Impact of changes in the Hadley circulation on regional rainfall

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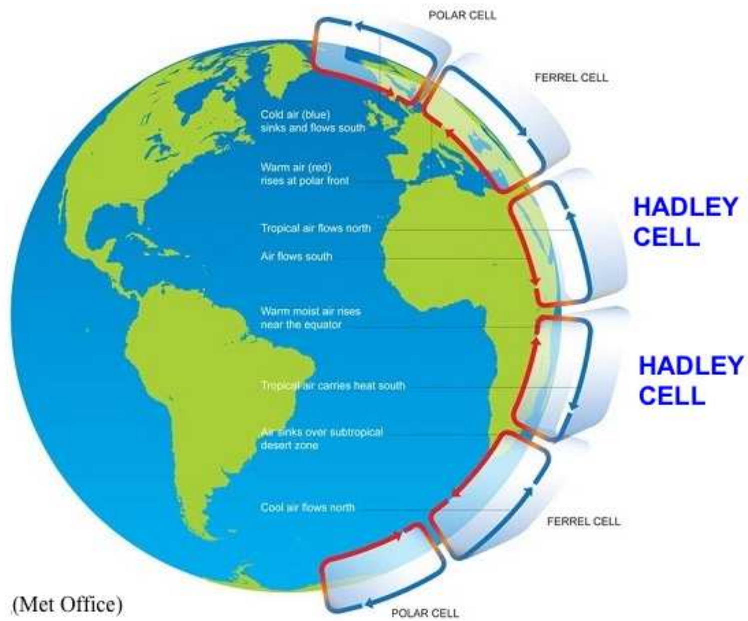
The Hadley circulation



(Met Office)

Zonal mean large-scale atmospheric circulation driven by differential heating

- moist, warm air rises near the equator
- poleward transport in the upper troposphere
- subsidence in the subtropics ($\sim$ 30°N and 30°S)
- transport towards the equator in the lower troposphere

Water vapour transport in the atmosphere

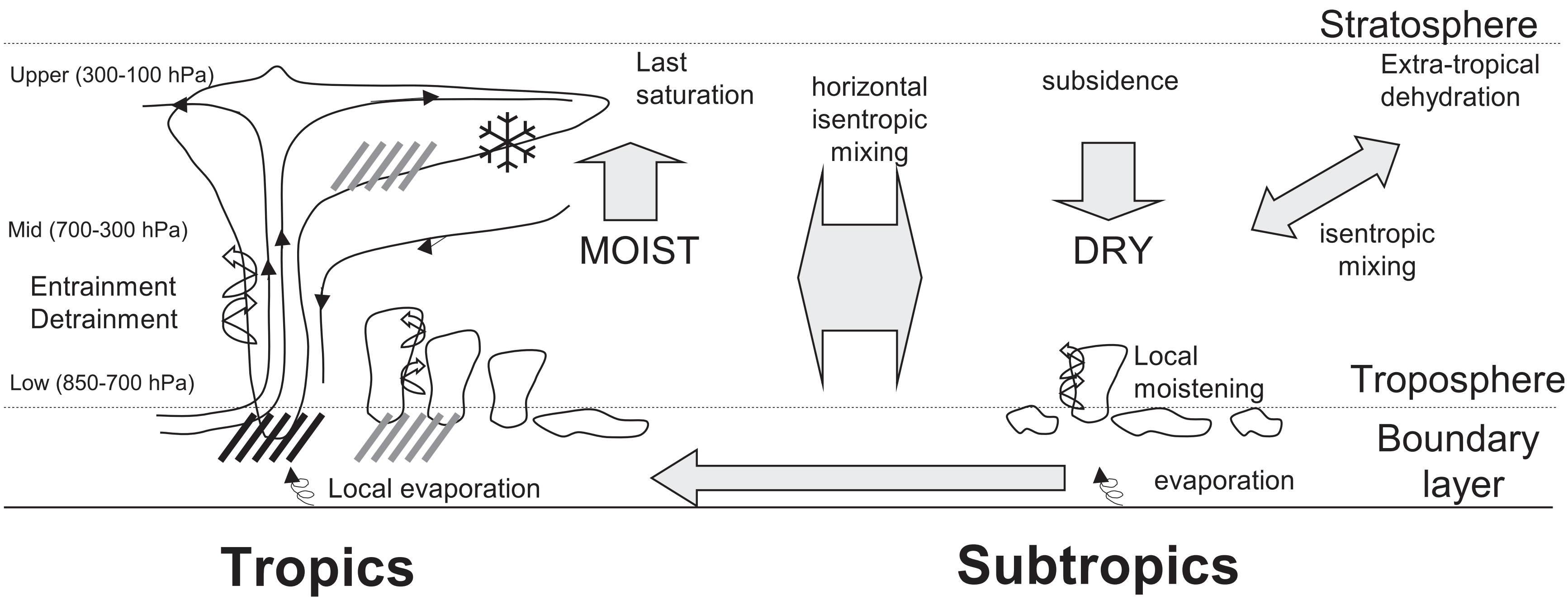


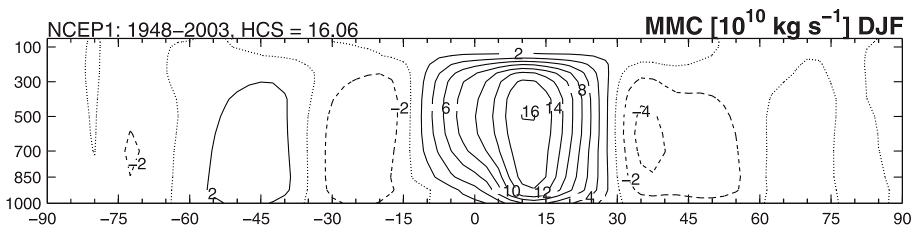
Figure 3. Schematic of the overturning circulation with emphasis on the mechanism controlling the humidity distribution in the subtropics.

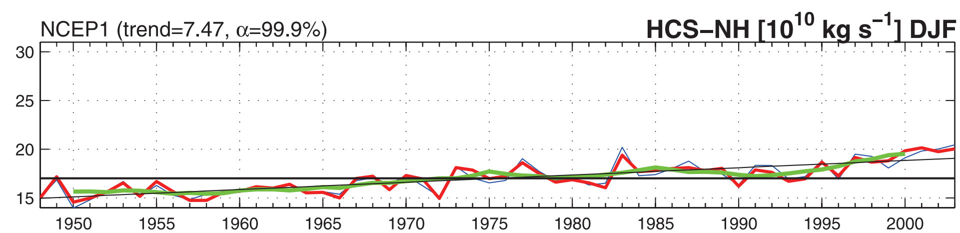
(Sherwood et al., Reviews of Geophysics, 2010)

- moisture source: surface evaporation, especially subtropical oceans
- transport towards the Inter-Tropical Convergence Zone (ITCZ) $\sim$ boundary between the northern and southern Hadley cells
- moisture rises up and condenses in ITCZ $\Rightarrow$ precipitation
- dry air moves away from the Tropics and subsides near the Subtropics

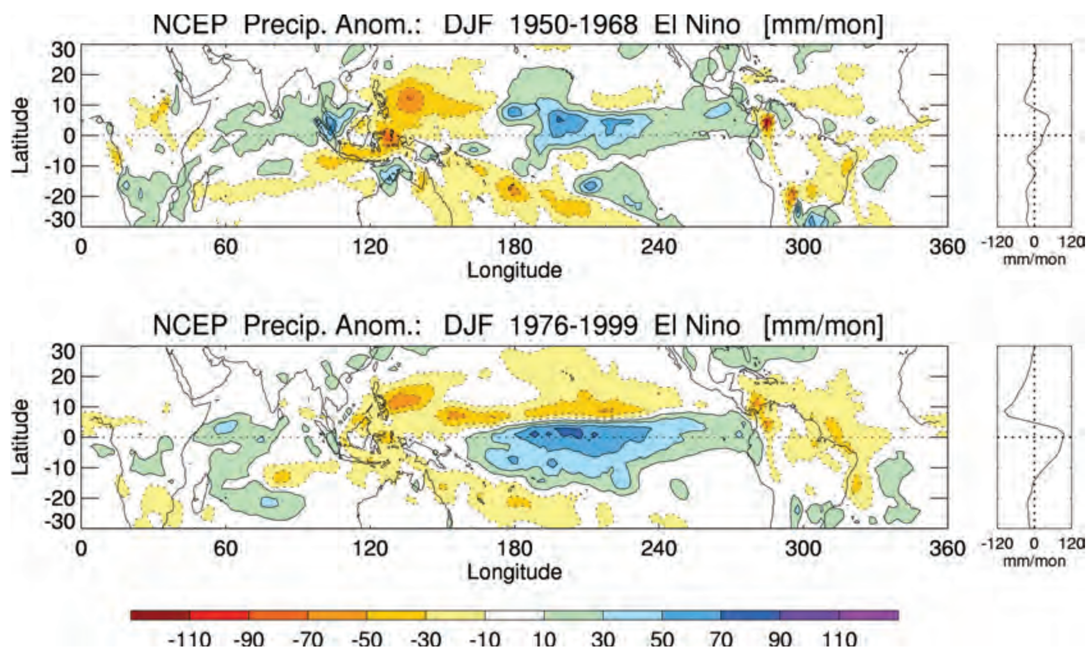
The changing Hadley cell: intensification

- strengthening of the northern hemisphere winter Hadley circulation
- Hadley cell strength index = maximum value of the stream function over 0°–30°N and from Dec to Feb *(Mitas & Clement, Geophys. Res. Lett, 2005)*





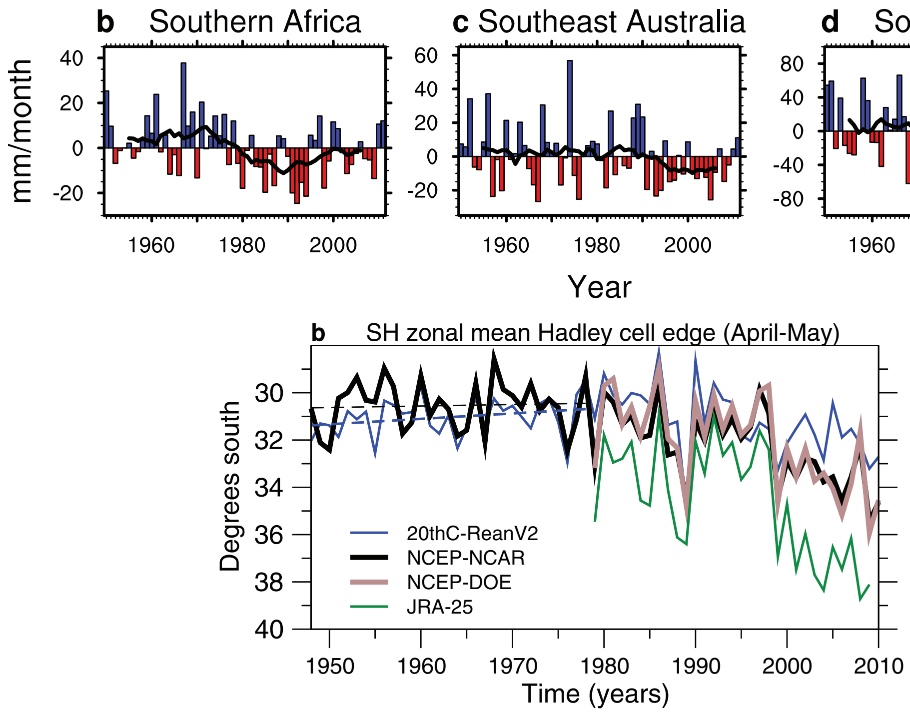
- anomalous rainfall pattern for the winter: belt of increased rainfall over the tropical Pacific shifted south *(Quan et al. 2005)*



- a cause and effect relation?

The changing Hadley cell: widening

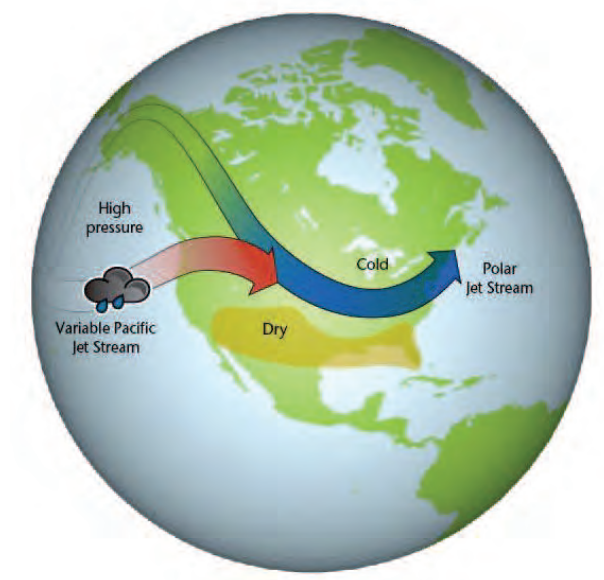
Rainfall reductions over Southern Hemisphere semi-arid regions: the role of subtropical dry zone expansion, Cai et al., Nature, 2012



- drying trend during Apr and May since the 1970s
- coincides with poleward expansion of the subtropical dry zone
- expansion not longitudinally uniform: no shift in southern Chile
- rainfall reductions attributable to poleward expansion of Hadley cell?

Teleconnection

The severe weather experienced in the UK and the North America during the winter of 2013/14 may have its root in the tropics *(Met Office, The Recent Storms and Floods in the UK, 2014)*



- persistent higher than normal rainfall over the tropics enhances Rossby wave response
- global atmospheric circulation perturbed
- buckling of the jet stream over the North Pacific brings severe weather to the UK and the North America

Models of different complexity

- investigations suggest how local rainfall pattern responds to changes in Hadley circulation but uncertainties remain
- need better understanding of the underlying physical mechanisms of such severe events $\Rightarrow$ better forecast
- build and study a hierarchy of models of increasing complexity:

GCM

comprehensive but non-trivial to disentangle the many processes in the models

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GCM	comprehensive but non-trivial to disentangle the many processes in the models
simplified GCM	bridge the gap, test robustness of idealised models
idealised mathematical model	capture the essential physics, study the effects of individual processes

Advection-condensation paradigm

$$\frac{\partial q}{\partial t} + \vec{u} \cdot \nabla q = S - C$$

- specific humidity of an air parcel:

$$q = \frac{\text{mass of water vapor}}{\text{total air mass}}$$

- saturation specific humidity $q_s(y)$, $y =$ latitude

$$q_s(y) = q_0 \exp(-\alpha y)$$

  - when $q > q_s$, condensation occurs
  - excessive moisture precipitates out, $q \to q_s$
  - $C =$ condensation sink in the rapid condensation limit

$$C : q(\vec{x}, t) \mapsto \min \left[q(\vec{x}, t) \, , \, q_s(\vec{x}) \right]$$

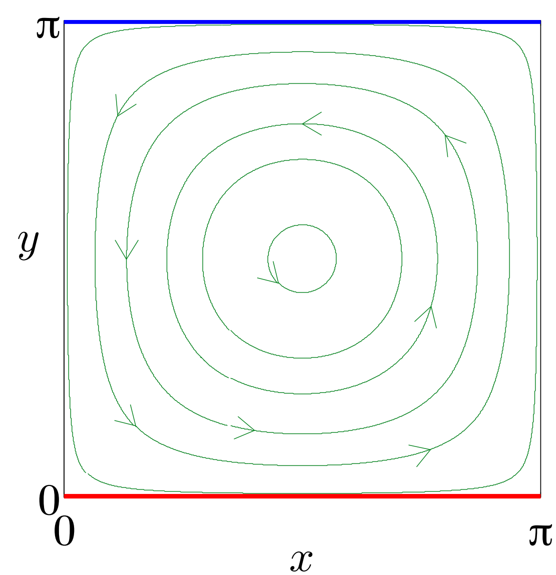
- $S =$ moisture source (evaporation)

Modelling the Hadley cell

- bounded domain: $[0,\pi]\times[0,\pi]$, reflective B.C.
- velocity of an air parcel: $\vec{\mathcal{U}}(t) = \vec{u} + \dot{W}(t)$
  - deterministic cellular flow:

$$\psi = -U\sin(x)\sin(y), \quad (u,v) = (-\psi_y, \psi_x)$$

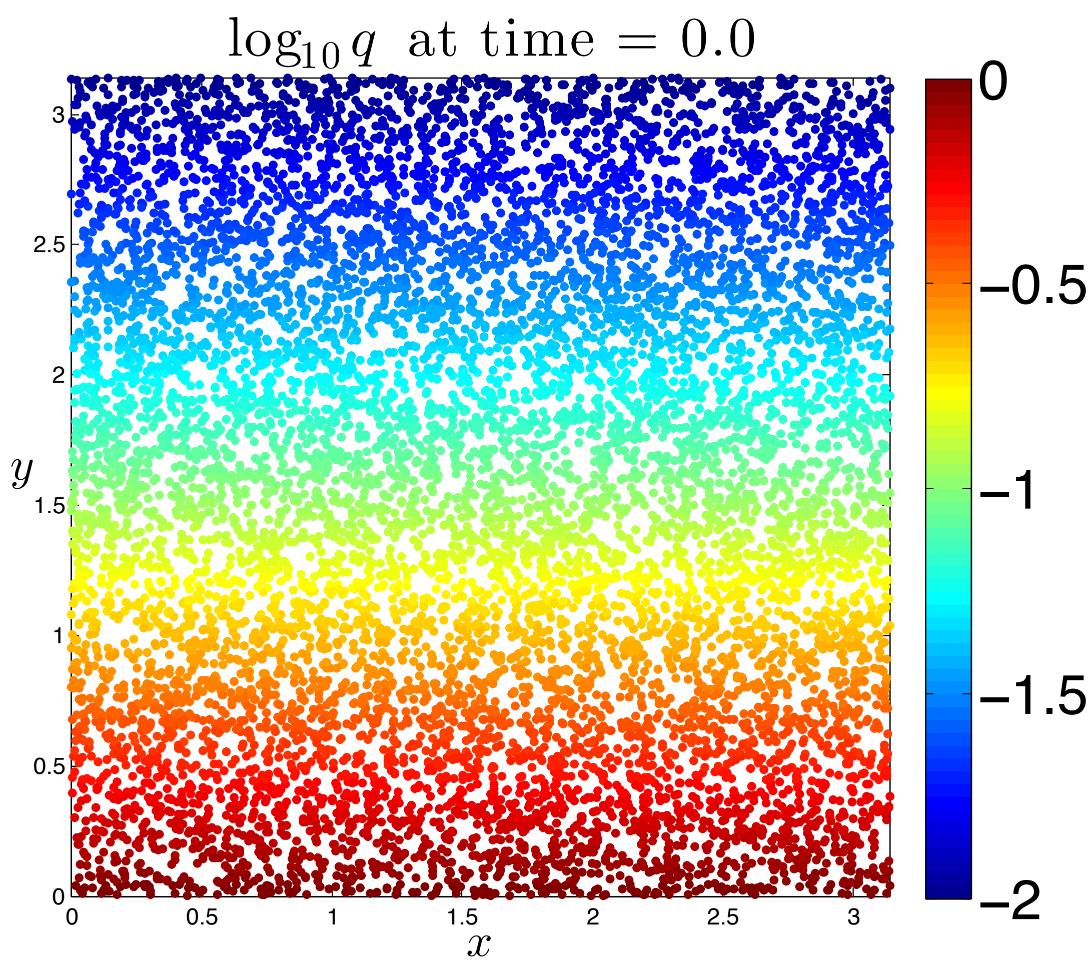
  - $W(t)$ is the Wiener process ($\dot{W}(t)$ = white noise)
- $q_s(y) = q_{\max}\exp(-\alpha y)$: $q_s(0) = q_{\max}$ and $q_s(\pi) = q_{\min}$
- resetting source: $Q = q_{\max}$ if particle hits $y = 0$



Stochastic system with source

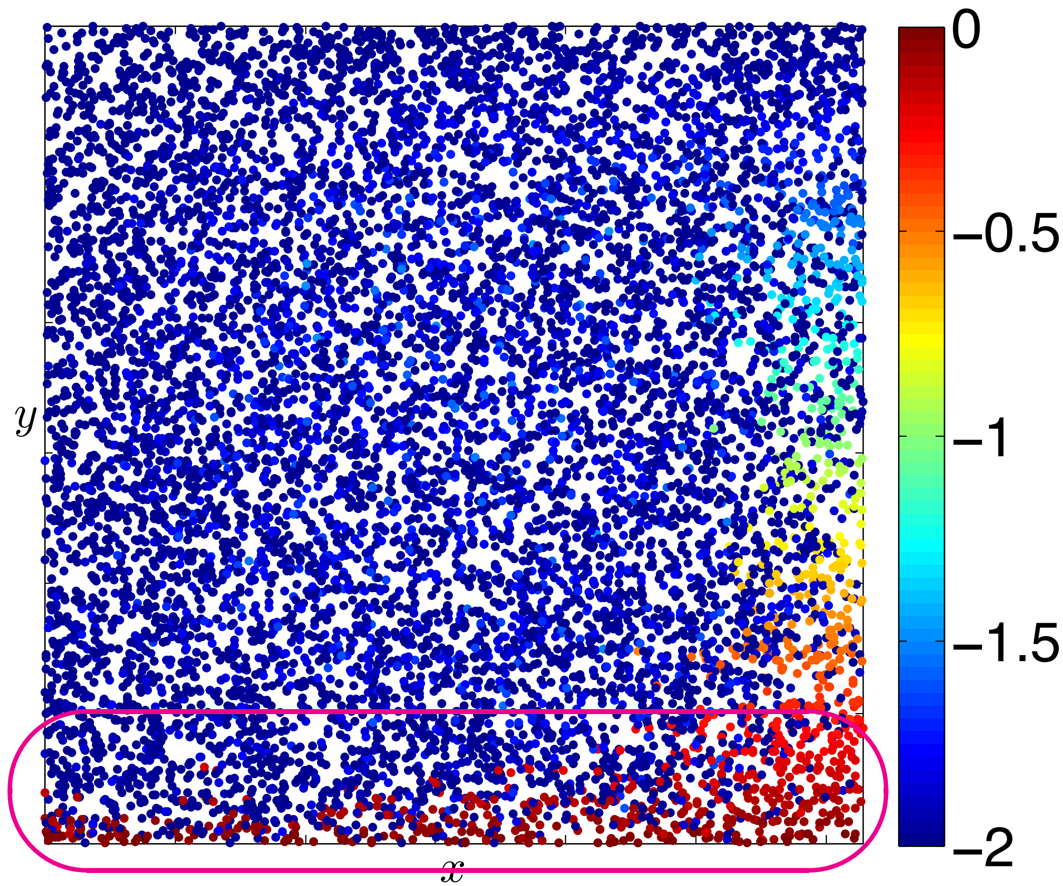
$$\mathrm{d}X(t) = u(X,Y)\,\mathrm{d}t + \sqrt{2\kappa}\,\mathrm{d}W_1(t)$$
$$\mathrm{d}Y(t) = v(X,Y)\,\mathrm{d}t + \sqrt{2\kappa}\,\mathrm{d}W_2(t)$$
$$\mathrm{d}Q(t) = [S(Y) - C(Q,Y)]\mathrm{d}t$$

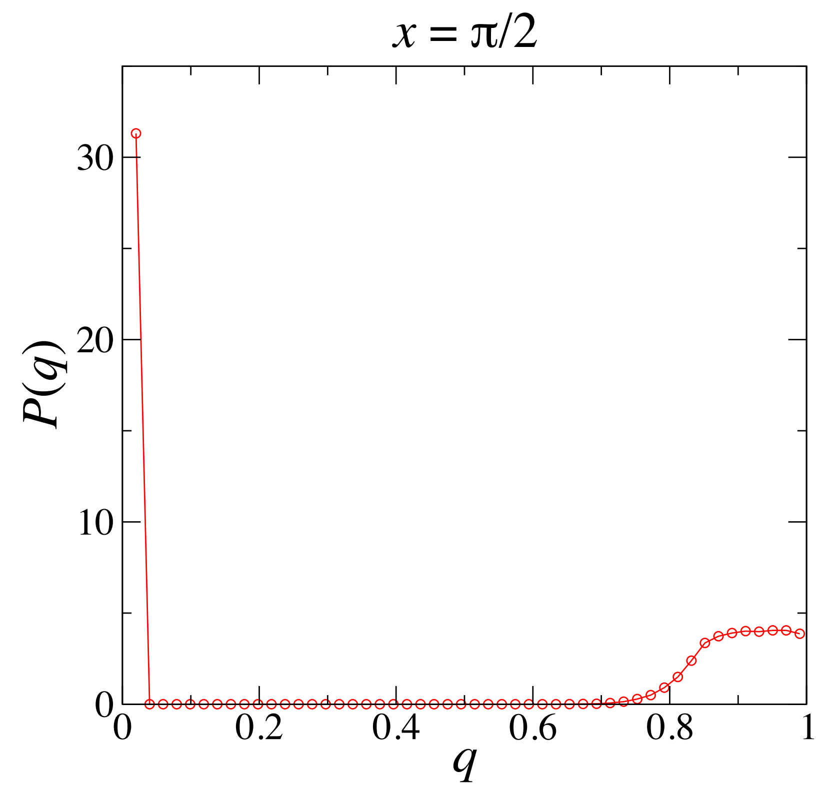
$$\psi = -U \sin x \sin y$$
$$u = -\psi_y$$
$$v = \psi_x$$



$$U = 1$$
$$\kappa = 10^{-2}$$

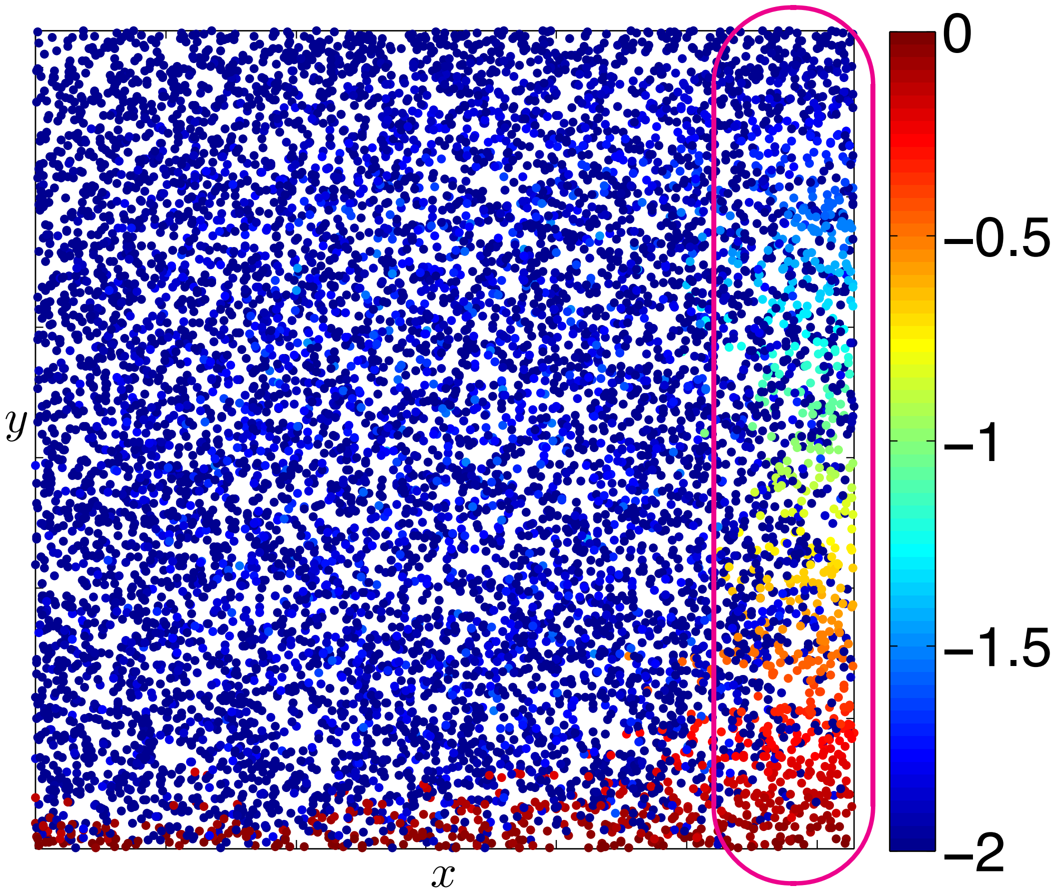
Source boundary layer

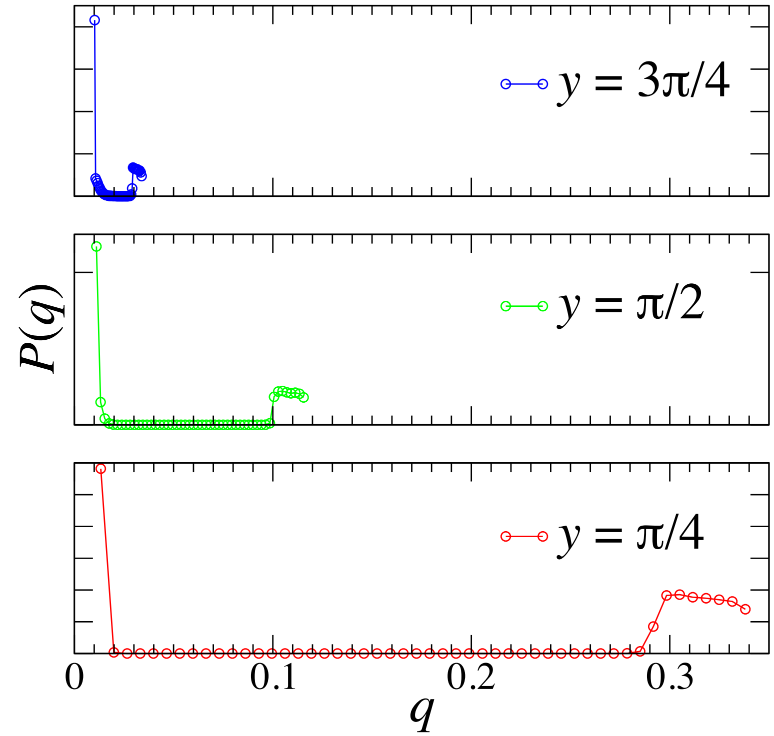




- **Bimodal distribution**: layer consists mainly of either:
  - $Q = q_{\min}$ from upstream of the flow and diffuse in from the domain interior
  - $Q \approx q_{\max}$ from the resetting source
- particles with $Q \approx q_{\max}$ spreading into the domain as x increases

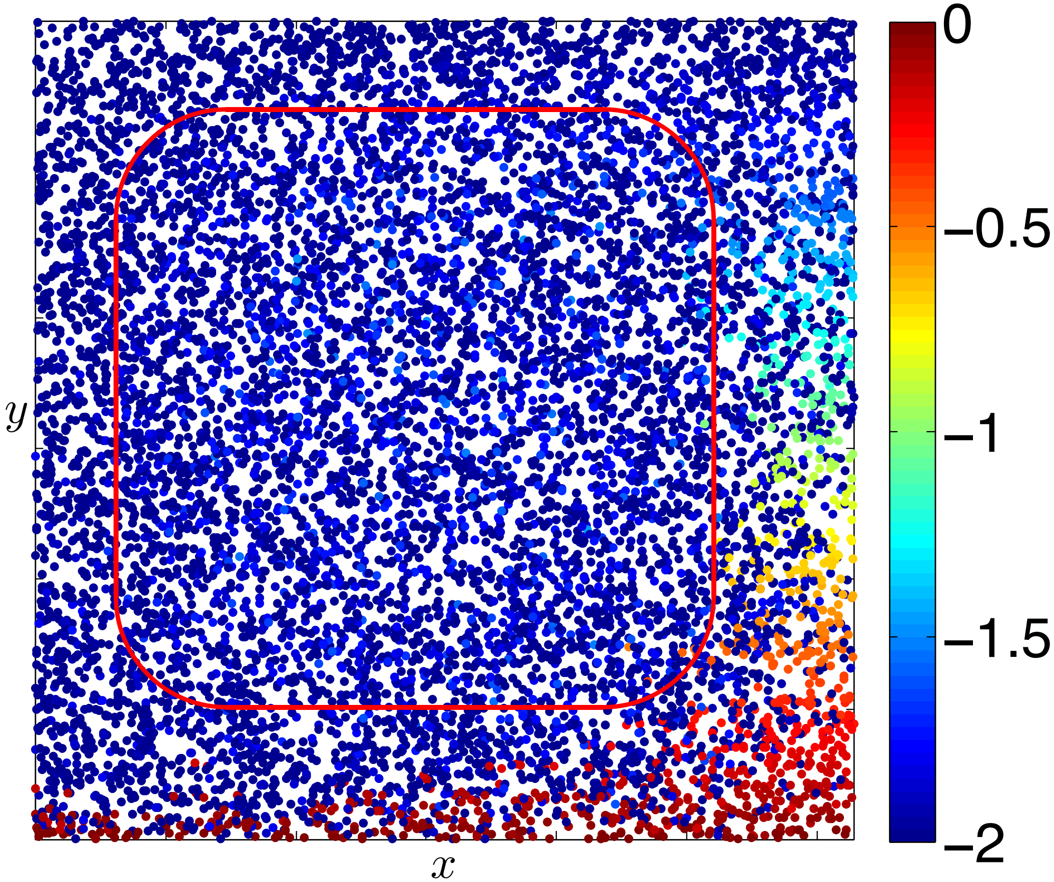
Condensation boundary layer

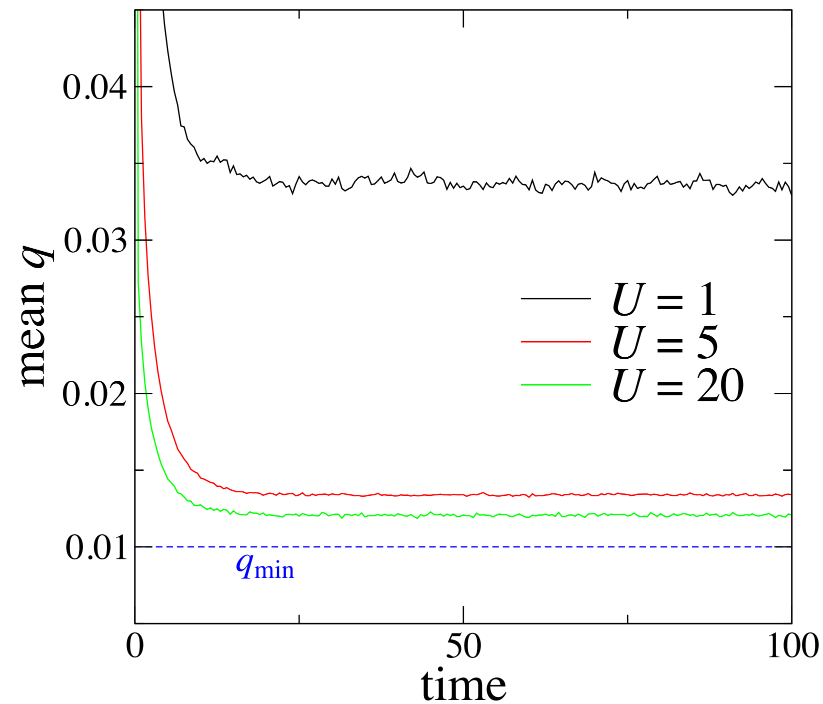




- moist particles move up into region of low $q_s(y)$
- at some fixed height y_1: mainly consists of $Q = q_{\min}$ (diffuse in from the interior) and $Q = q_s(y_1)$ — **Bimodal distribution**
- condensation $\Rightarrow$ localized rainfall over a narrow $O(\epsilon^{1/2})$ region

Interior region





- a homogeneous region of very dry air $Q \approx q_{\min}$ is created in the domain interior
- the vortex "shields" the source from the interior
- interior effectively undergoing stochastic drying

Steady-state problem

Steady-state Fokker-Planck equation for $P(x, y, q)$:

$$\epsilon^{-1}\vec{u} \cdot \nabla P - \partial_q[(S - C)P] = \nabla^2 P, \quad \epsilon = \kappa/(UL) \ll 1$$

Rapid condensation limit:

$$P \neq 0 \text{ for } x, y \in [0, \pi] \text{ and } q \in [q_{\min}, q_s(y)]$$

$$P(x, y, q_s(y)) = 0$$

Resetting boundary source:

$$P(x, 0, q) = \pi^{-1}\delta(q - q_{\max})$$

Hence,

$$\epsilon^{-1}\vec{u} \cdot \nabla P = \nabla^2 P$$

which implies boundary layer of thickness $O(\epsilon^{1/2})$

Matched asymptotics

1. domain interior:

$$P = \pi^{-2} \delta(q - q_{\min})$$

2. source boundary layer:

$$P = G(x, y)\, \delta(q - q_{\min}) + H(q, x, y)$$

3. condensation boundary layer:

$$P = G(x, y)\, \delta(q - q_{\min}) + [\pi^{-2} - G(x, y)]\, \delta(q - q_s(y))$$

In the $O(\epsilon^{1/2})$ boundary layers, introducing coordinates (Childress 1979):

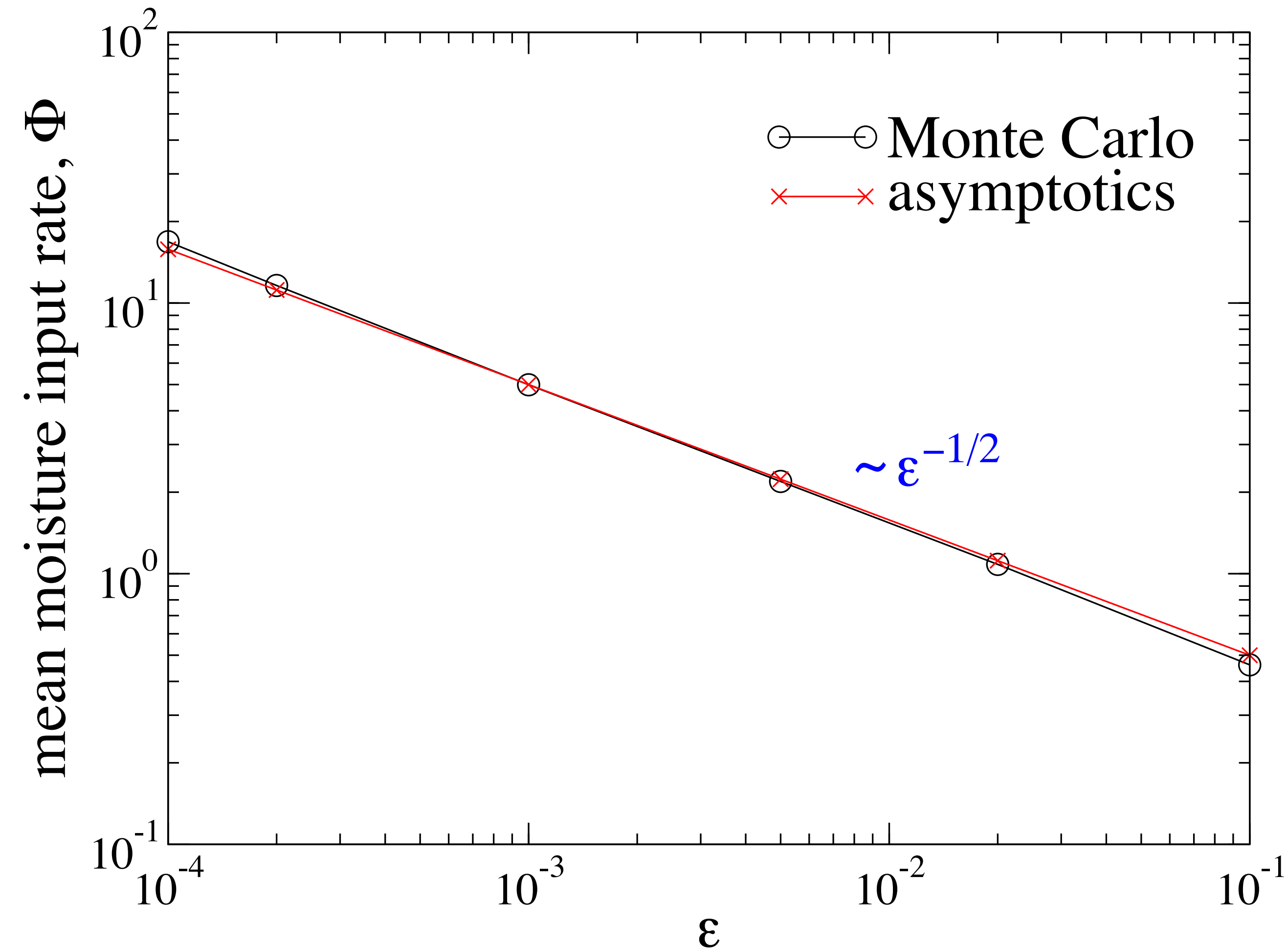
$$\zeta = \epsilon^{-1/2}\psi \quad \text{and} \quad \sigma = \int |\nabla\psi|\, \mathrm{d}l\,, \quad l = \text{arclength}$$

Equation for $G(\sigma, \zeta)$ reduces to:

$$\partial_\sigma G = \partial_{\zeta\zeta} G$$

Mean moisture input rate Φ

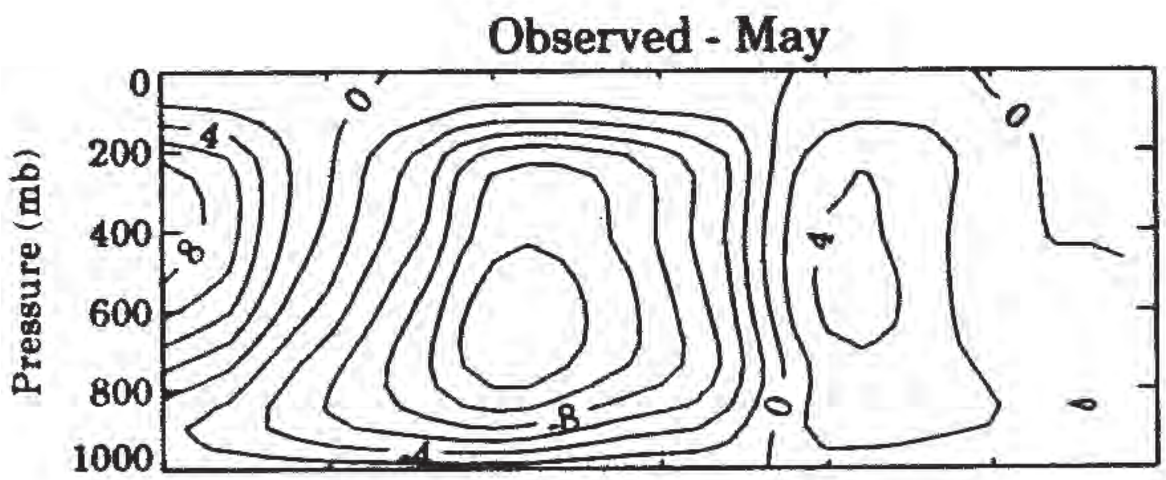
$$\Phi = \epsilon^{-1/2}\sqrt{\frac{8\kappa}{\pi}}(q_{\max} - q_{\min})$$

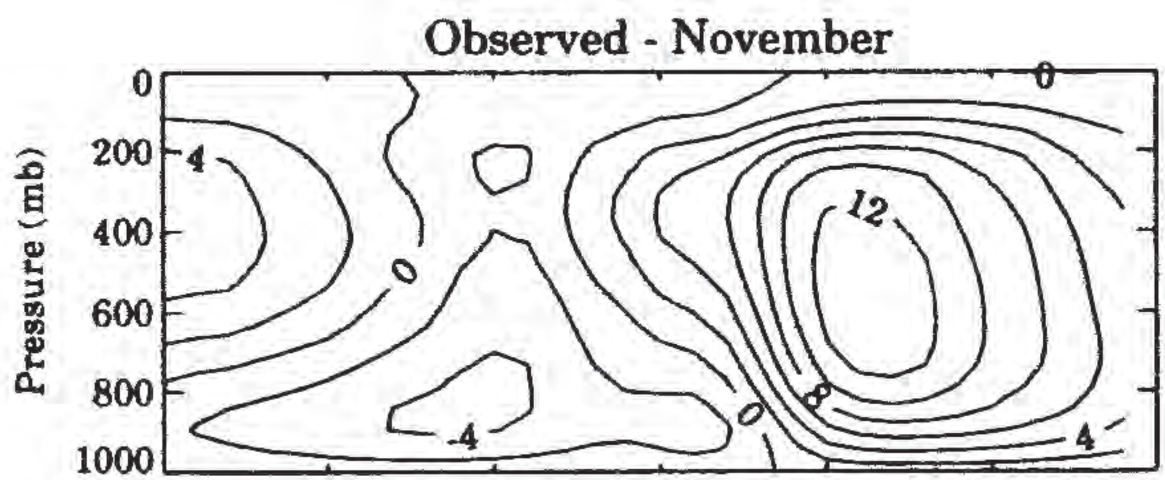


Other diagnostics: horizontal rainfall profile, vertical moisture flux, …etc

Modelling the seasonal variations

The strength and the boundary between the two Hadley cells also undergoes seasonal oscillations *(Oort & Yienger 1996)*

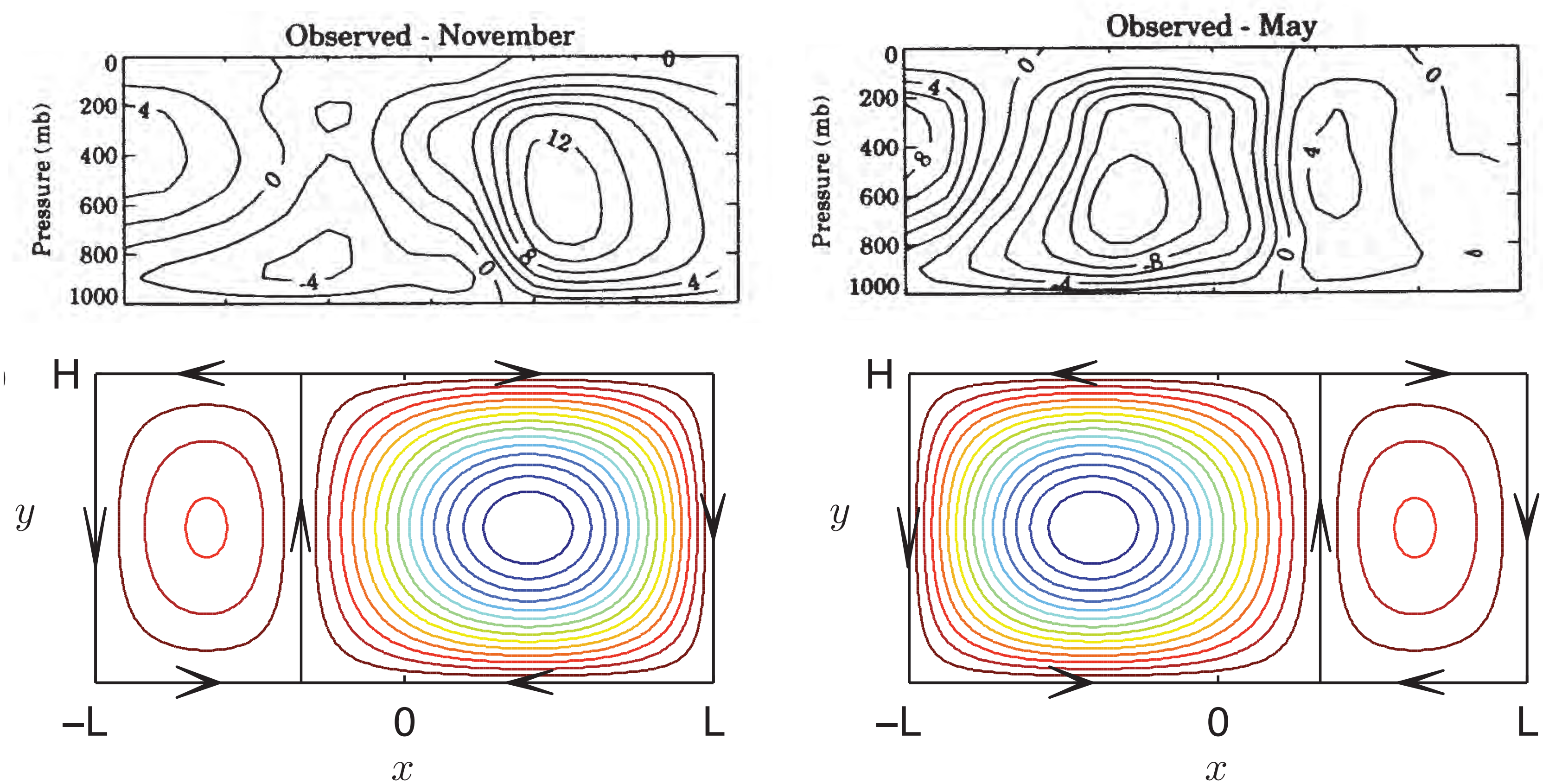




Modelling the seasonal variations

The model of Bowman & Cohen *(J. Atoms. Sci., 1997)* captures 78% of the variance in the observed climatological monthly mean Hadley circulation:

$$\psi(\vec{x},t) = UL\left[\sin(\frac{\pi x}{L})\sin(\frac{\pi y}{H}) + \delta\cos(\frac{\pi x}{2L})\sin(\frac{\pi y}{H})\sin(\frac{2\pi t}{\tau})\right]$$



A simplified moist GCM

The model of Frierson et al. *(J. Atmos. Sci., 2006)*:

- moisture does not affect radiative transfer
- a simple diffusive boundary layer
- mixed-layer ocean boundary condition: a slab of water of specified heat capacity with no horizontal transport.

A model at this complexity level is more controllable than a full atmospheric model and thus makes a better connection to idealised mathematical models.