Particle diffusion in magnetohydrodynamic turbulence: effects of a guiding magnetic field

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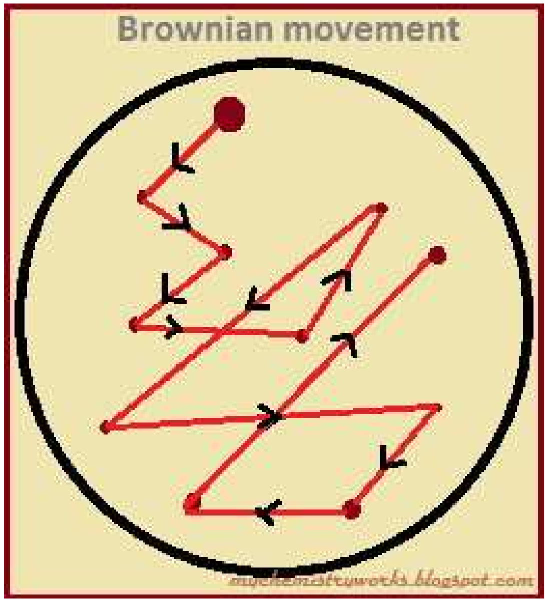
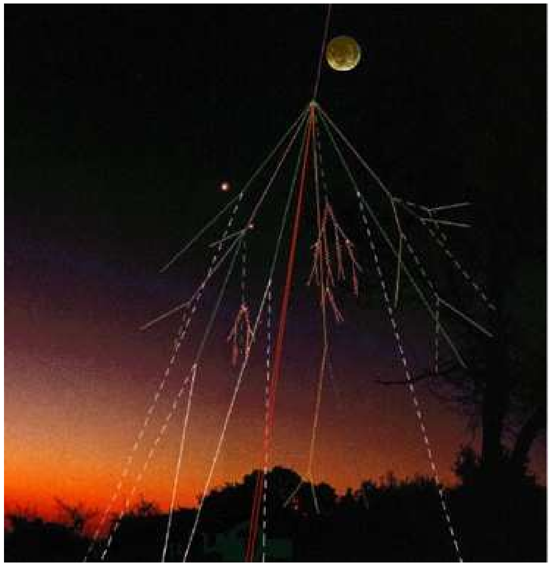
Particle transport in fluids





- Brownian motion observed under the microscope
- dispersion of pollutants in the atmosphere
- cosmic ray propagation through the interstellar medium
- tracing particle trajectories gives alternative view of the structure of the fluid flow — the Lagrangian viewpoint

Single-particle turbulent diffusion

- mean squared displacement:

$$\langle|\Delta\vec{X}(t)|^2\rangle\,, \quad \Delta\vec{X}(t) = \vec{X}(t) - \vec{X}(0)$$

- Taylor's formula (1921) for large t:

$$\vec{X}(t) = \vec{X}(0) + \int_0^t \mathrm{d}\tau\,\vec{V}(\tau)$$

$$\langle|\Delta\vec{X}(t)|^2]\rangle = 2\,t\int_0^\infty \mathrm{d}\tau\,\langle\vec{V}(\tau)\cdot\vec{V}(0)\rangle = 2tD$$

assume system is homogeneous and stationary and the integral exists

- Lagrangian velocity correlation:

$$C_L(\tau) = \langle\vec{V}(\tau)\cdot\vec{V}(0)\rangle$$

- diffusion coefficient:

$$D = \int_0^\infty \mathrm{d}\tau\,\langle\vec{V}(\tau)\cdot\vec{V}(0)\rangle$$

MHD turbulence

- Motion of a electrically conducting fluid:

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{1}{\rho_0}\nabla p + (\nabla \times \vec{B}) \times \vec{B} + \nu \nabla^2 \vec{u} + \vec{f}$$

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$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{u} \times \vec{B}) + \eta \nabla^2 \vec{B}$$

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$\vec{f}$: random forcing at the largest scales

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- Evolution of passive tracer particles:

$$\frac{\mathrm{d}\vec{X}(t)}{\mathrm{d}t} = \vec{u}(\vec{X}(t), t) = \vec{V}(t)$$

$$\vec{X}(0) = \vec{\alpha}$$

- Field-guided MHD turbulence:

$$\vec{B}(\vec{x}, t) = B_0 \hat{z} + \vec{b}(\vec{x}, t)$$

Previous work: the 2D case

ON THE EFFECTS OF A WEAK MAGNETIC FIELD ON TURBULENT TRANSPORT

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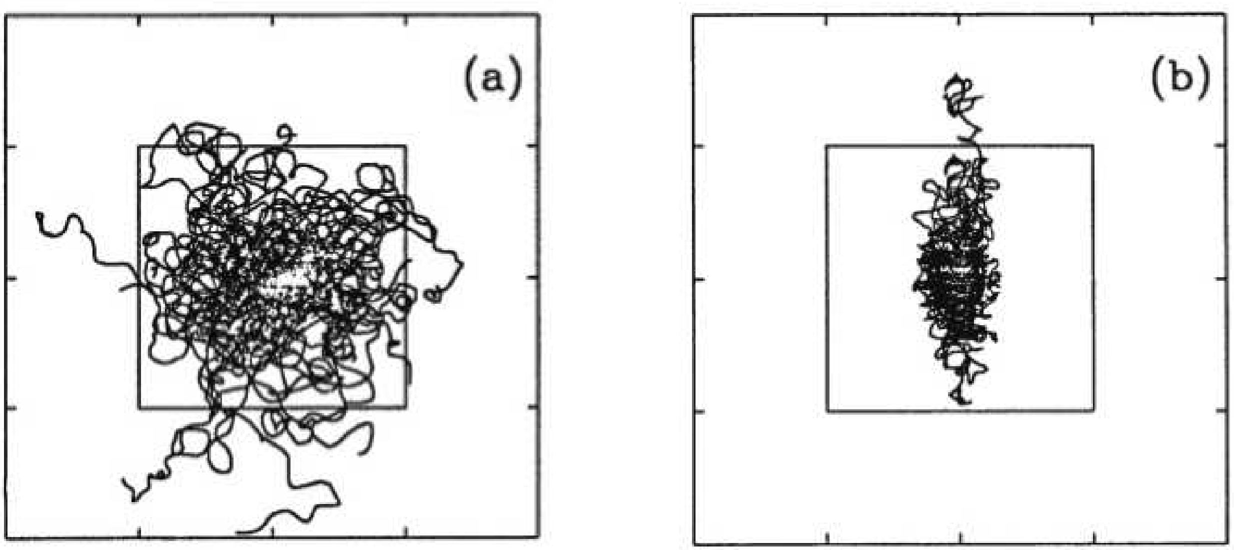
ABSTRACT

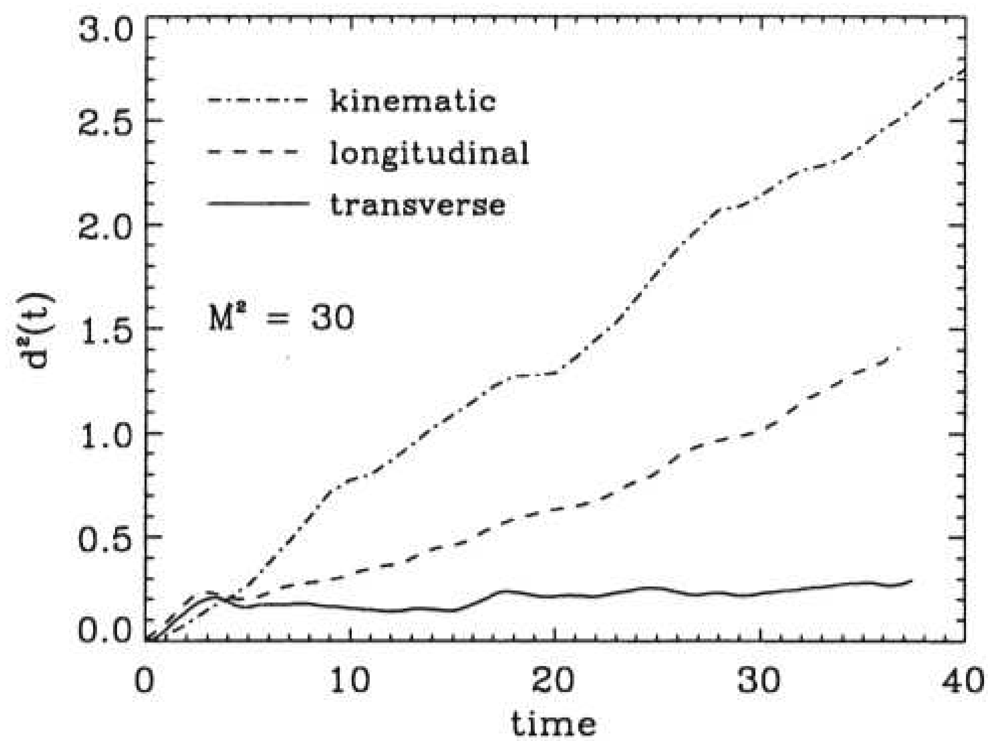
We discuss the effects of a weak large-scale magnetic field on turbulent transport. We show by means of a series of two-dimensional numerical experiments that turbulent diffusion can be effectively suppressed by a (large-scale) magnetic field whose energy is small compared to equipartition. The suppression mechanism is associated with a subtle modification of the Lagrangian energy spectrum, and it does not require any substantial reduction of the turbulent amplitude. We exploit the relation between diffusion and random walking to emphasize that the effect of a large-scale magnetic field is to induce a long-term memory in the field of turbulence. The implications for the general case of three-dimensional transport are briefly discussed.
Subject headings: diffusion — MHD — turbulence

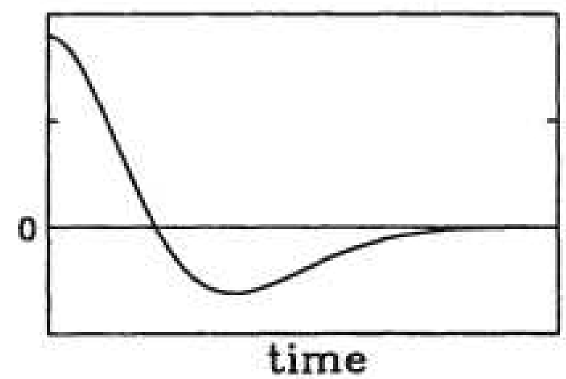
1. transport suppressed in direction $\perp$ to $B_0\hat{y}$ when $B_0 > B_0^*$



Previous work: the 2D case

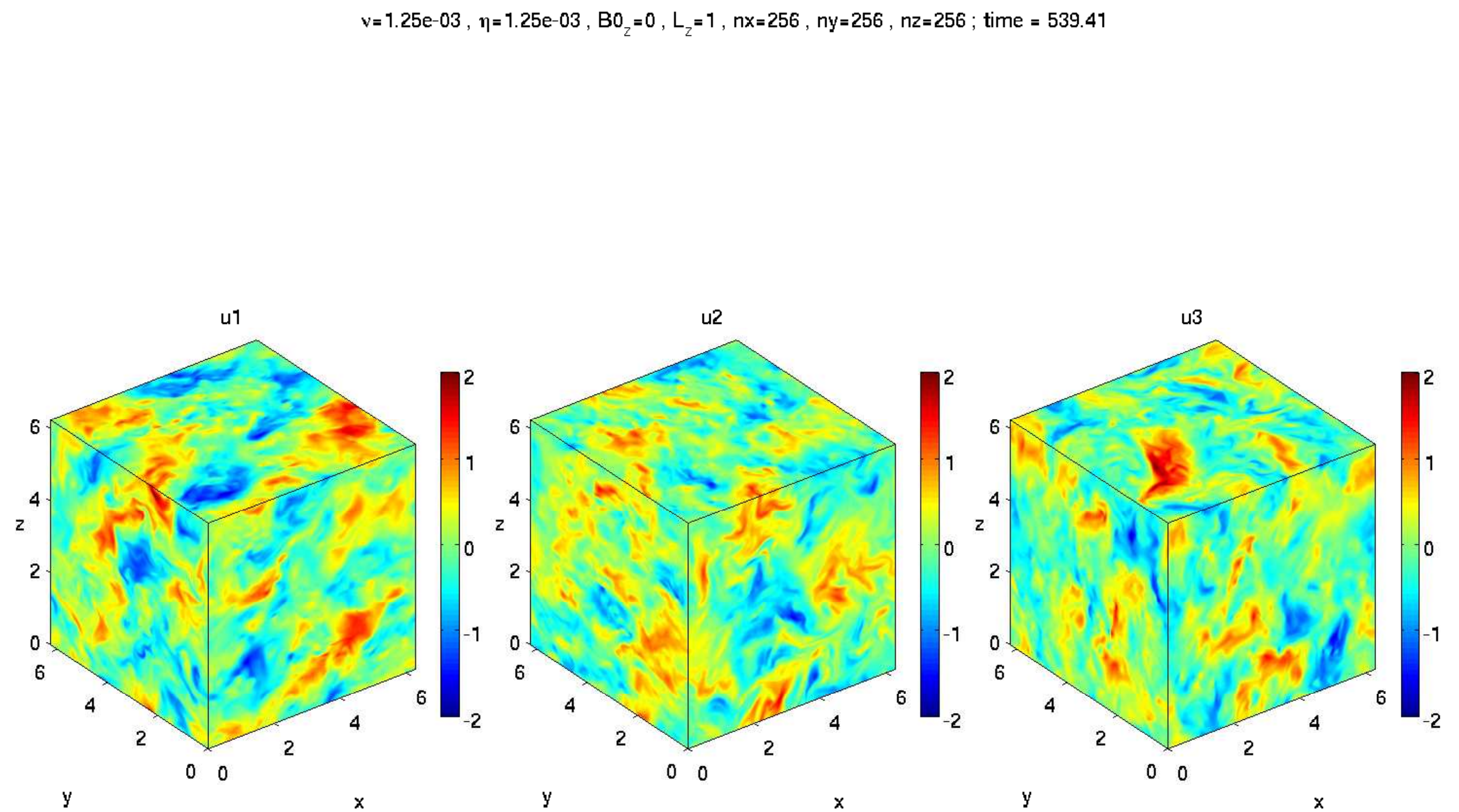
2. as $Re_m = UL/\eta$ increases, the critical B_0^* decreases
3. the system has long-term memory: slow decay of $C_L(\tau)$



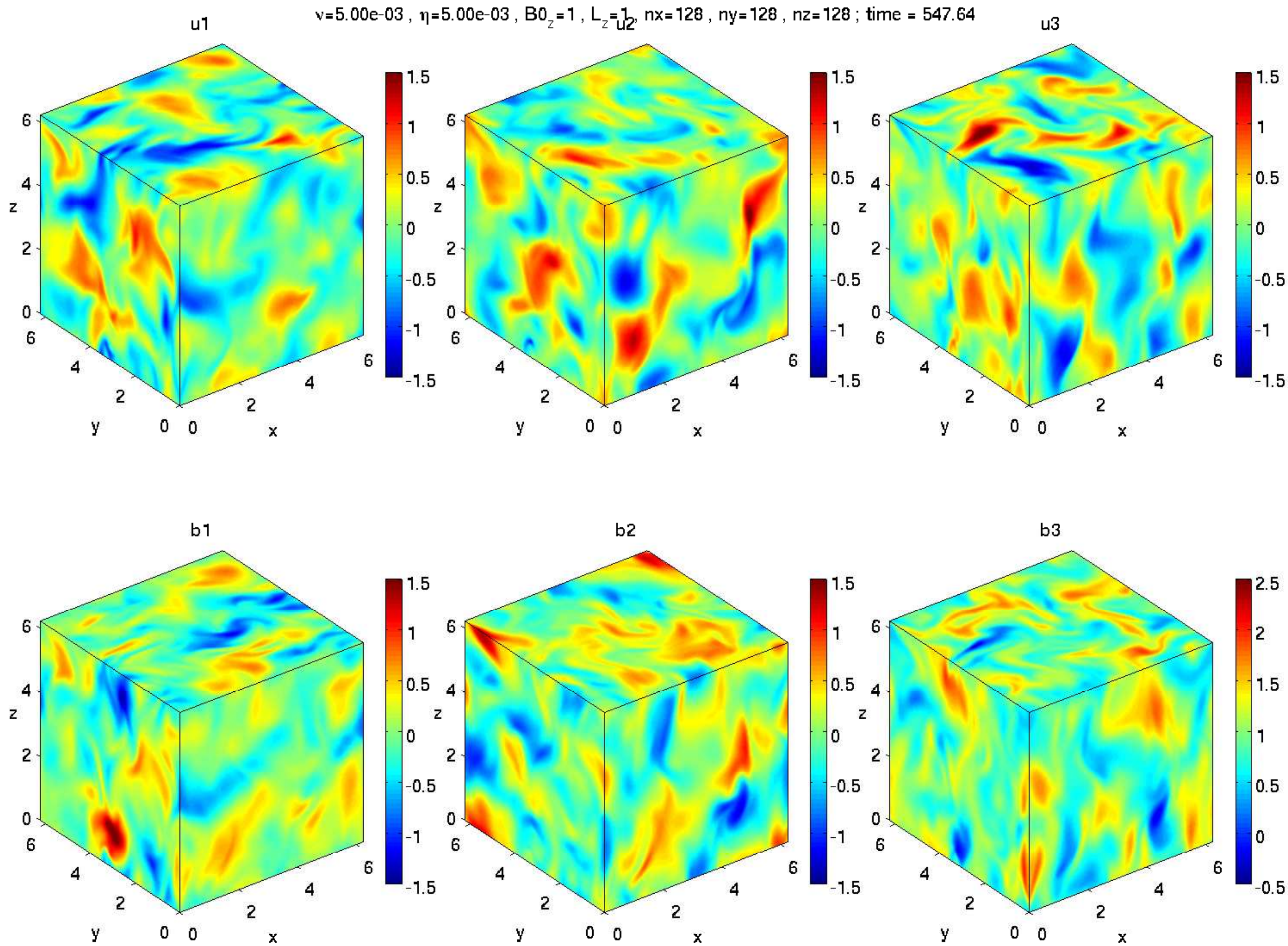


- Whether such suppression of turbulent diffusion occurs in 3D is not clear.

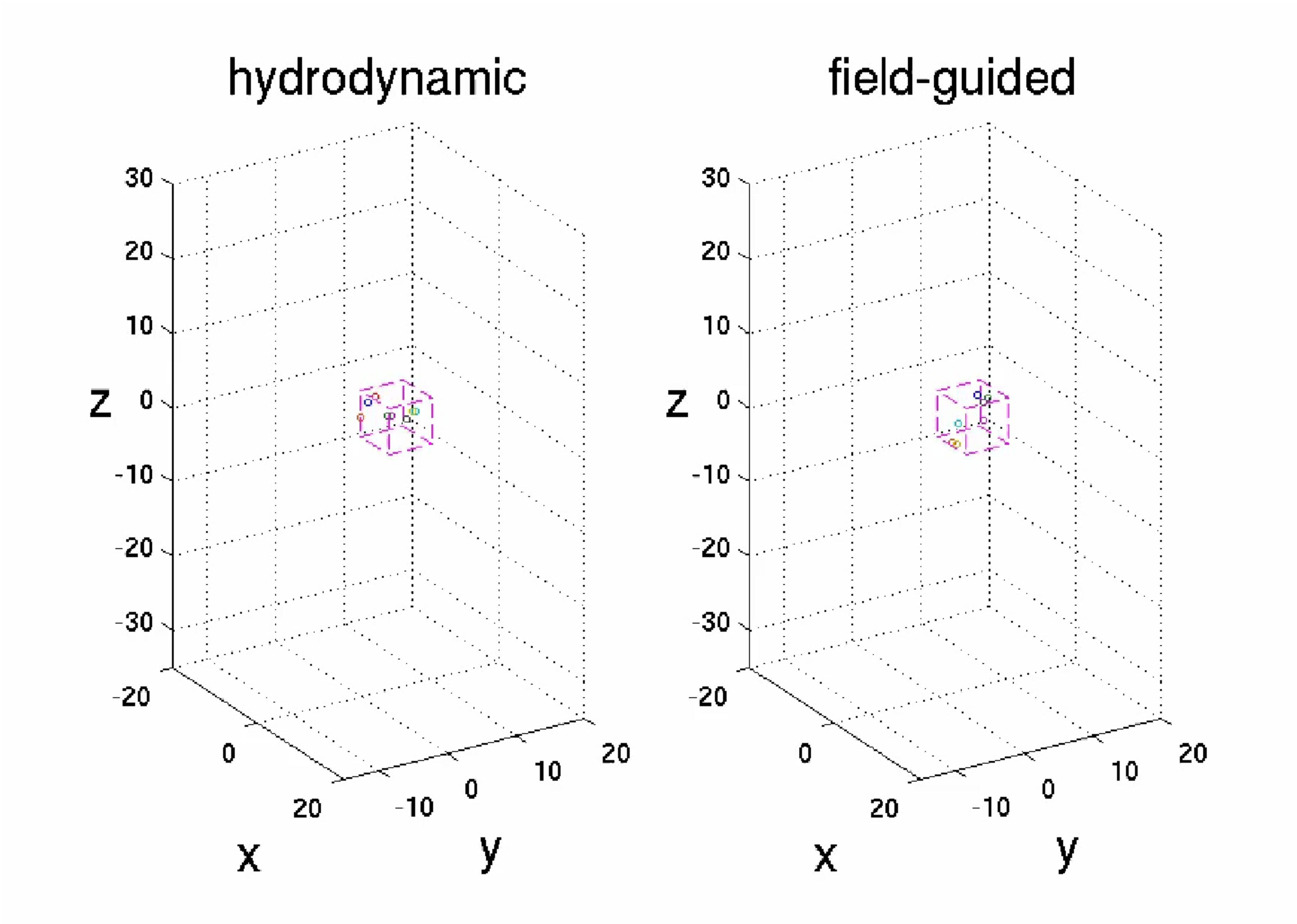
The hydrodynamic case, $\vec{B} = 0$



The field-guided case, $\vec{B} = B_0 \hat{z}$



Particle tracking



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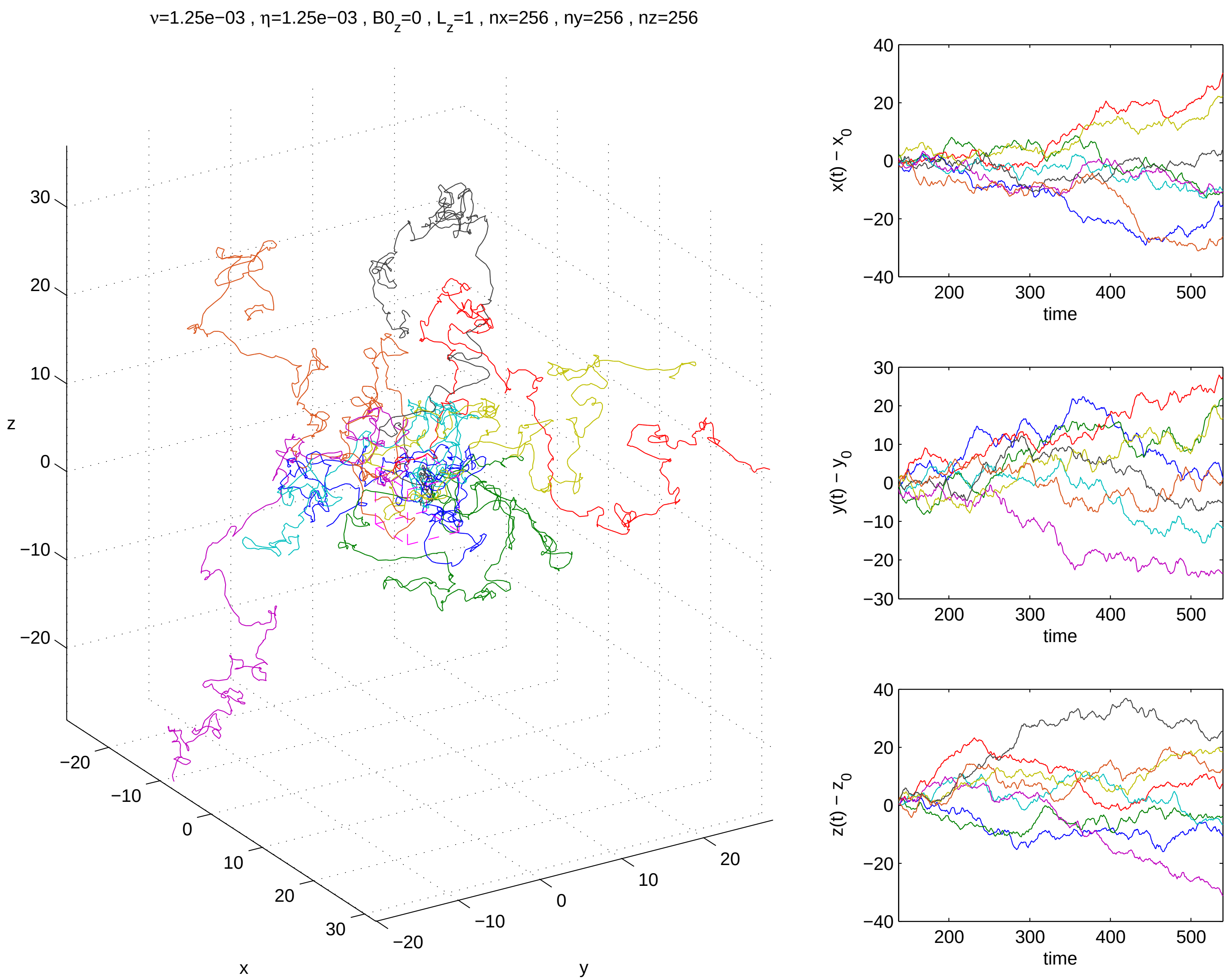
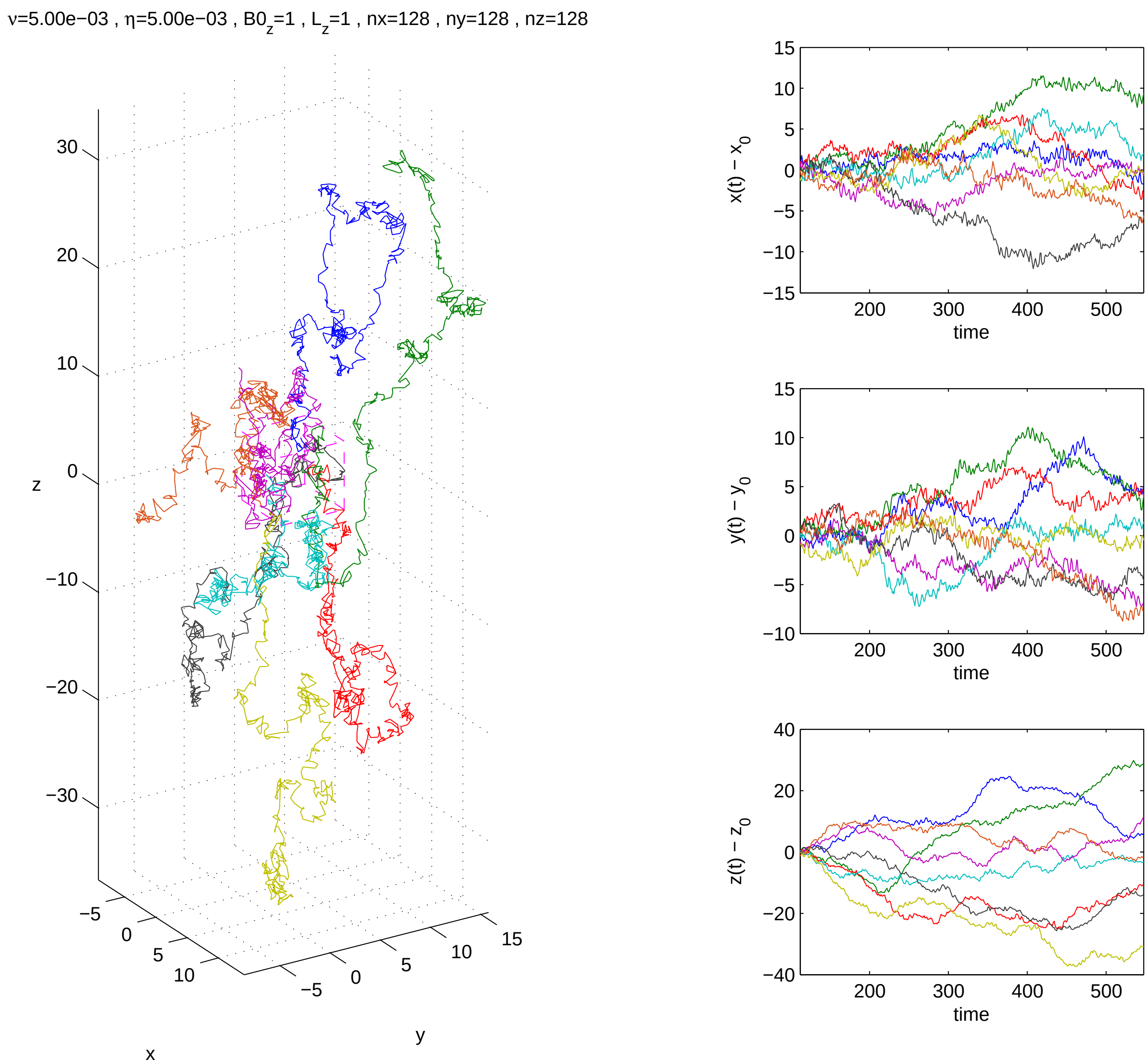
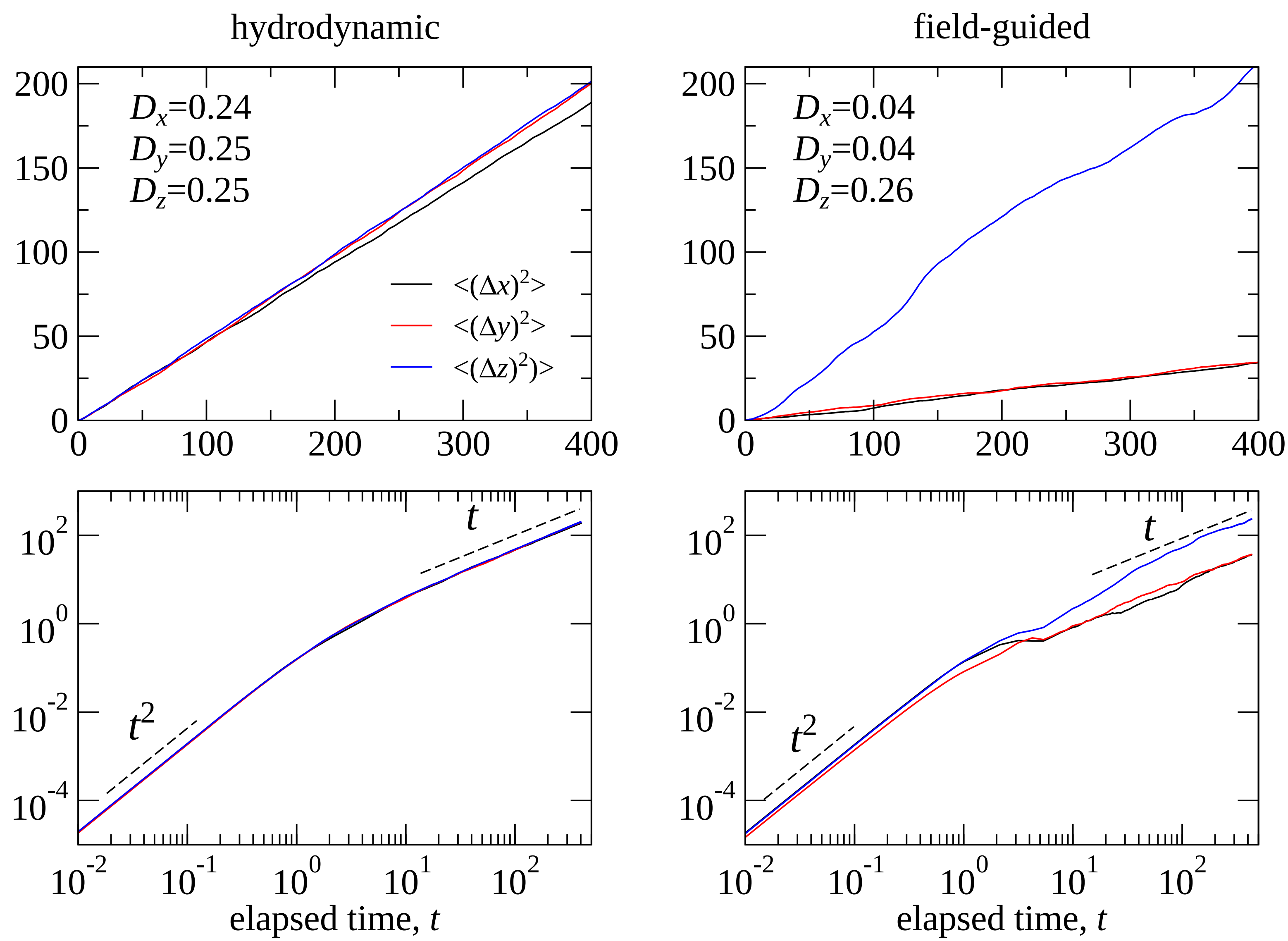


The field-guided case, $\vec{B} = B_0 \hat{z}$



- transport suppressed in the field-perpendicular direction!

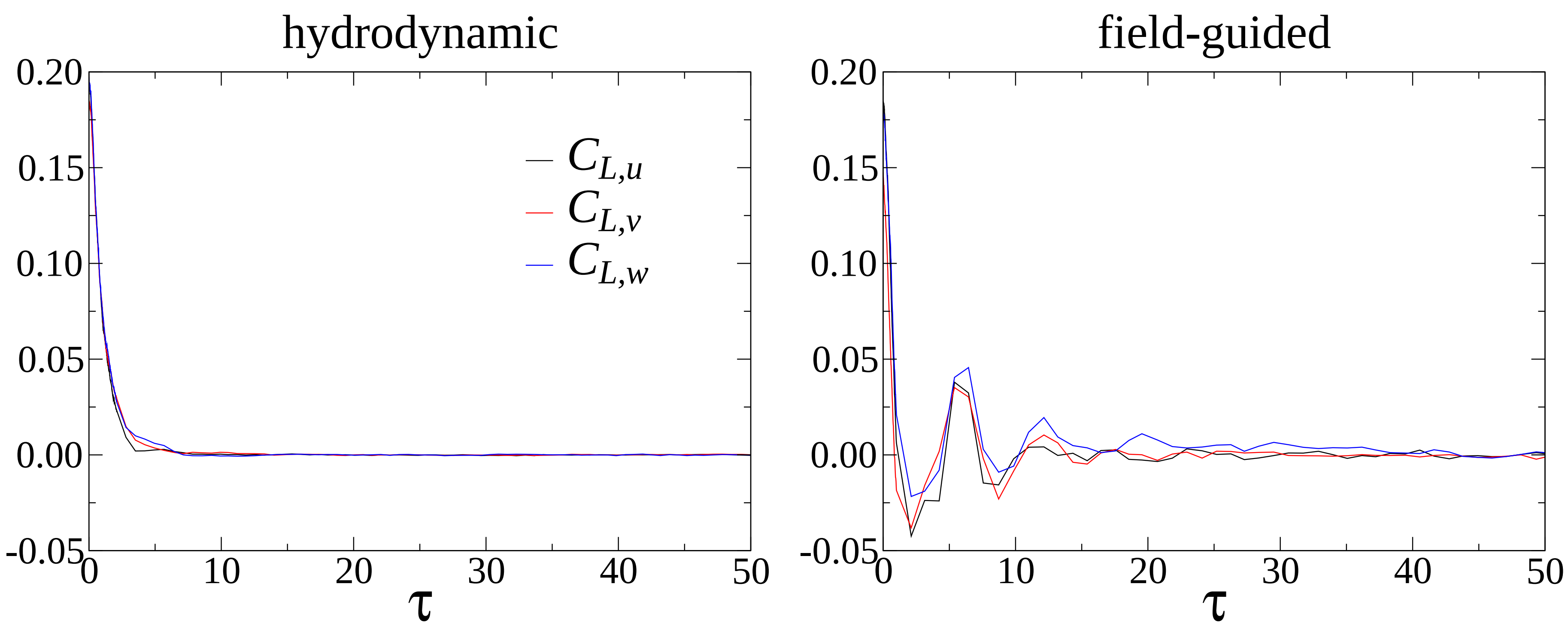
Scaling of mean-squared displacement



- ballistic limit: $\sim t^2$ at small time
- diffusive scaling: $\sim t$ at large time, $\langle(\Delta x)^2\rangle \sim 2D_x t$, etc

Lagrangian velocity correlation function

$$C_L(\tau) = \langle \vec{V}(\tau) \cdot \vec{V}(0) \rangle$$



- hydrodynamic: $\sim \exp(-\tau)$, short correlation time
- field-guided: oscillatory, long correlation time

Summary

- study single-particle diffusion in 3D MHD turbulence
- strong field-guided case versus the hydrodynamics case
- transport shows diffusive scaling at large time
- suppression of turbulent diffusion transport in the field-perpendicular direction
- Check Re_m dependence? What is the suppression mechanism in 3D?