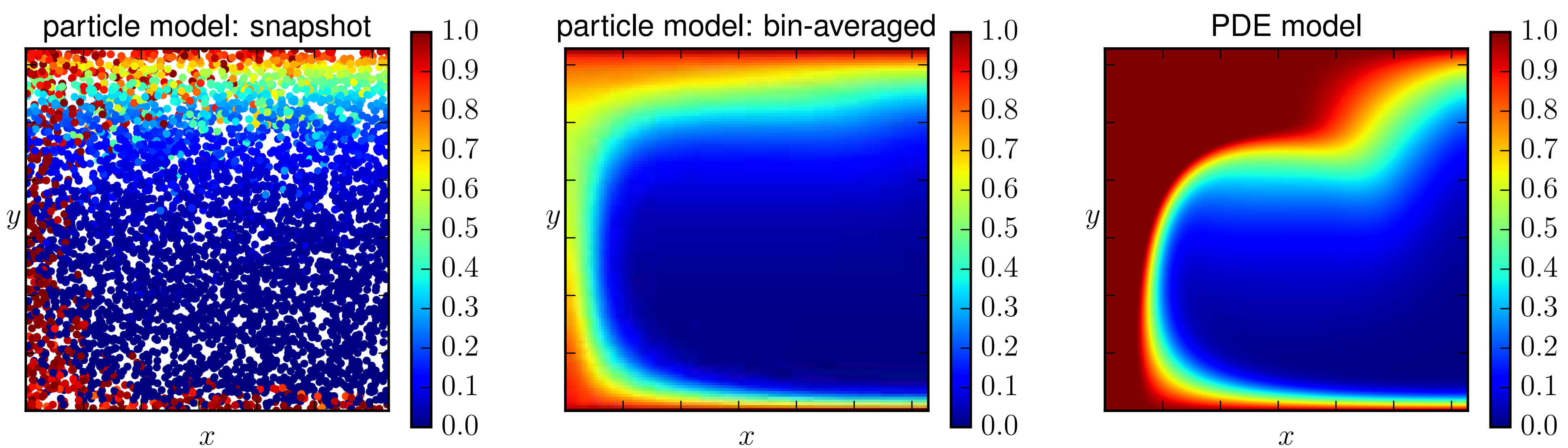


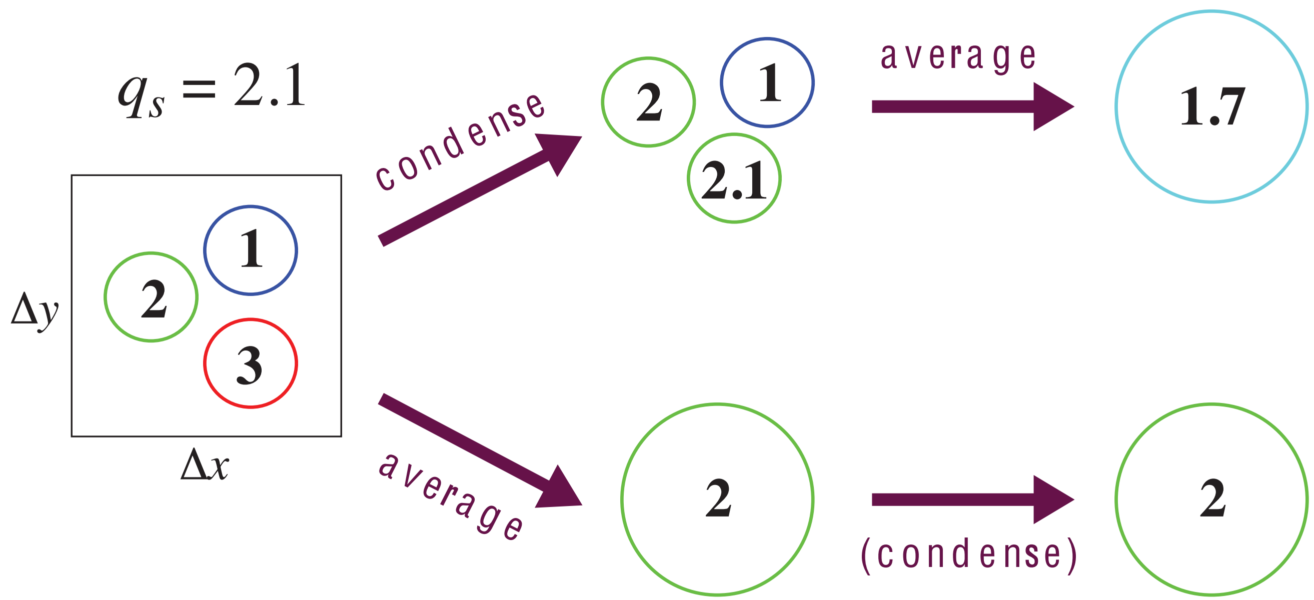
Why PDE models saturate the domain?

- $C_{\mathrm{PDE}}(\bar{q}) = \tau_c^{-1}(\bar{q} - q_s)H(\bar{q} - q_s)$, $\quad H$: Heaviside step function
- Fokker-Planck: $\partial_t P + \vec{u} \cdot \nabla P + \partial_q[(S - C)P] = \kappa_b \nabla^2 P$

$$\bar{q}(x,y,t) = \pi^2 \int_{q_{\min}}^{q_{\max}} q' P(x,y,q',t) \mathrm{d}q'$$

$$\bar{C} = \pi^2 \int_{q_s(y)}^{q_{\max}} (q' - q_s) P(x,y,q',t) \mathrm{d}q$$

- condensation and averaging do not commute



Parametrization of condensation

$$\frac{\partial \bar{q}}{\partial t} + \vec{u} \cdot \nabla \bar{q} = \kappa_q \nabla^2 \bar{q}, \quad \bar{q} \to C(\bar{q}, q_s)$$

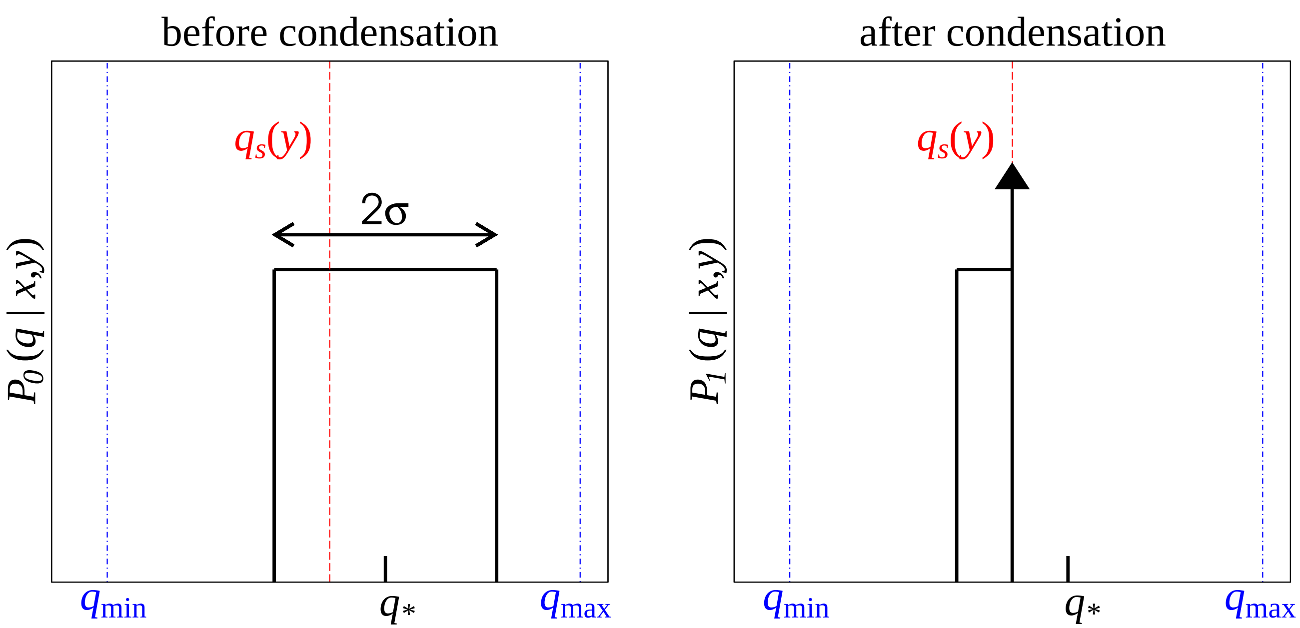
- at a grid point (x, y) and time t, after advection and diffusion steps

$$\text{let's say} \quad \bar{q}(x, y, t) = q_*$$

- imagine there is a distribution $P_0(q|x, y)$ such that

$$q_* = \int q' P_0(q'|x, y)\, \mathrm{d}q'$$

$$\text{then,} \;\; \bar{q}(x, y, t + \Delta t) = \int q' P_1(q'|x, y)\, \mathrm{d}q'$$

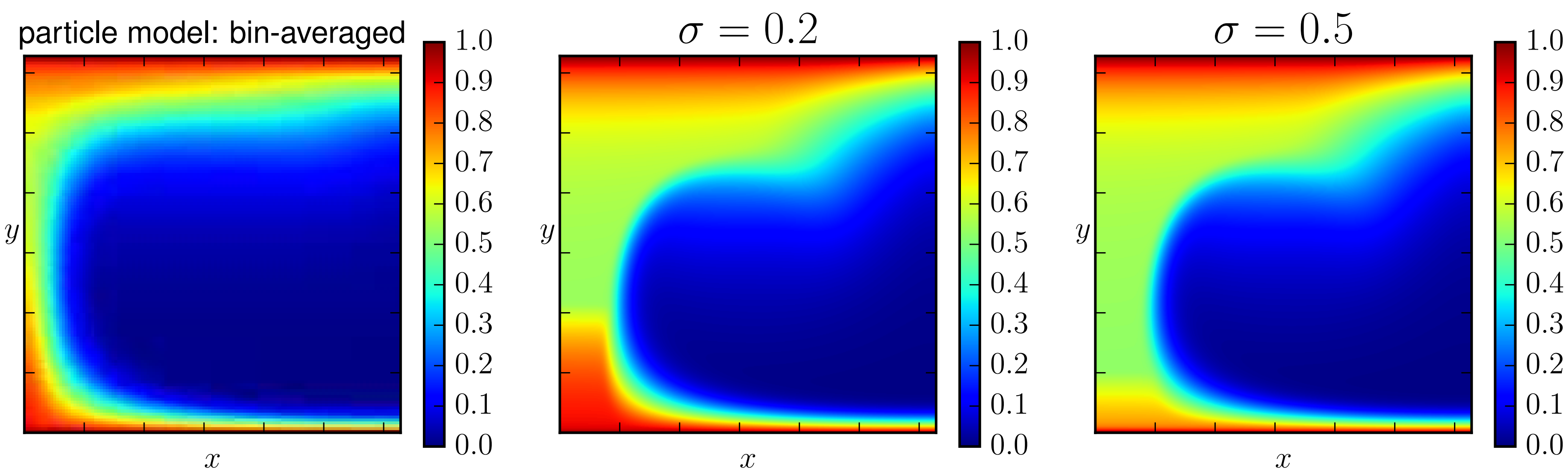


Test results

- $P_0(q|x,y)$: a top hat distribution of width 2σ
- as a test, prescribe a constant σ
- for $\bar{q} - \sigma < q_s < \bar{q} + \sigma$, condensation occurs as:

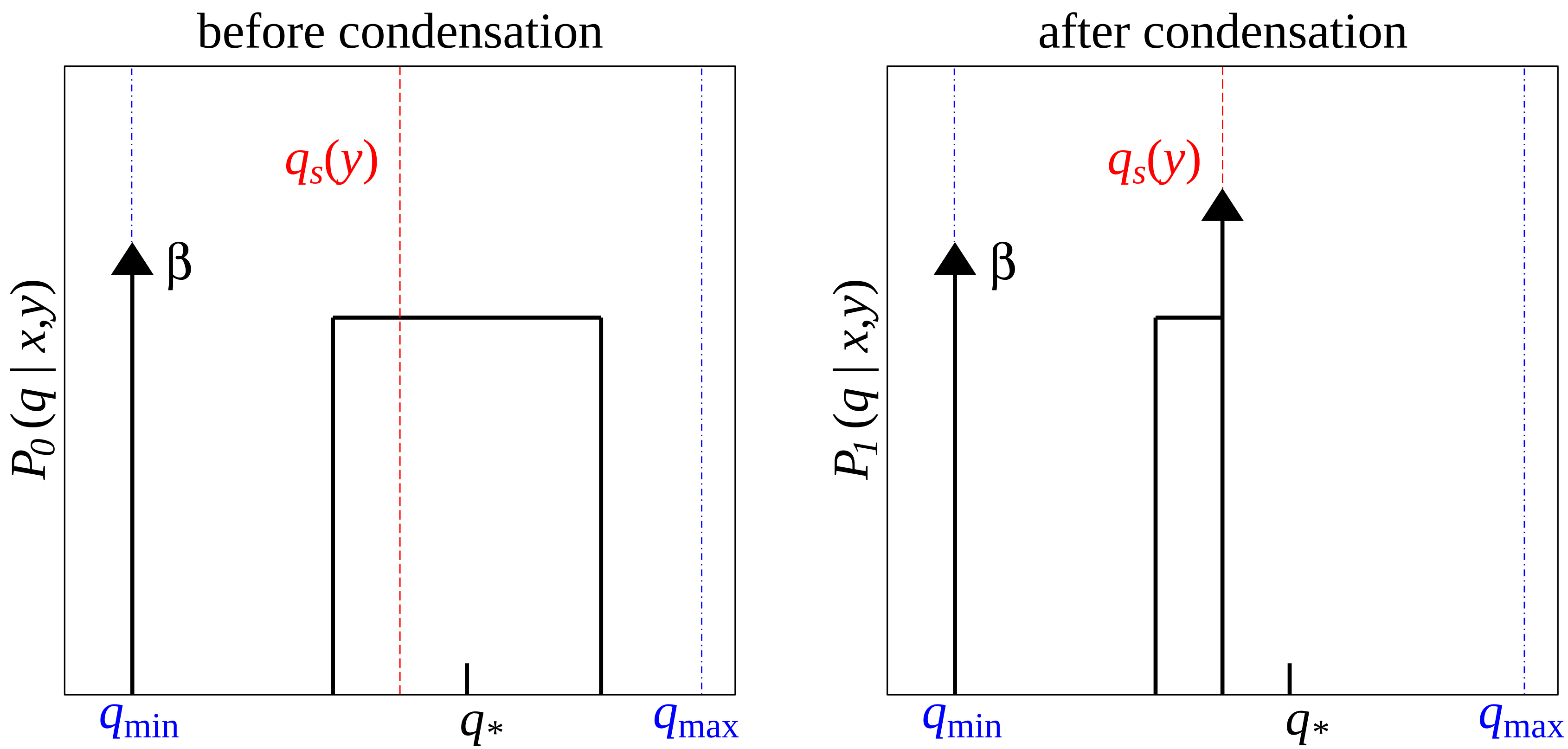
$$\bar{q} \to \bar{q} - \frac{[\bar{q} + \sigma - q_s]^2}{4\sigma}$$

$$\underline{\kappa_q = 0.01}$$



Parametrization with dry peak

- subsidence of dry air parcels is important
- include a dry peak of amplitude β in $P_0(q|x,y)$

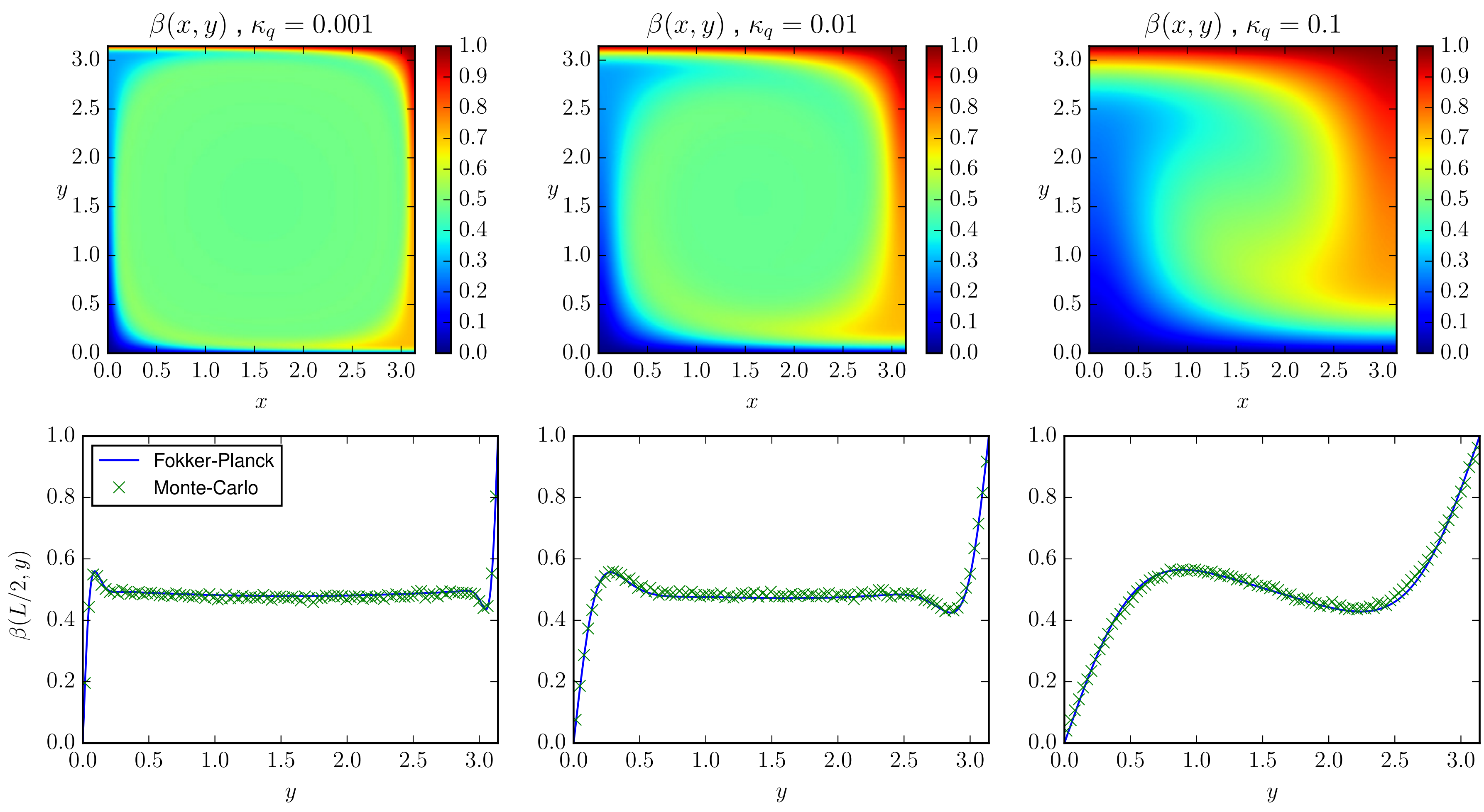


Amplitude of dry peak

$$\beta(x,y) = \pi^2 \rho(x,y)\,, \qquad P(q_{\min}, x, y, t) = \delta(q - q_{\min})\rho(x,y)$$

$$\frac{\partial \rho}{\partial t} + \vec{u}\cdot\nabla\rho = \kappa_q \nabla^2 \rho$$

$$\rho(x,0,t) = 0\,, \qquad \rho(x,\pi,t) = \pi^{-2}$$



Results with dry peak