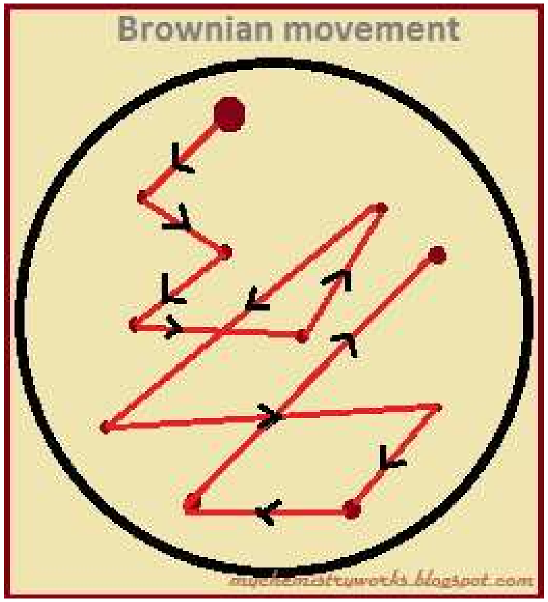
Particle diffusion in strong field-guided magnetohydrodynamic turbulence

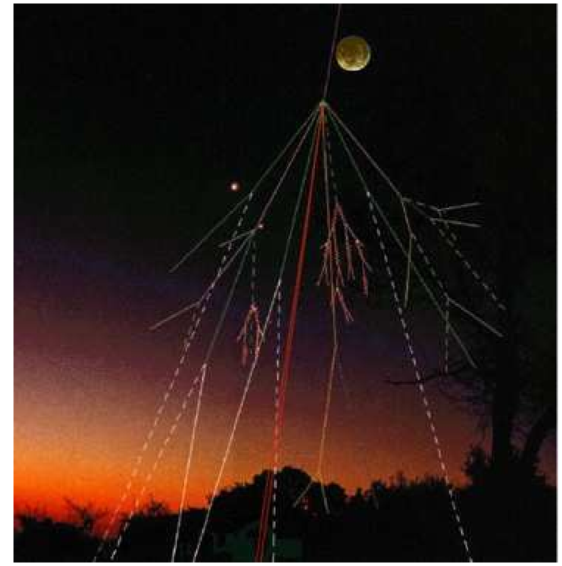
Yue-Kin Tsang

Centre for Astrophysical and Geophysical Fluid Dynamics
Mathematics, University of Exeter

Joanne Mason

Particle transport in fluids



- Brownian motion observed under the microscope
- dispersion of pollutants in the atmosphere
- cosmic ray propagation through the interstellar medium
- tracing particle trajectories gives alternative view of the structure of the fluid flow — the Lagrangian viewpoint

Single-particle turbulent diffusion

- mean squared displacement:

$$\langle |\Delta\vec{X}(t)|^2\rangle\,, \quad \Delta\vec{X}(t) = \vec{X}(t) - \vec{X}(0)$$

- Taylor's formula (1921) for large t:

$$\vec{X}(t) = \vec{X}(0) + \int_0^t \mathrm{d}\tau\, \vec{V}(\tau)$$

$$\langle |\Delta\vec{X}(t)|^2]\rangle = 2\,t \int_0^\infty \mathrm{d}\tau\, \langle \vec{V}(\tau)\cdot\vec{V}(0)\rangle = 2tD$$

assume system is homogeneous and stationary and the integral exists

- Lagrangian velocity correlation:

$$C_L(\tau) = \langle \vec{V}(\tau)\cdot\vec{V}(0)\rangle$$

- diffusion coefficient:

$$D = \int_0^\infty \mathrm{d}\tau\, \langle \vec{V}(\tau)\cdot\vec{V}(0)\rangle$$

MHD turbulence

- Motion of a electrically conducting fluid:

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{1}{\rho_0}\nabla p + (\nabla \times \vec{B}) \times \vec{B} + \nu \nabla^2 \vec{u} + \vec{f}$$

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$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{u} \times \vec{B}) + \eta \nabla^2 \vec{B}$$

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$\vec{f}$: random forcing at the largest scales

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- Evolution of passive tracer particles:

$$\frac{\mathrm{d}\vec{X}(t)}{\mathrm{d}t} = \vec{u}(\vec{X}(t), t) = \vec{V}(t)$$

$$\vec{X}(0) = \vec{\alpha}$$

- Field-guided MHD turbulence:

$$\vec{B}(\vec{x}, t) = B_0 \hat{z} + \vec{b}(\vec{x}, t)$$

Previous work: the 2D case

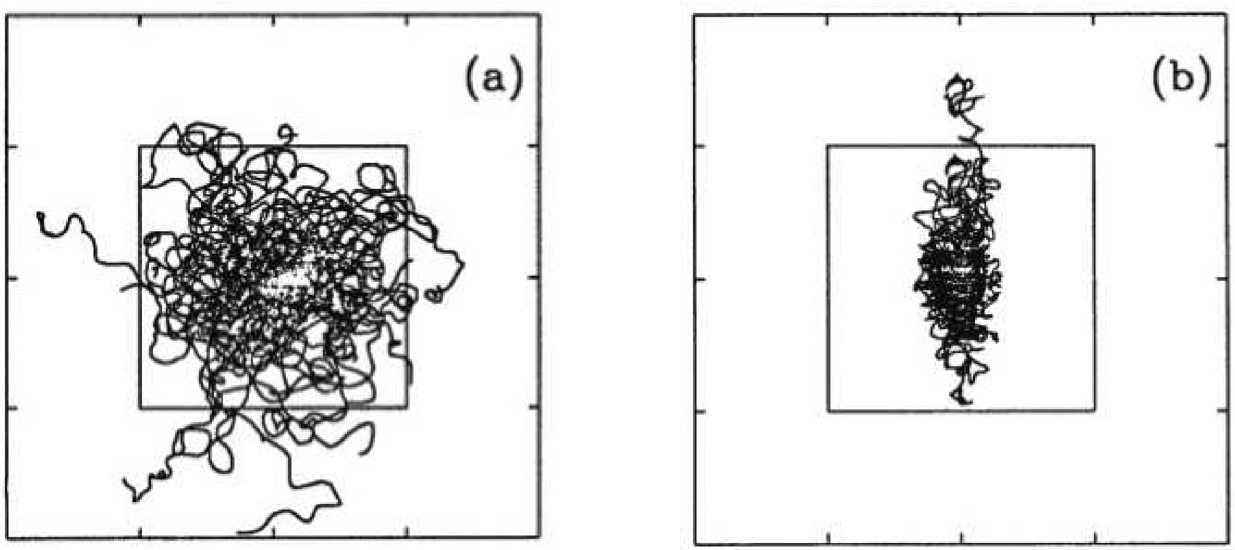
ON THE EFFECTS OF A WEAK MAGNETIC FIELD ON TURBULENT TRANSPORT

FAUSTO CATTANEO
Department of Astronomy and Astrophysics, University of Chicago, 5640 South Ellis Avenue, Chicago, IL 60637
Received 1993 August 19; accepted 1994 April 22

ABSTRACT

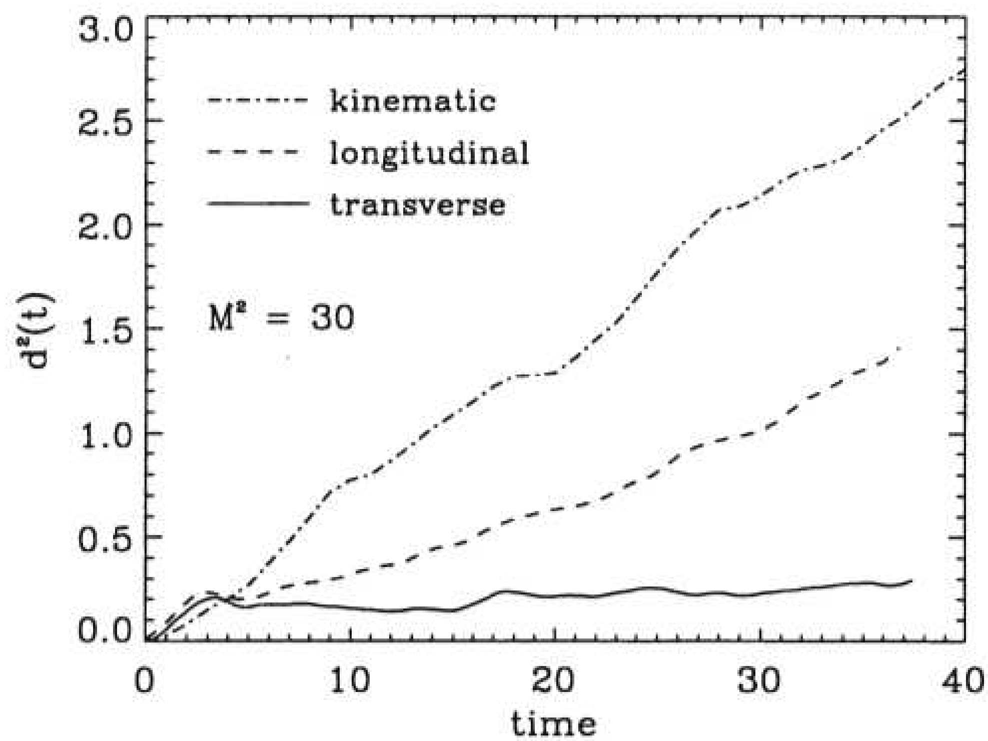
We discuss the effects of a weak large-scale magnetic field on turbulent transport. We show by means of a series of two-dimensional numerical experiments that turbulent diffusion can be effectively suppressed by a (large-scale) magnetic field whose energy is small compared to equipartition. The suppression mechanism is associated with a subtle modification of the Lagrangian energy spectrum, and it does not require any substantial reduction of the turbulent amplitude. We exploit the relation between diffusion and random walking to emphasize that the effect of a large-scale magnetic field is to induce a long-term memory in the field of turbulence. The implications for the general case of three-dimensional transport are briefly discussed.
Subject headings: diffusion — MHD — turbulence

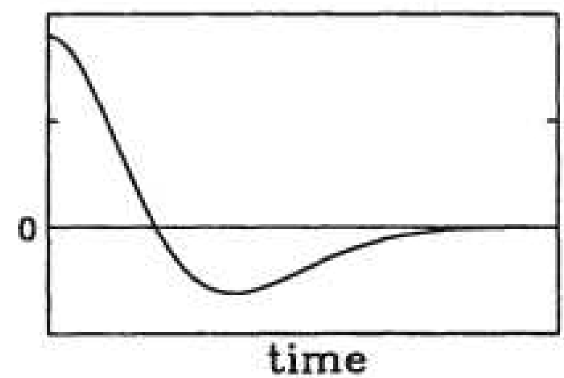
1. transport suppressed in direction $\perp$ to $B_0\hat{y}$ when $B_0 > B_0^*$



Previous work: the 2D case

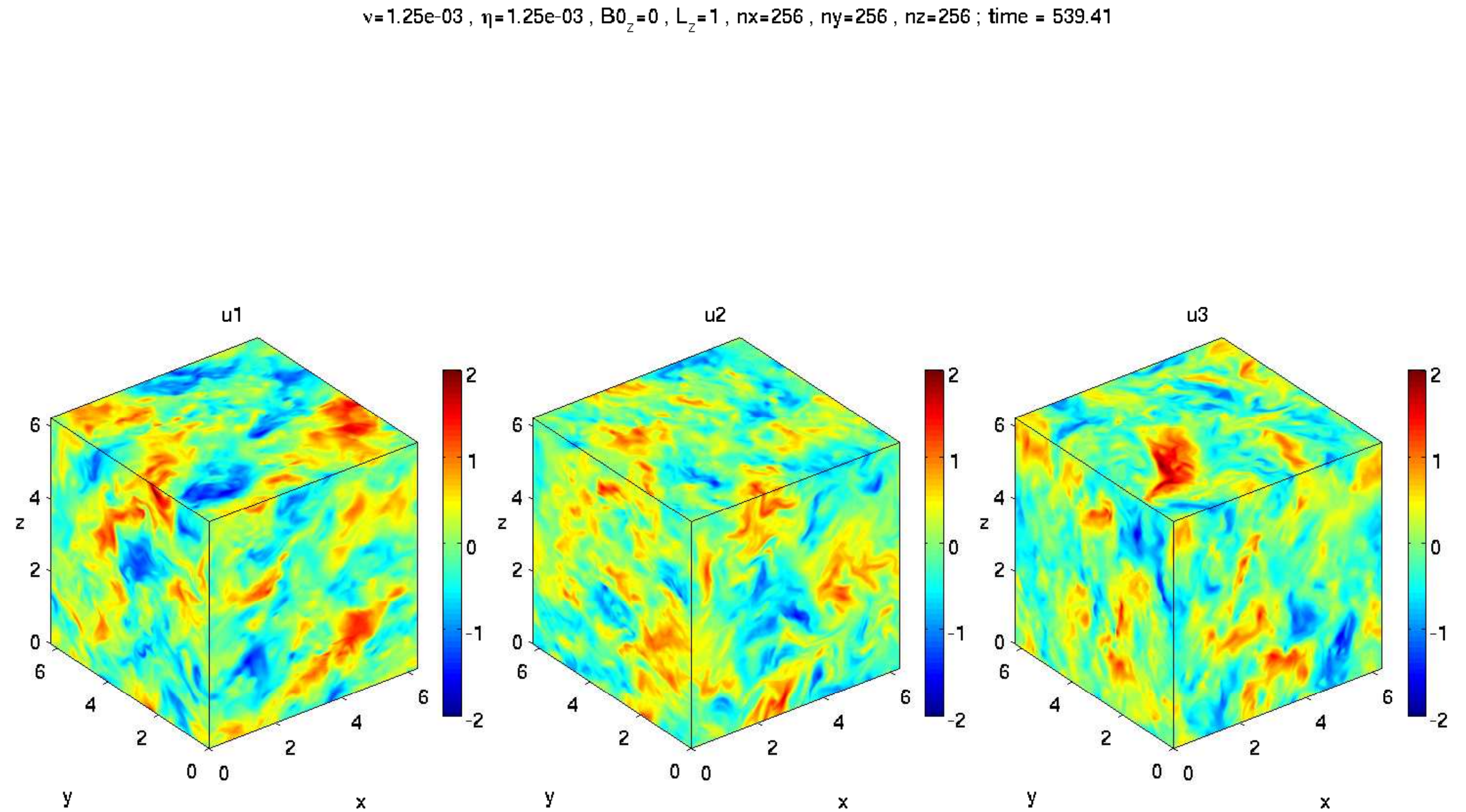
2. as $Re_m = UL/\eta$ increases, the critical B_0^* decreases
3. the system has long-term memory: slow decay of $C_L(\tau)$



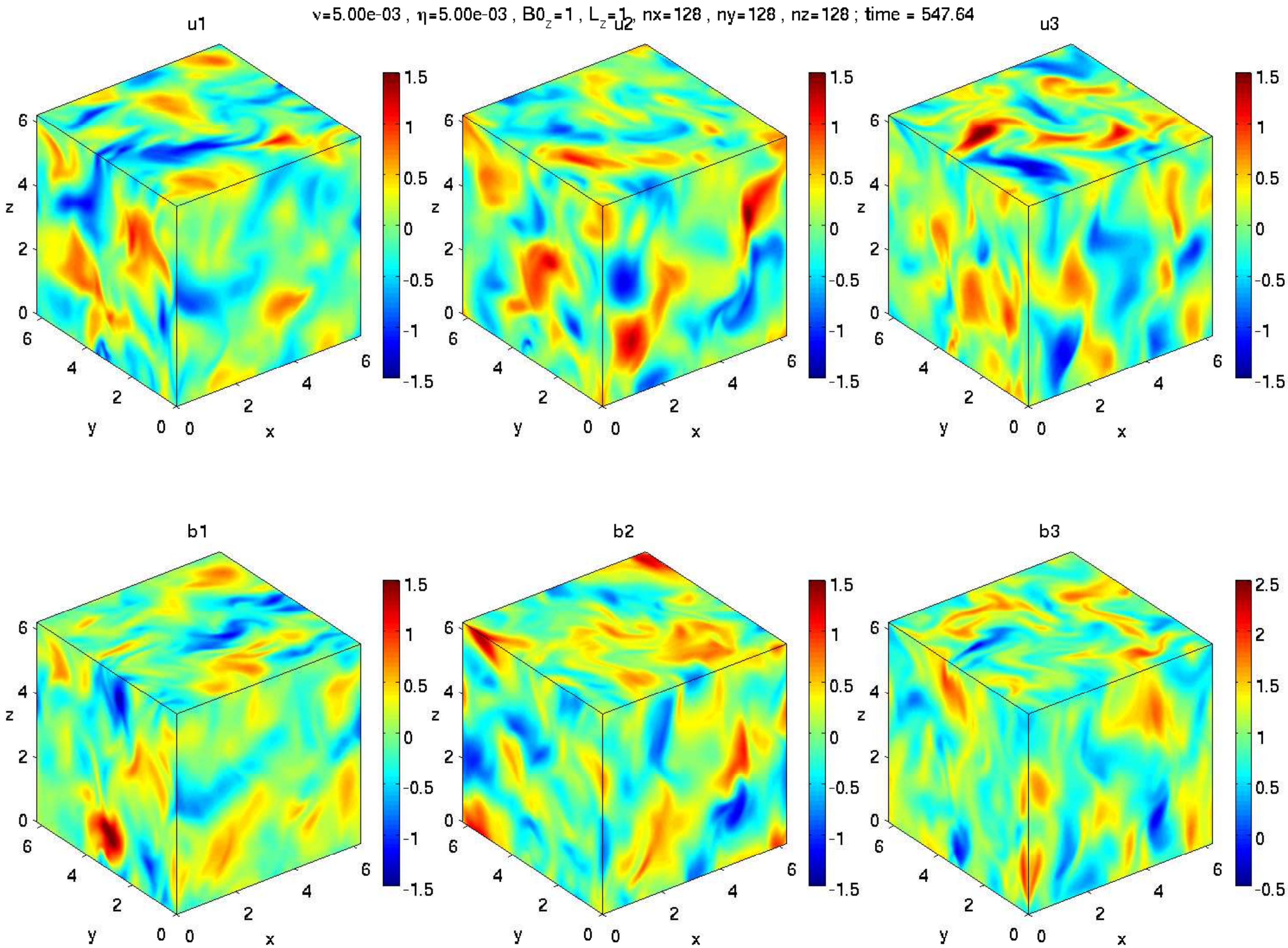


- Whether such suppression of turbulent diffusion occurs in 3D is not clear. (Cattaneo, Vainshtein: ApJ91,94; Gruzinov, Diamond: POP96, PRL97)

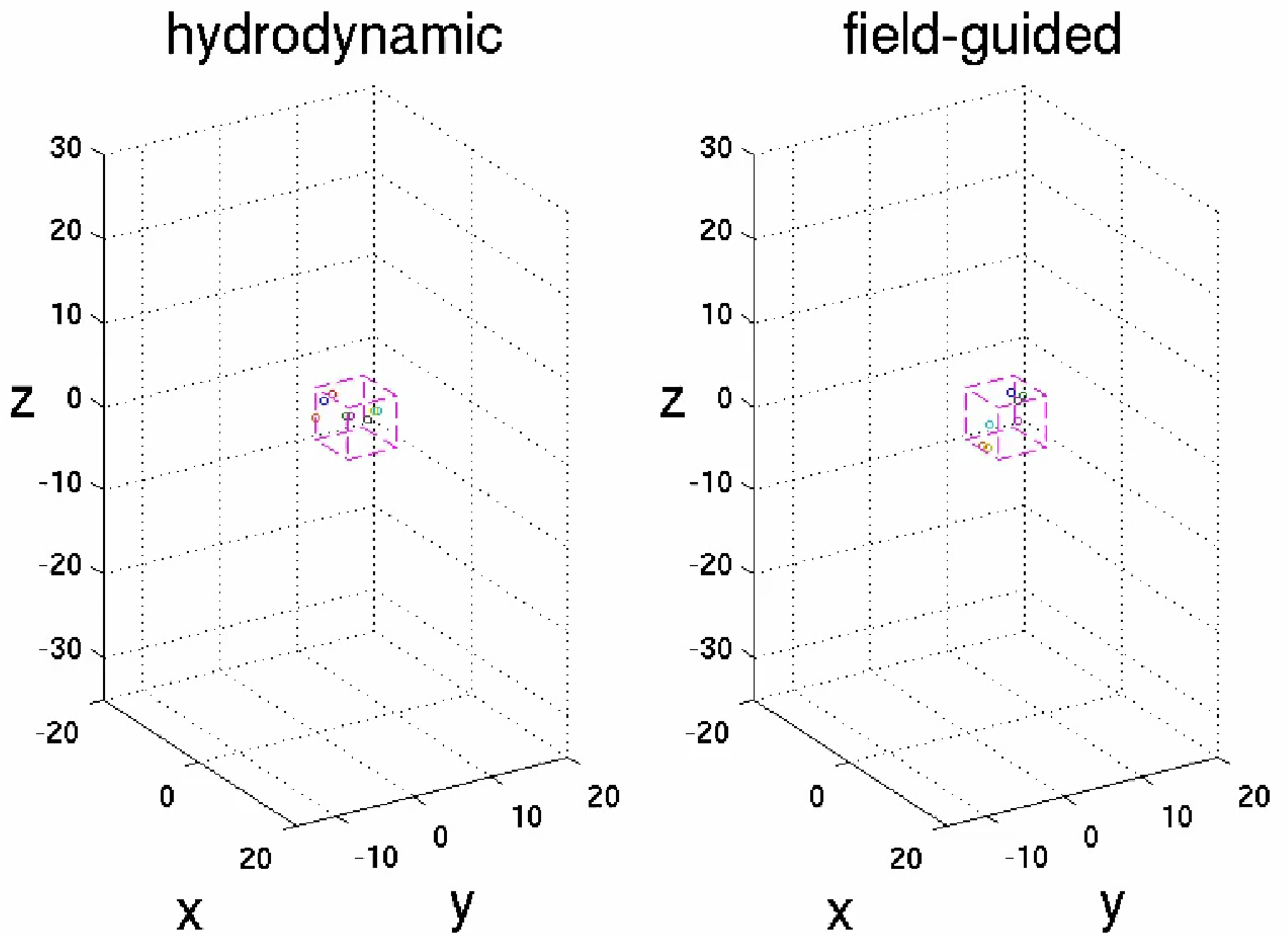
The hydrodynamic case, $\vec{B} = 0$



The field-guided case, $\vec{B} = B_0\hat{z}$



Particle tracking



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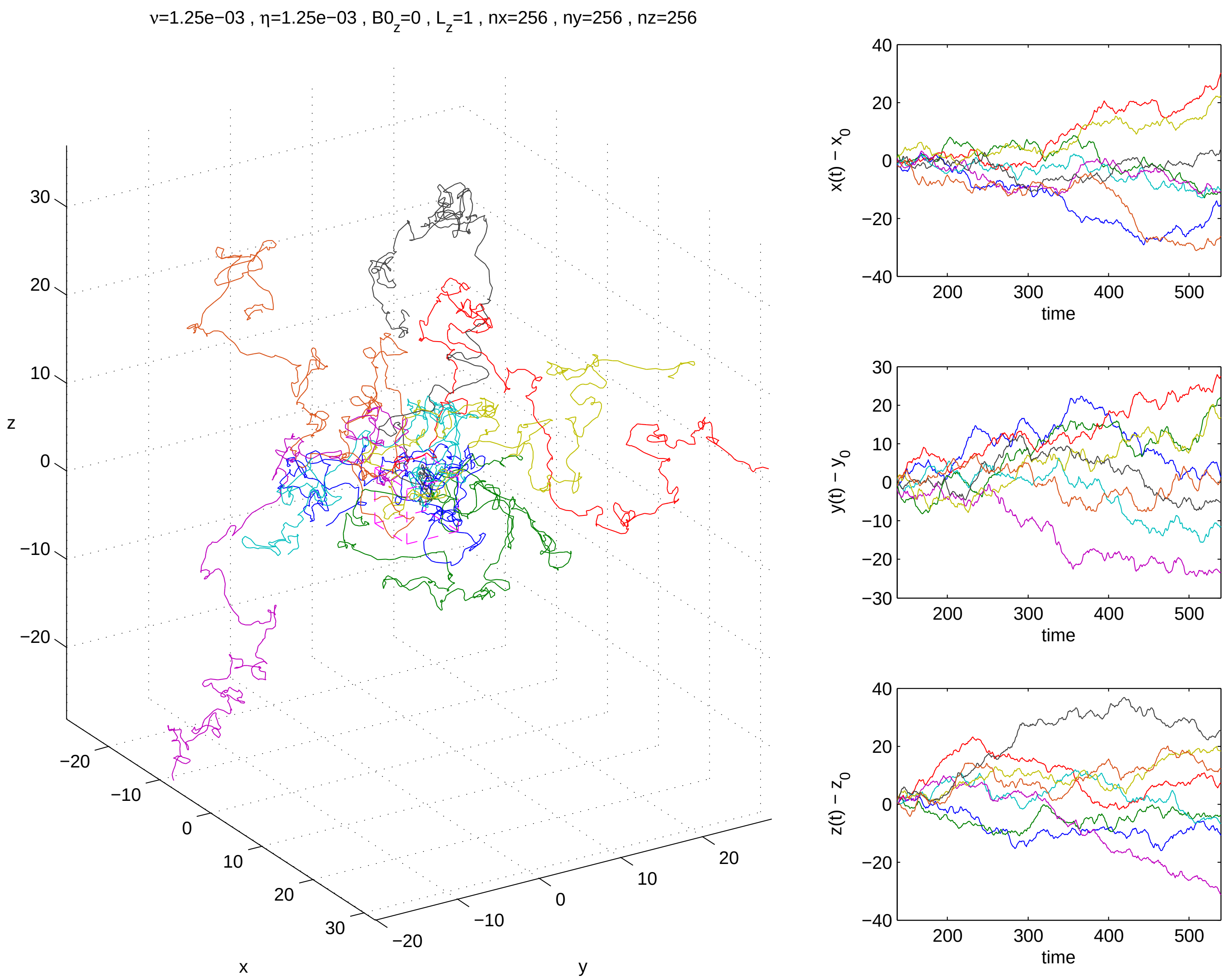
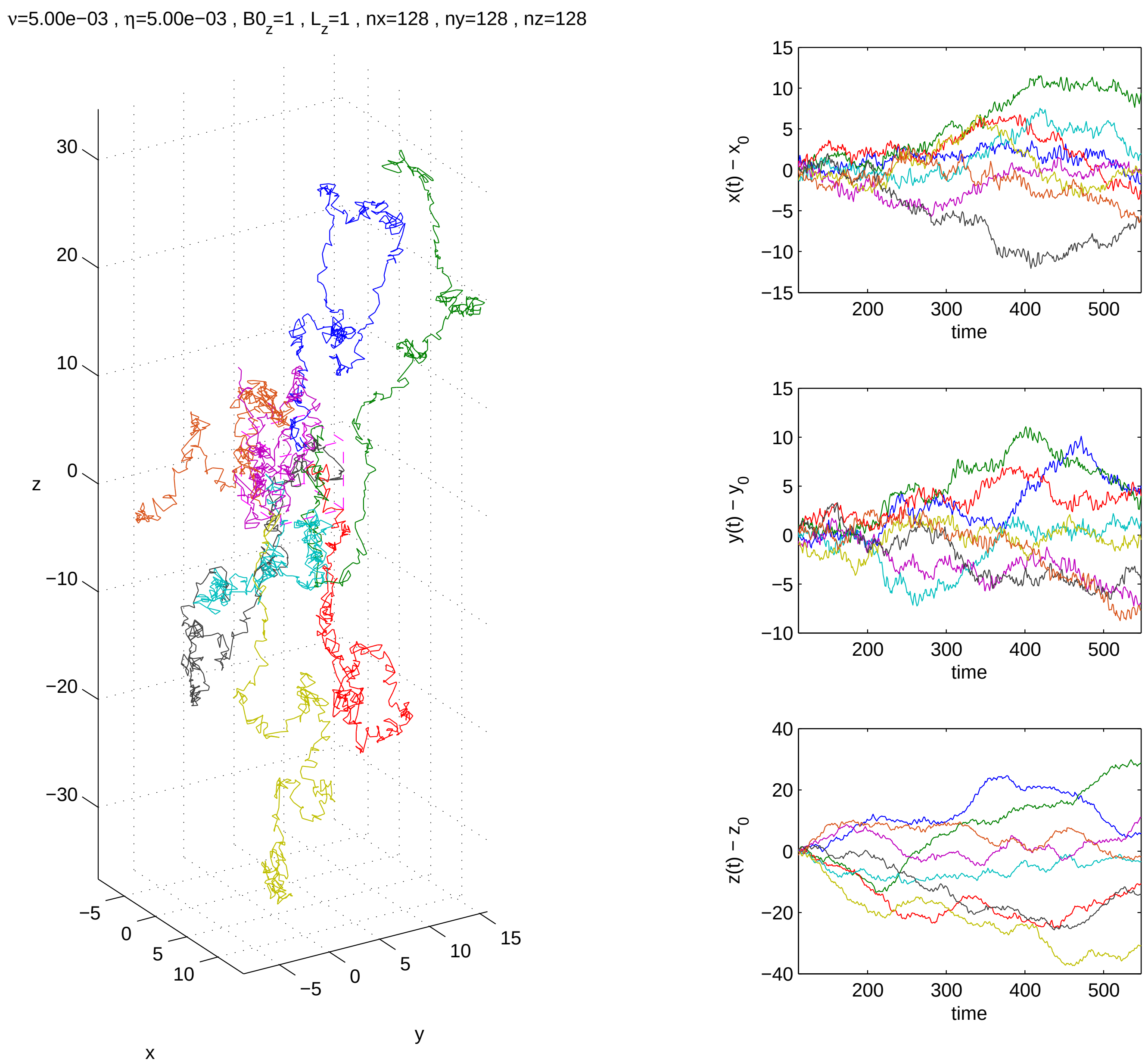
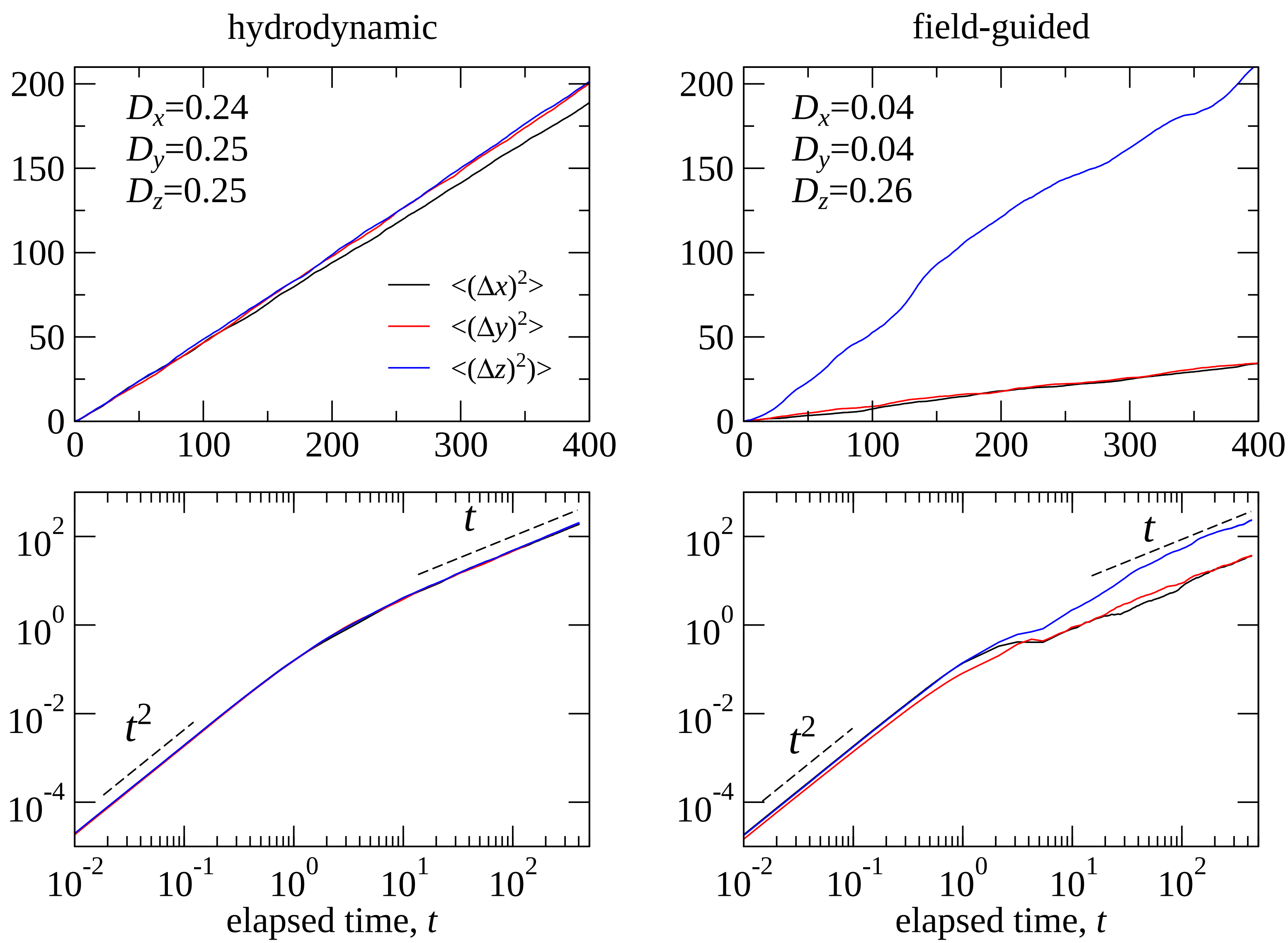


The field-guided case, $\vec{B} = B_0 \hat{z}$



- transport suppressed in the field-perpendicular direction!

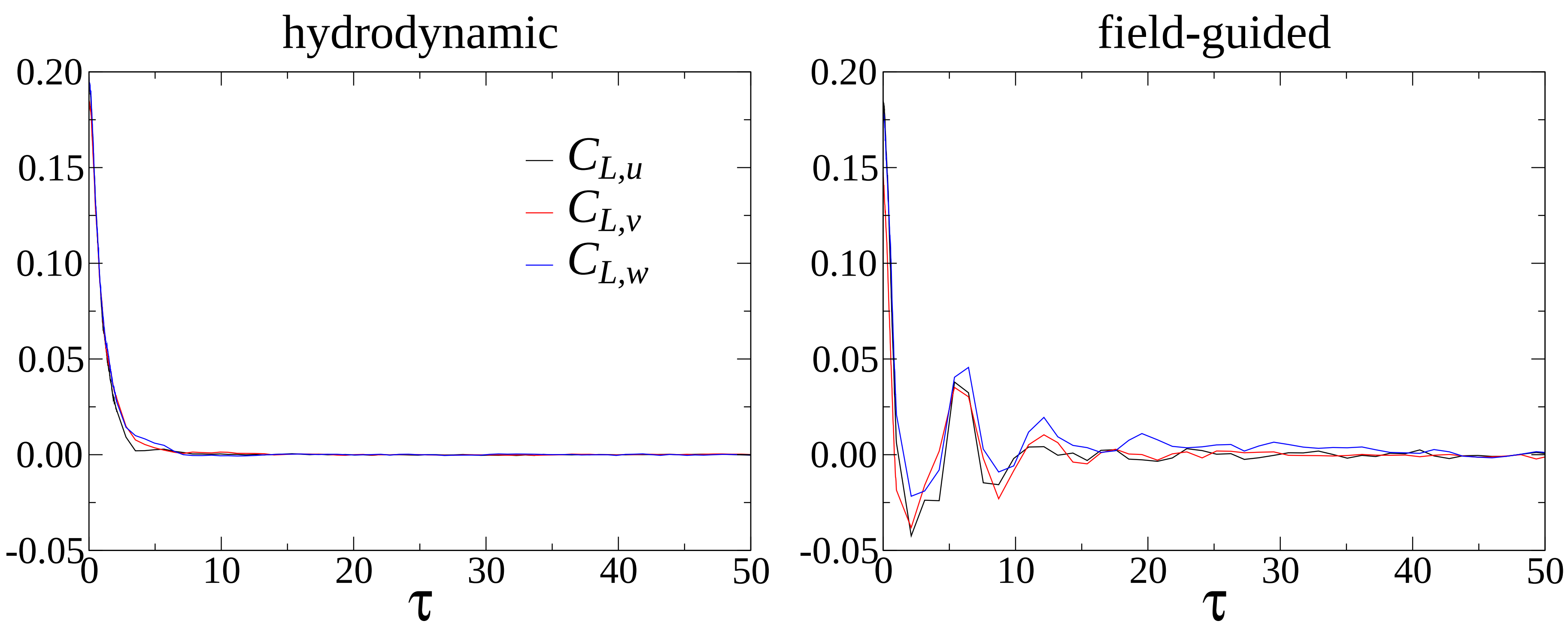
Scaling of mean-squared displacement



- ballistic limit: $\sim t^2$ at small time
- diffusive scaling: $\sim t$ at large time, $\langle(\Delta x)^2\rangle \sim 2D_x t$, etc

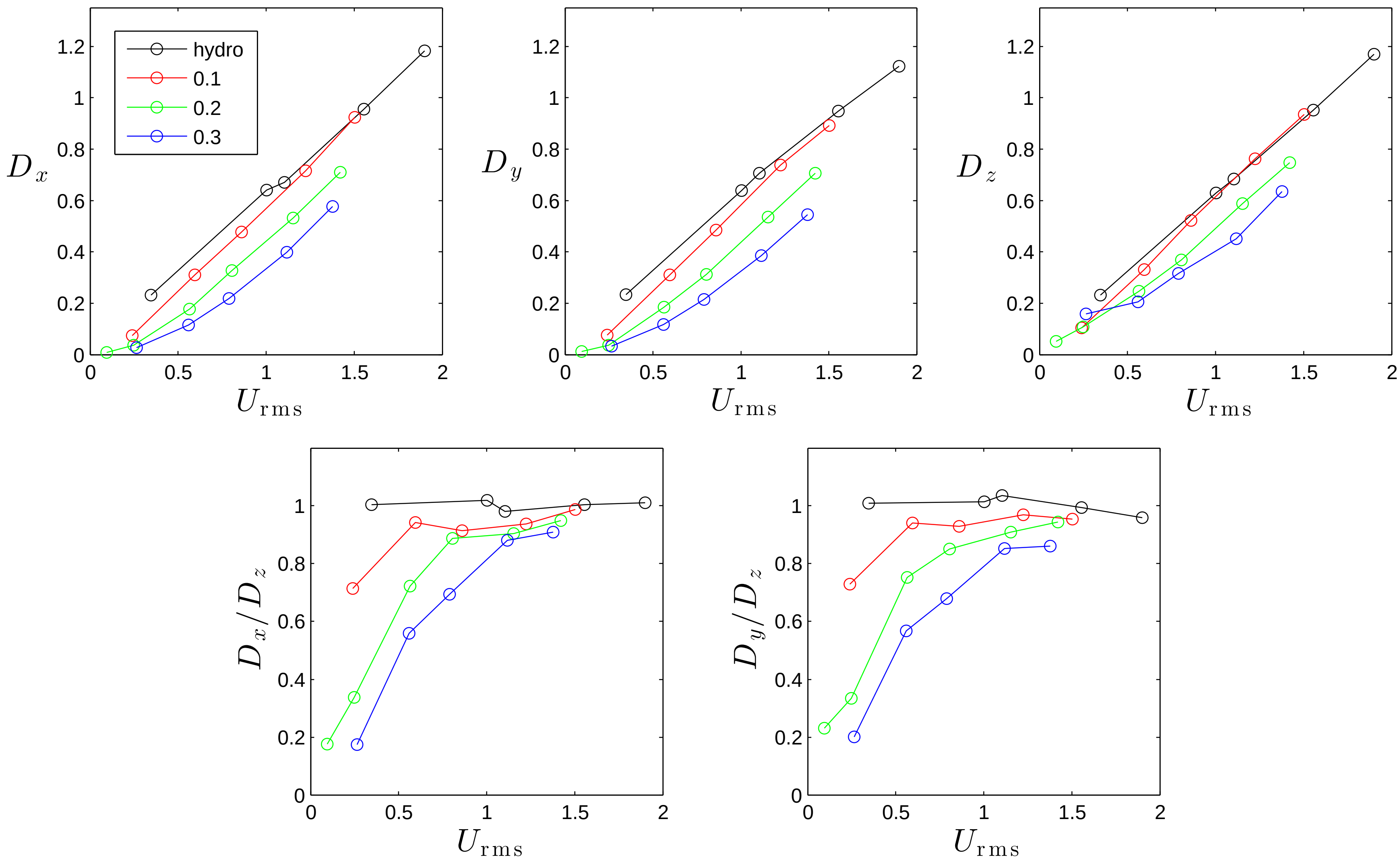
Lagrangian velocity correlation function

$$C_L(\tau) = \langle \vec{V}(\tau) \cdot \vec{V}(0) \rangle$$



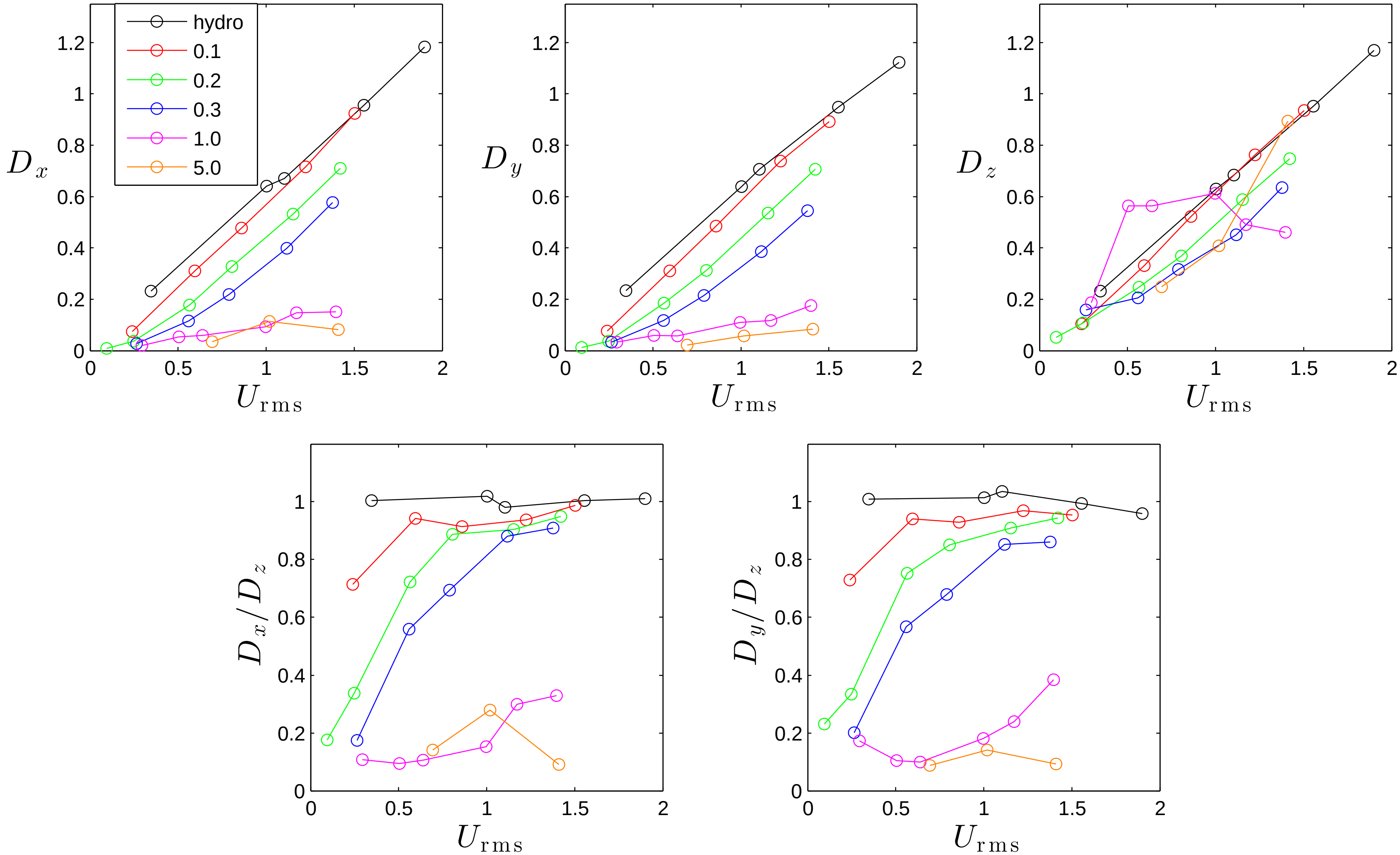
- hydrodynamic: $\sim \exp(-\tau)$, short correlation time
- field-guided: oscillatory, long correlation time
- how things depend on the guided-field strength B_0?

Diffusivity at different (weak) $B_{0z} \lesssim U_{\rm rms}$



- diffusion is reduced by B_{0z}, including the z-direction
- anisotropic suppression: $D_x, D_y \lesssim D_z$
- strong $U_{\rm rms}(\gtrsim B_{0z})$ reduces the anisotropy in D's

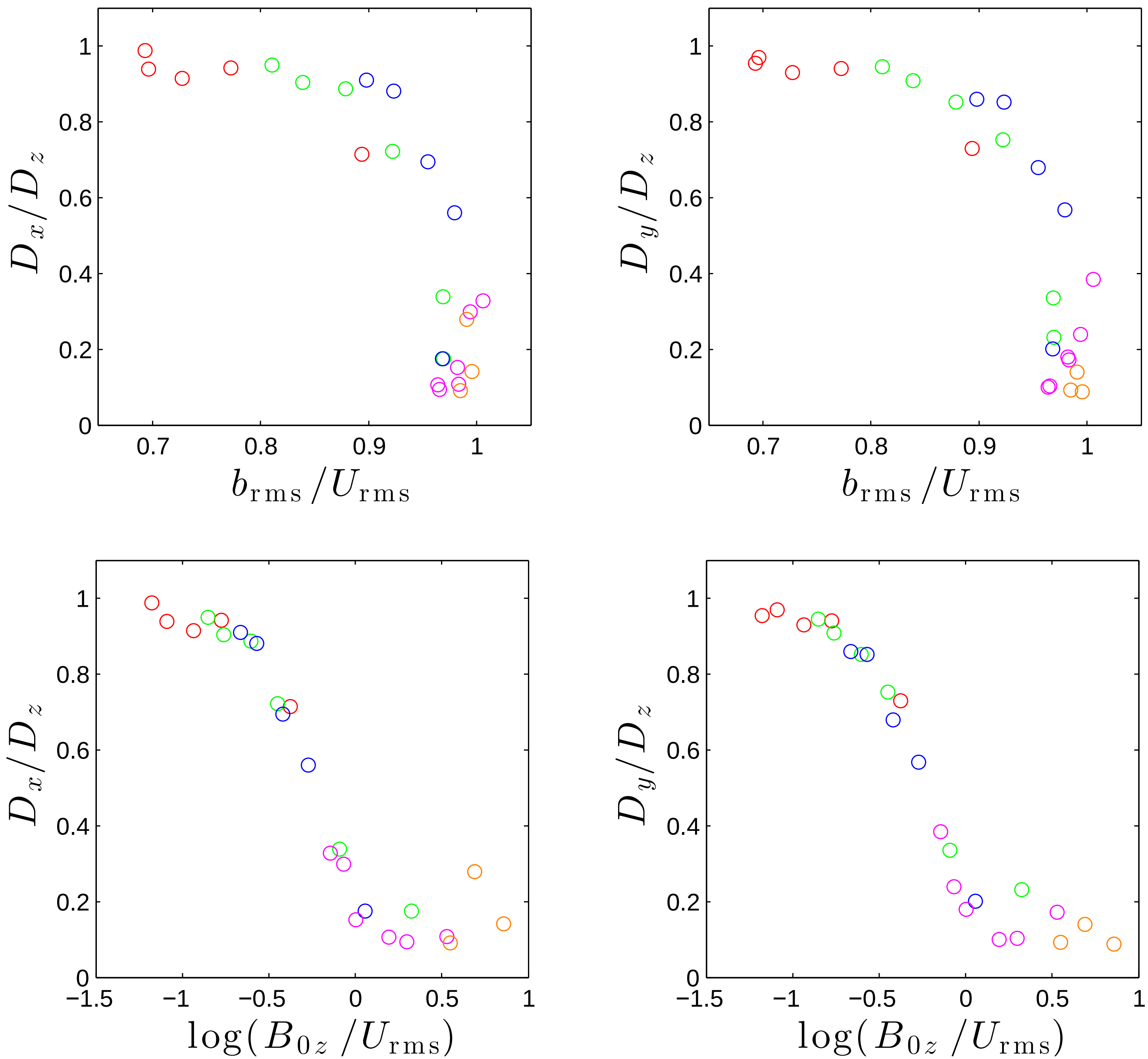
Diffusivity at different B_{0z}



At strong guided-field strength, $B_{0z} \gtrsim U_{\rm rms}$

- D_x, D_y are strong suppressed, anomalous behavior of D_z
- D_x/D_z , $D_y/D_z \ll 1$ for the values of $U_{\rm rms}$ studied

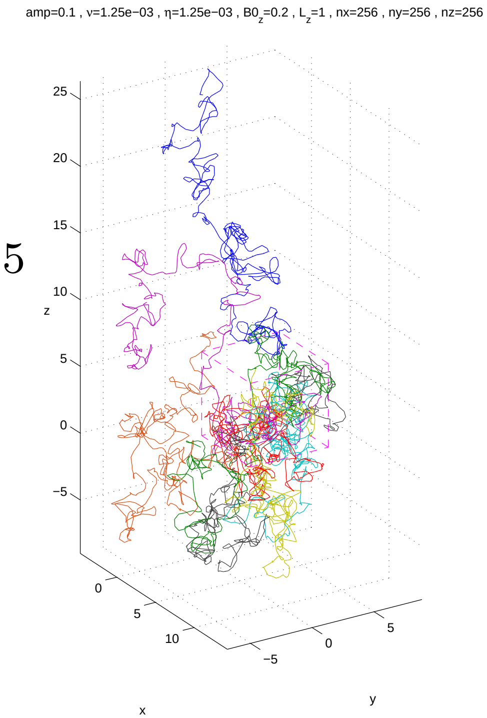
Anisotropic turbulent diffusion

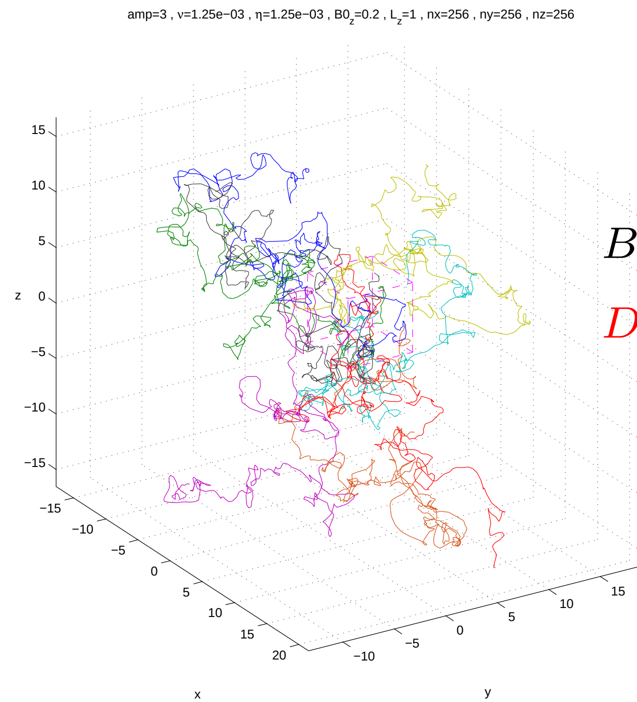


Particle trajectories

$B_{0z} = 0.2, U_{\rm rms} = 0.25$

$D_x/D_z = 0.34$



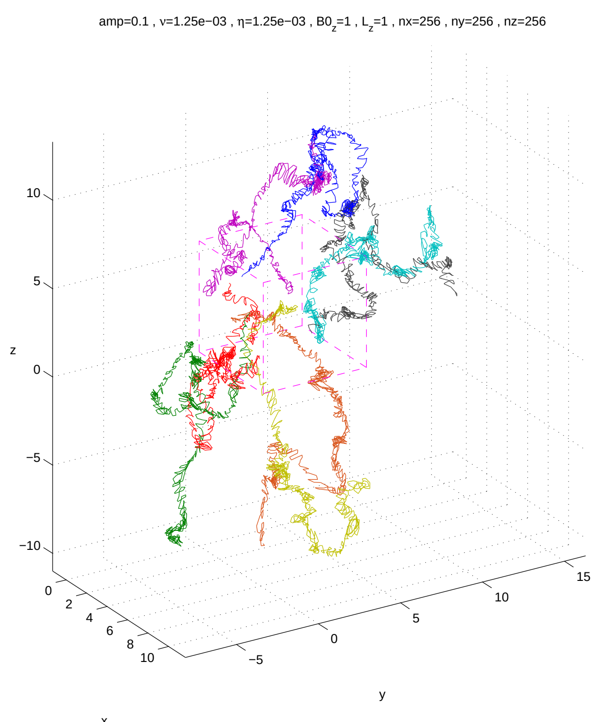


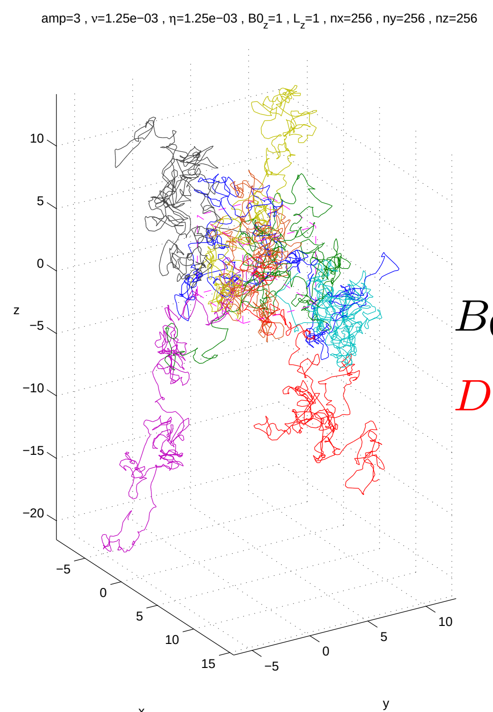
$B_{0z} = 0.2, U_{\rm rms} = 1.42$

$D_x/D_z = 0.95$

$B_{0z} = 1.0, U_{\rm rms} = 0.29$

$D_x/D_z = 0.24$

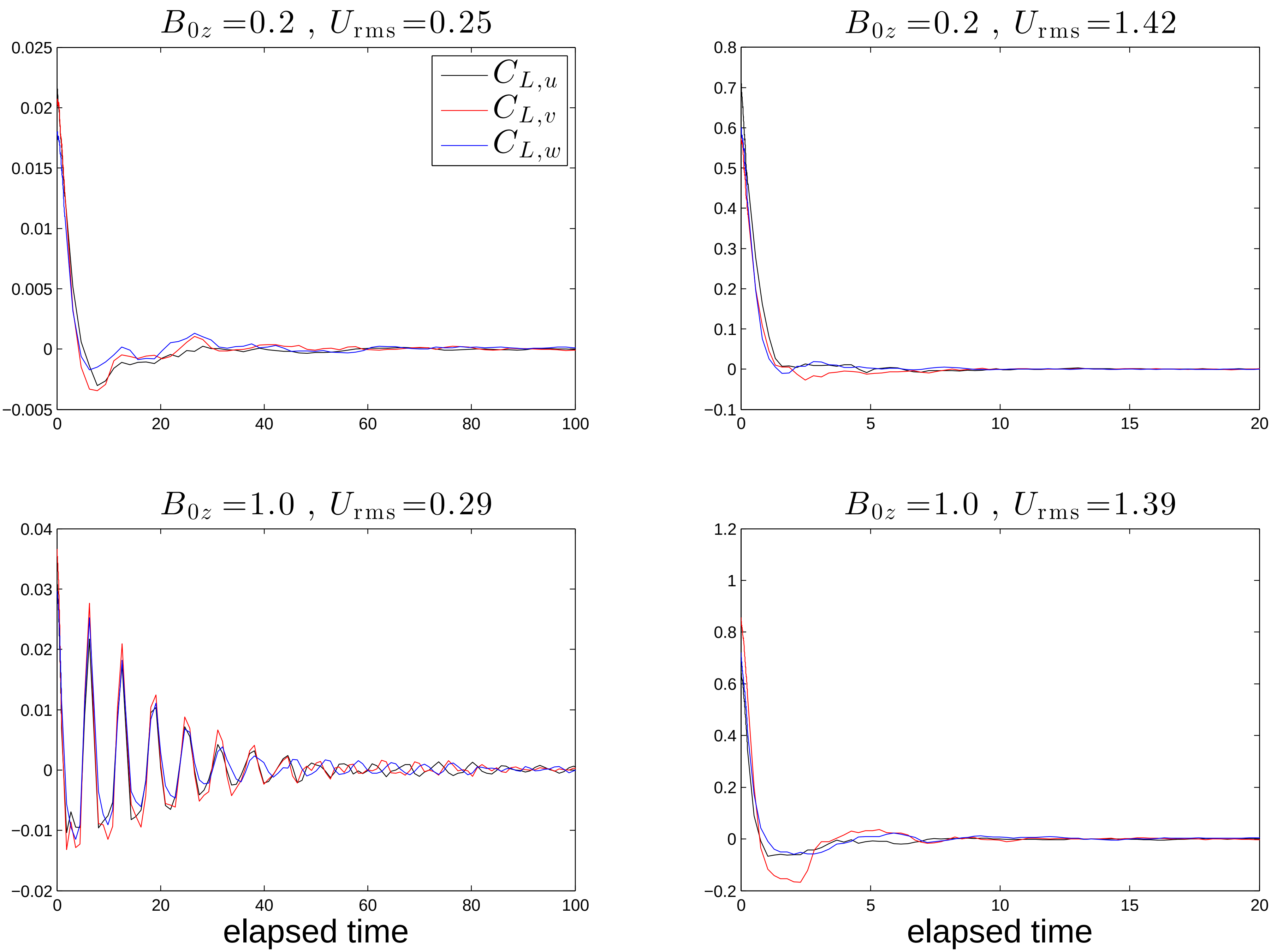




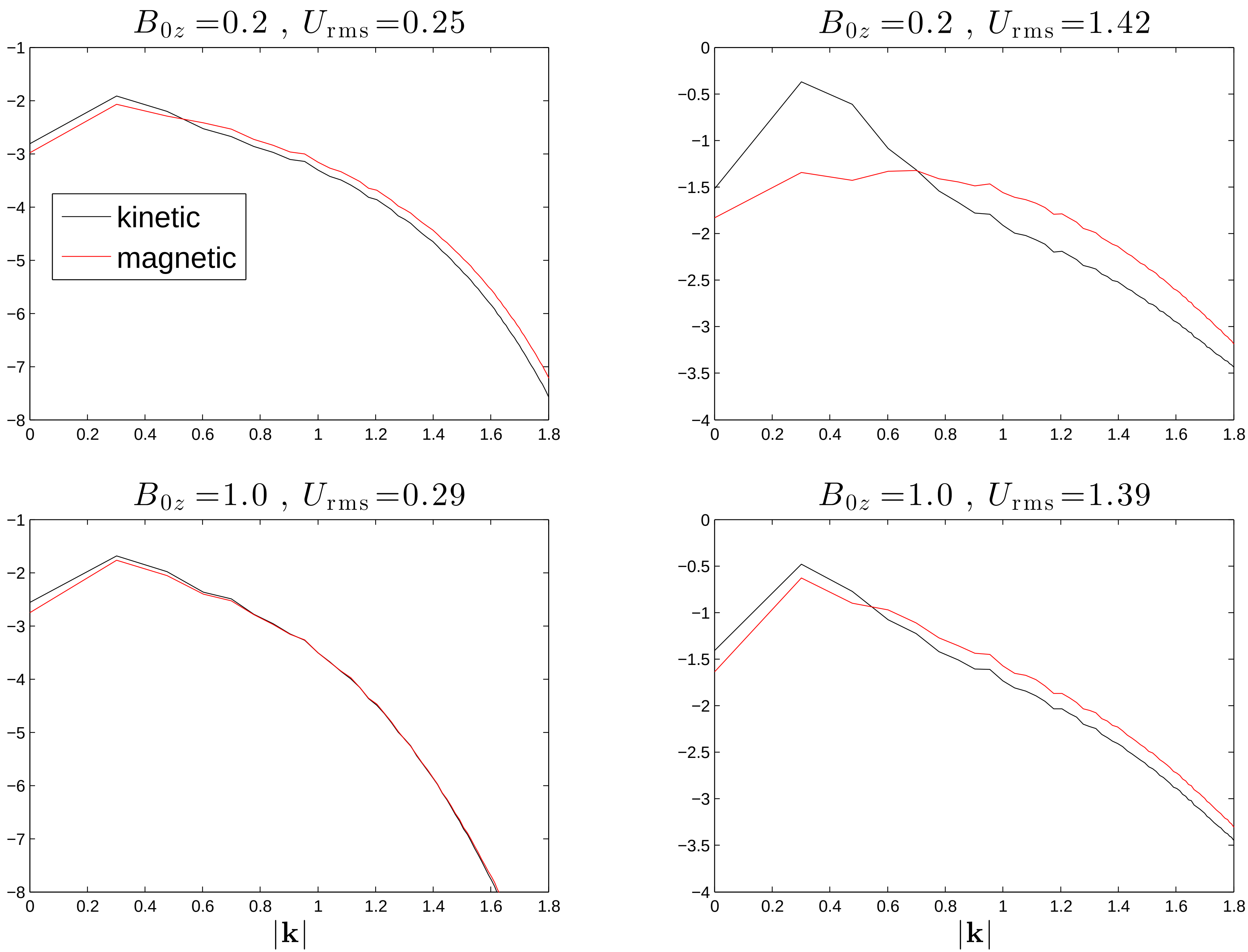
$B_{0z} = 1.0, U_{\rm rms} = 1.39$

$D_x/D_z = 0.34$

Lagrangian velocity correlation



Energy spectrum



A possible physical picture . . .

- wave induces memory into the system

  wave frequency: $\tau_A^{-1} \sim B_{0z}$

- background turbulence removes memory

  decorrelation time: τ_u

- a competition between τ_A and τ_u

- anisotropic diffusion:
  - $\tau_A \ll \tau_u$
  - $b_{\rm rms}/U_{\rm rms} \approx 1$
  - $E_u(k) \approx E_b(k)$

- further investigation required

Summary

- study single-particle diffusion in 3D MHD turbulence
- transport mostly shows diffusive scaling at large time
- anisotropic suppression of turbulent diffusion by a guided-field $(D_x \, , D_y \lesssim D_z)$
- what is the suppression mechanism in 3D (vs 2D)?
- calculate D from homogenization theory