Multifractality in Detrended Human Heart Beat Increment

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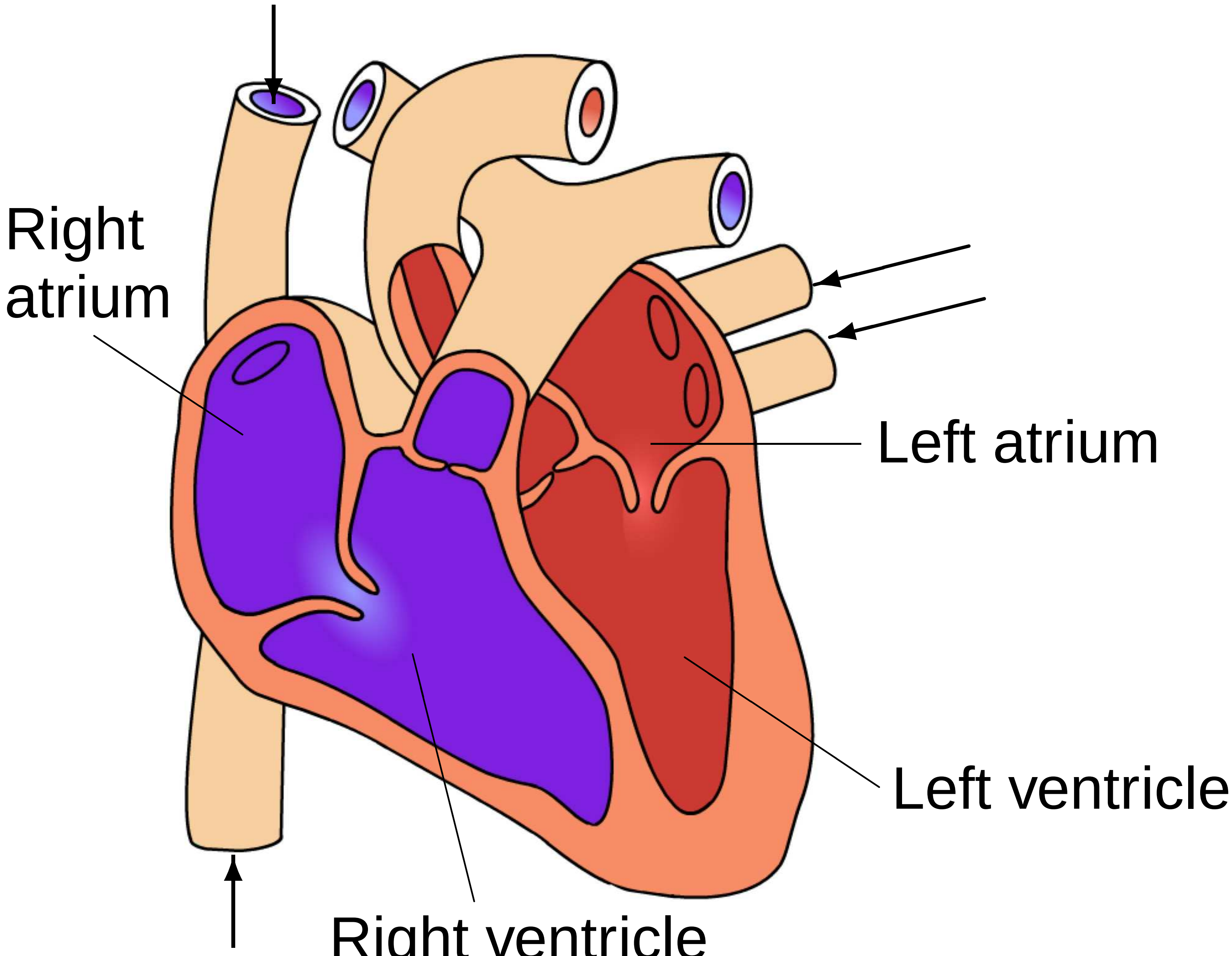
The Cardiac Pump



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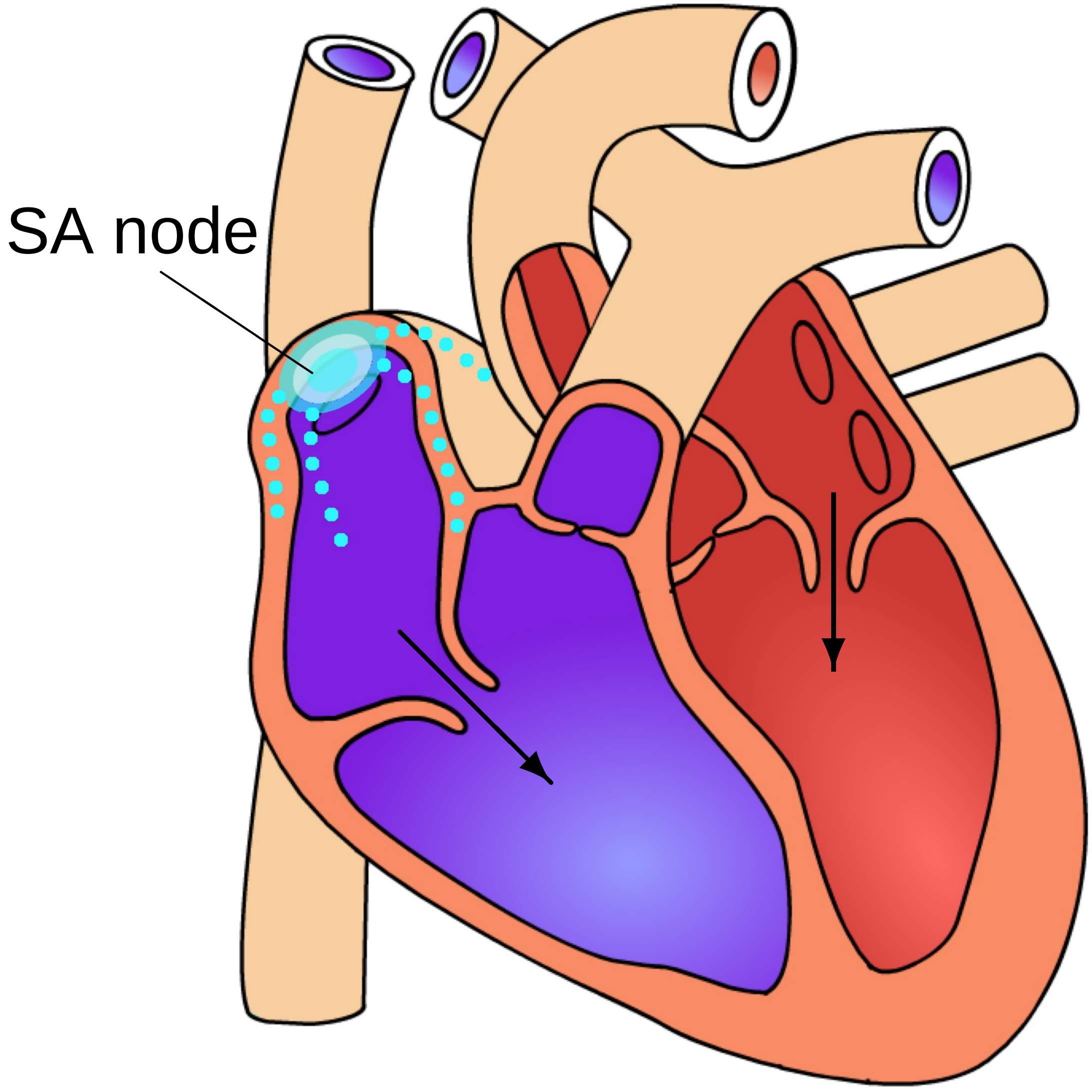
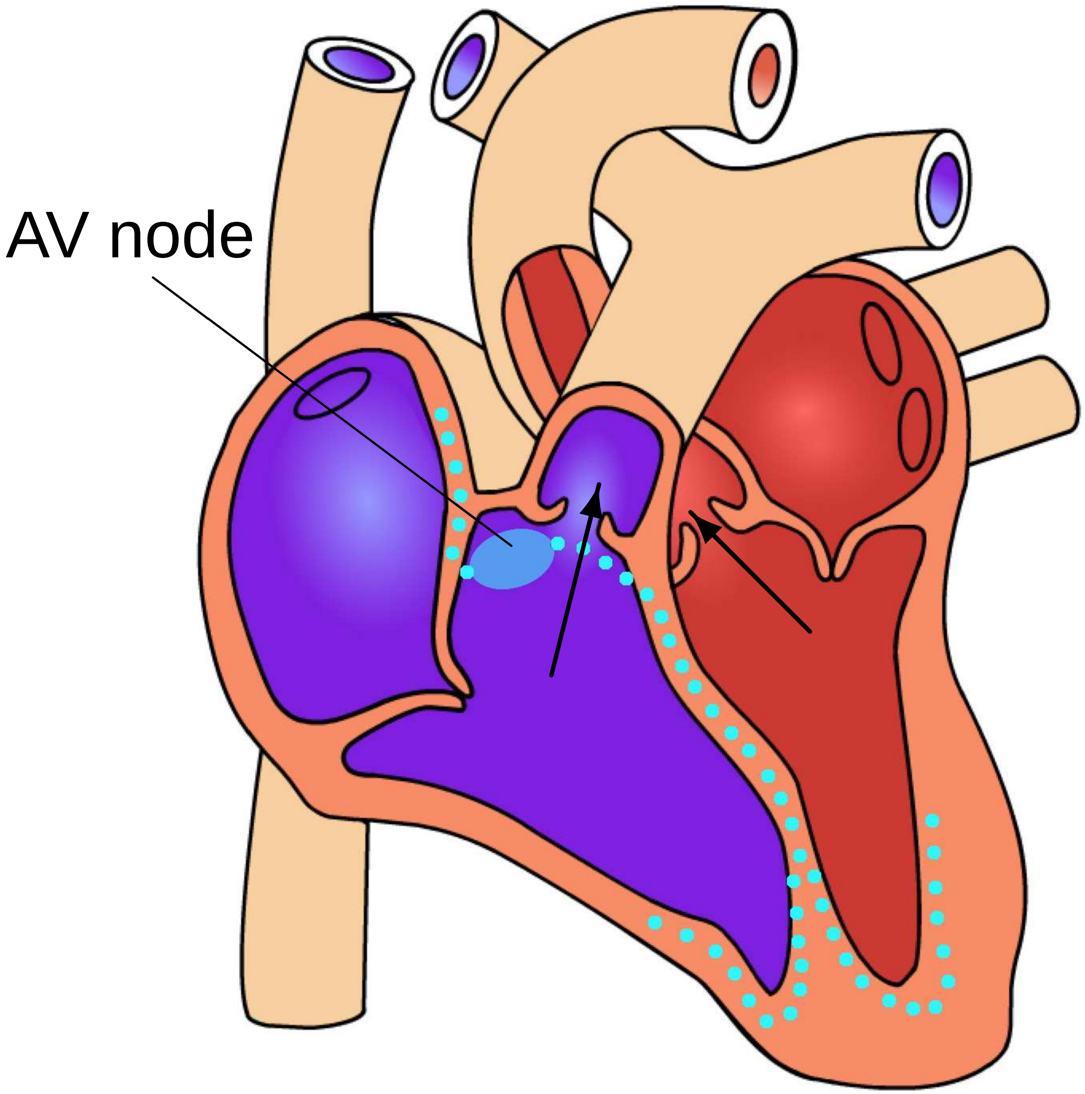


electric impulse initiated in the sinoatrial (SA) node spreads to the atria — atrial contraction

The Cardiac Pump



electric impulse initiated in the sinoatrial (SA) node spreads to the atria
— atrial contraction

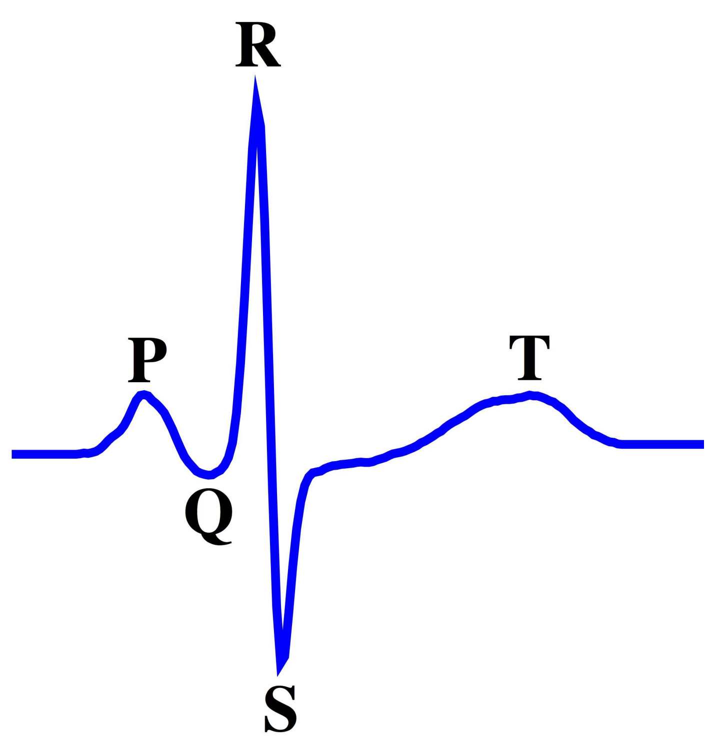
impulse reaches the atrioventricular (AV) node and is conducted to the ventricles
— ventricular contraction

delay in the passage of the impulse occurs in the AV node

Heart Beat Intervals

- Electrocardiogram (ECG)

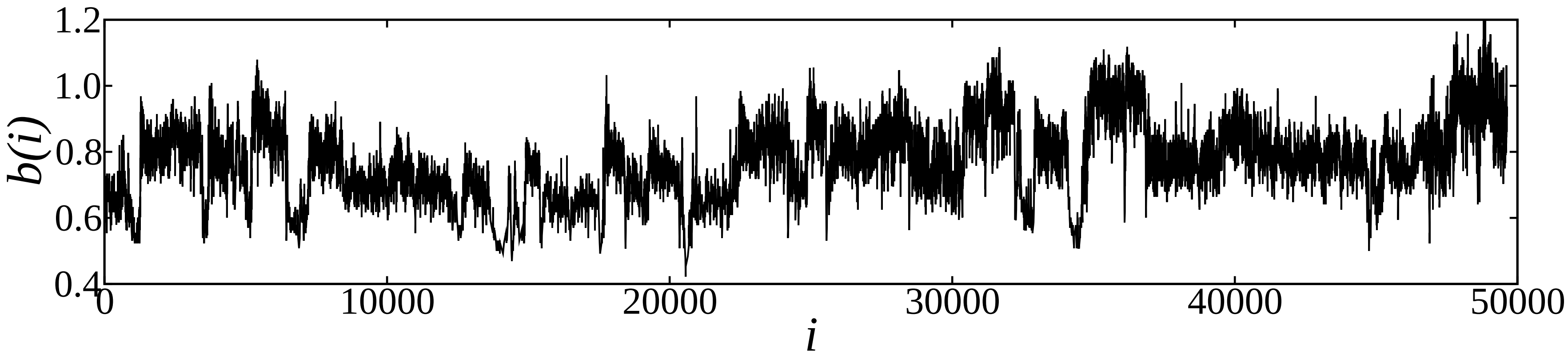
  - P-wave : atrial activation
  - QRS complex : ventricular activation
  - T-wave : recovery phase of ventricular

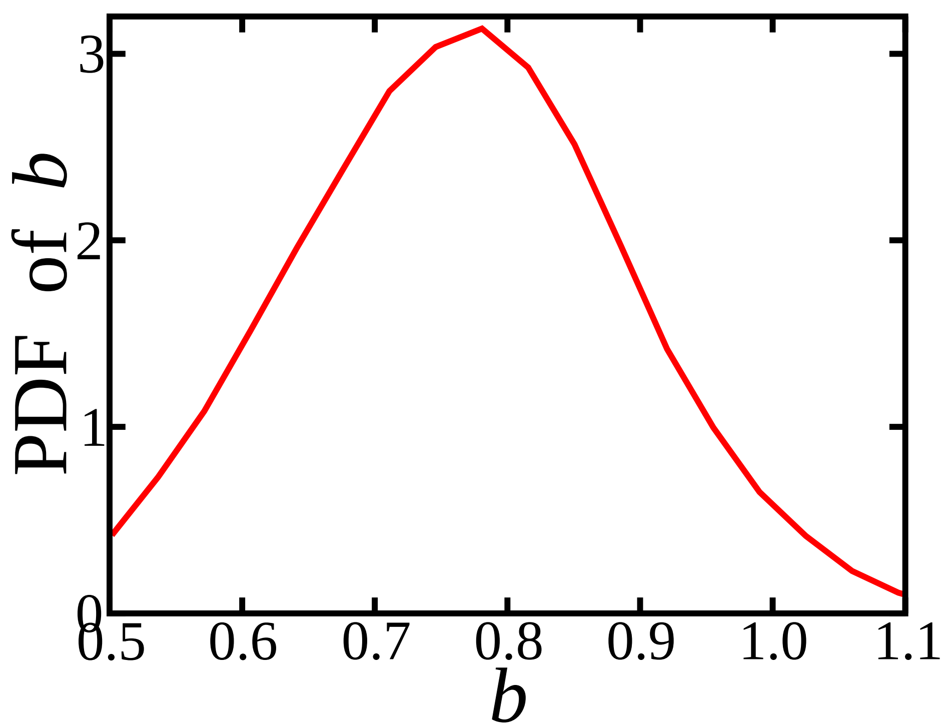


- RR-intervals (RRi) — measure of heart rate

Heart Rate Variability (HRV)

Time series of RRi : $b(i)$





Reasons to study HRV

- To understand how the autonomic nervous system (ANS) control heart rate
  - sympathetic branch of the ANS increases heart rate and parasympathetic branch decreases heart rate
  - both branches are active and interacts, parasympathetic effects usually dominate
  - parasympathetic branch affects heart rate with a much shorter delay
- Abnormalities in HRV have prognostic significance
  - healthy human RRi shows multifractality
  - multifractality lost in congestive heart failure condition

P. Ch. Ivanov et al., Multifractality in human heartbeat dynamics, Nature **399**, *461 (1999)*

Multifractal

A timeseries $X(i)$ has different statistical properties at different scales.

$$\Delta_n X(i) \equiv X(i+n) - X(i)$$

(1) Probability density function of $\Delta_n X(i)$

$P_n(\Delta_n X)$ changes shape with n

(2) Structure function

$$S_q(n) \equiv \langle |\Delta_n X|^q \rangle \sim n^{\zeta_q}$$

ζ_q is a nonlinear function of q

Multifractal

(1) ⇔ (2) Let $Y = \dfrac{\Delta_n X}{\langle |\Delta_n X|^2 \rangle^{1/2}}$

$P_n(\Delta_n X)$ has same shape for different n

$\Leftrightarrow$ $\bar{P}_n(Y)$ is independent of n

$\langle |Y|^q \rangle = \int |Y|^q \, \bar{P}_n(Y)\, dY$ is independent of n

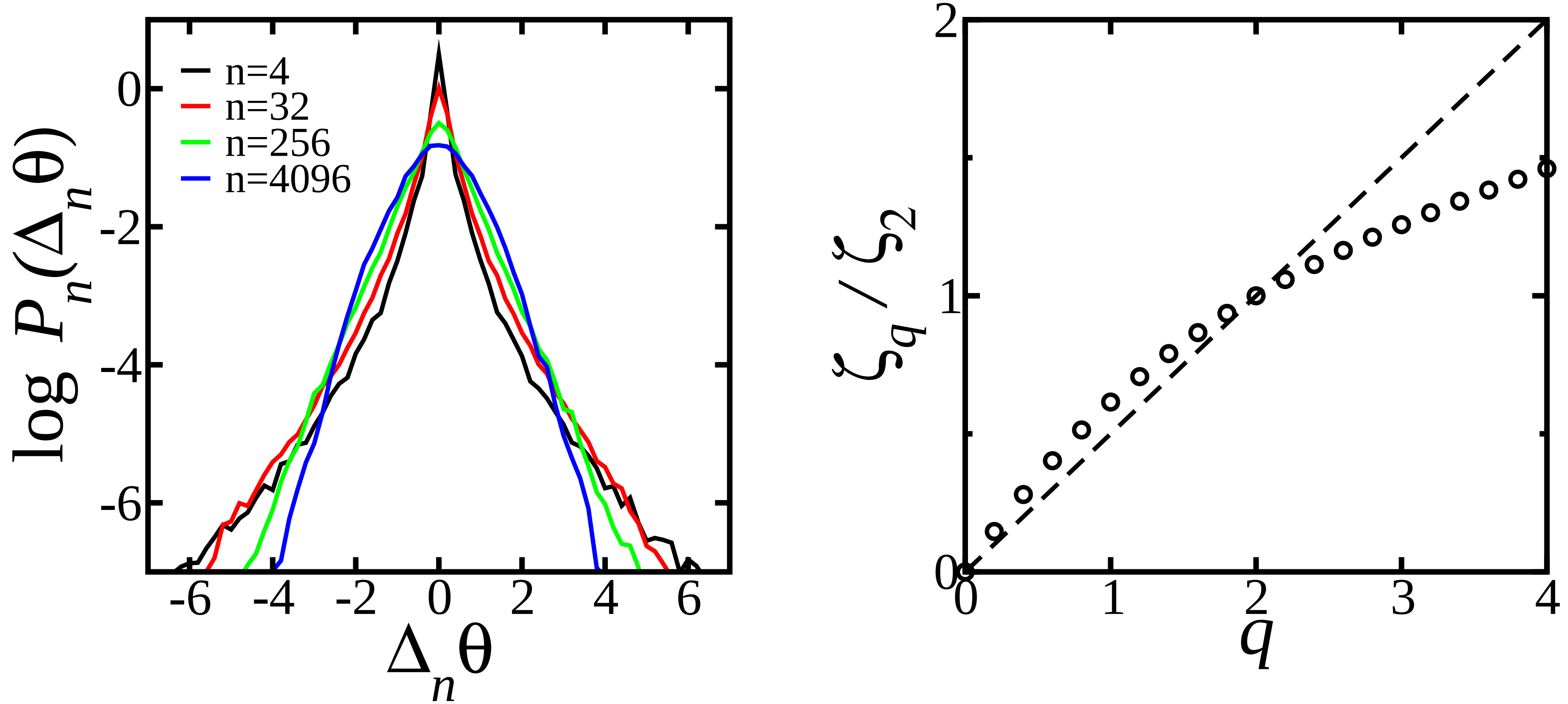
$$\langle |Y|^q \rangle = \frac{\langle |\delta_n X|^q \rangle}{\langle |\delta_n X|^2 \rangle^{q/2}} = \frac{S_q(n)}{[S_2(n)]^{q/2}} \sim n^{\zeta_q - q\zeta_2/2}$$

$$\zeta_q = \frac{q}{2}\zeta_2$$

scale-invariant $P_n(\Delta_n X) \iff \zeta_q \propto q$

An Example from Fluid Turbulence

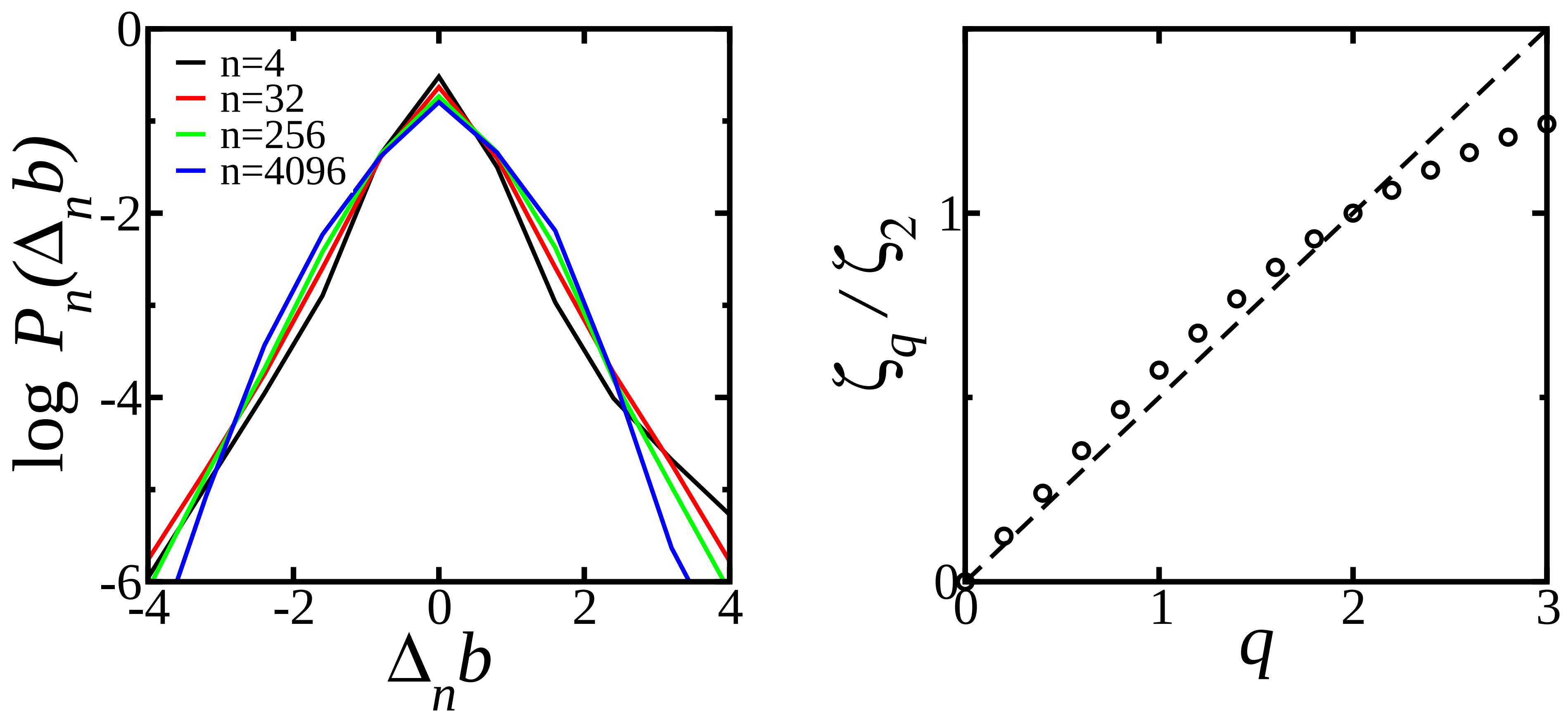
Multifractal scaling in temperature increments $\Delta_n\theta$ from turbulent thermal convective experiments:



[data from *B. Castaing et al., J. Fluid Mech.* **204**,*1 (1989)*]

Multifractality in Healthy Heart Rate

Healthy heart rate (RRi) increments $\Delta_n b$ from data of daytime normal sinus rhythm:



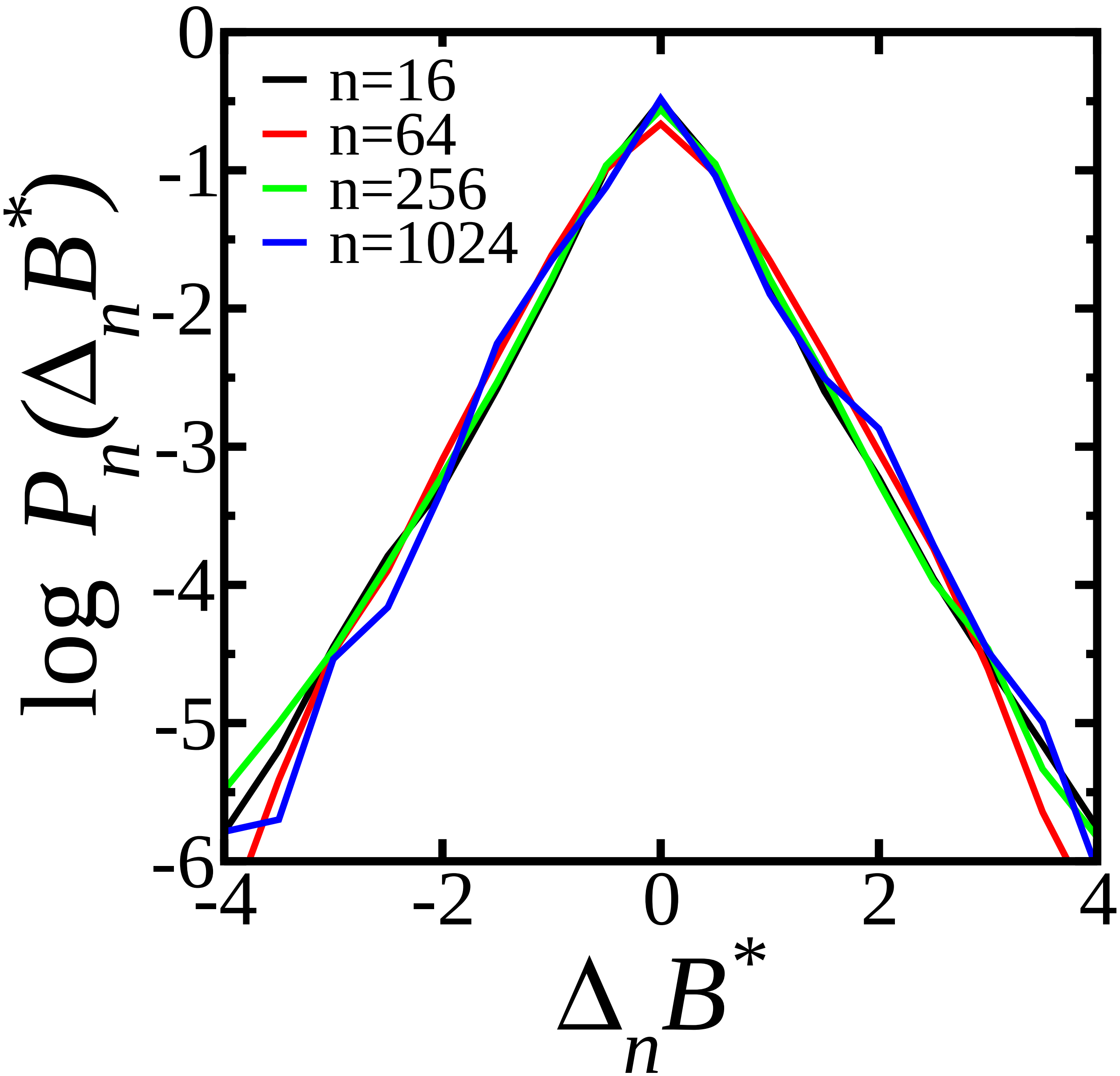
[data from *http://physionet.org*]

Scale-invariant Detrended RRi ?

- "scale-invariance in the PDF of detrended healthy human heart rate increments"
- A detrend procedure for non-stationary timeseries:
  1. $B(i) = \sum_{j=1}^{i} b(j)$
  2. divide $B(i)$ into segments of size $2n$
  3. fit $B(i)$ in each segment with the best d-th order polynomial, $p_d^{(n)}(i)$
  4. $B^*(i) = B(i) - p_d^{(n)}(i)$ ("trend" removed)
  5. $\Delta_n B^*(i) = B^*(i+n) - B^*(i)$
- $P_n(\Delta_n B^*)$ is scale-invariant

K. Kiyono et al., Phys. Rev. Lett. **93**,*17 (2004)*

PDF of $\Delta_n B^*$ in Healthy Heart Rate



But isn't healthy heart rate multifractal !?

Detrended Heart Rate $b^*(i)$

- $\Delta_n B^*(i)$ is actually related to the sum of $b(i)$

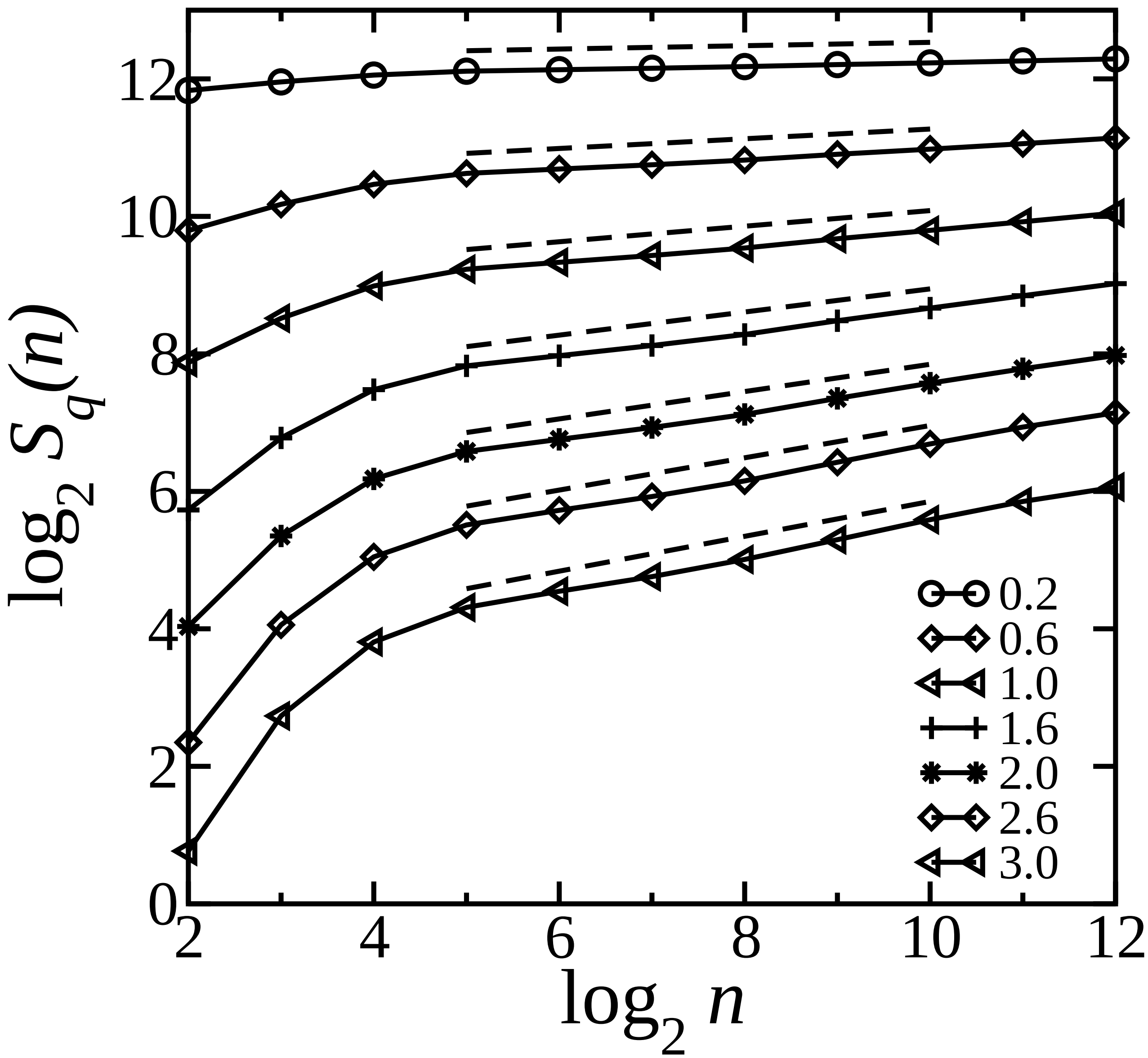
$$\begin{aligned}\Delta_n B^*(i) &= B^*(i+n) - B^*(i) \\ &= B(i+n) - B(i) - [p_d^{(n)}(i+n) - p_d^{(n)}(i)] \\ &= \sum_{j=i+1}^{i+n} b(j) - \Delta_n p_d^{(n)}(i)\end{aligned}$$

- A natural definition of detrended heart rate $b^*(i)$ is

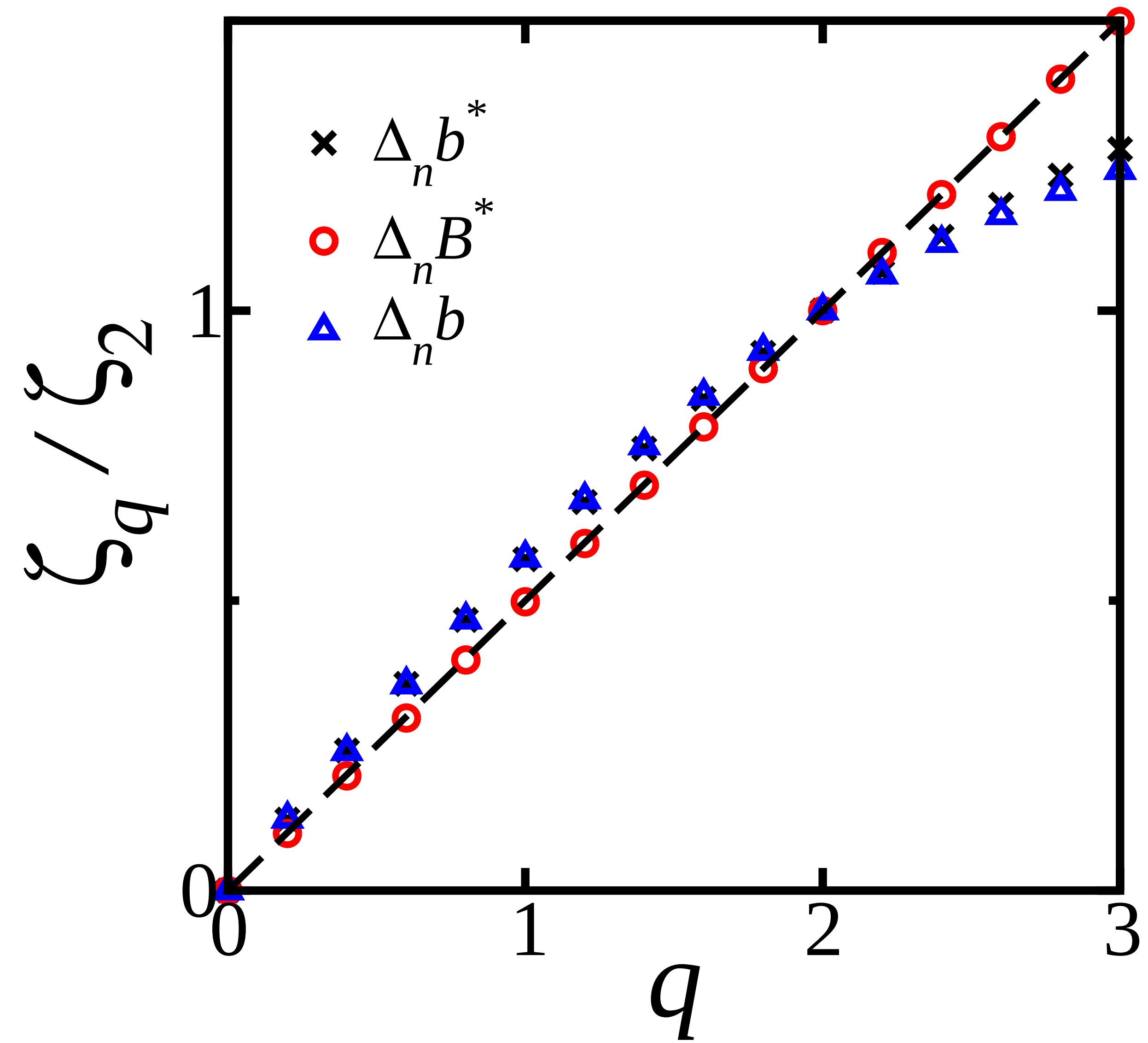
$$\begin{aligned}B^*(i) &\equiv \sum_{j=1}^{i} b^*(j) \\ \Rightarrow \qquad \Delta_n B^*(i) &= \sum_{j=i+1}^{i+n} b^*(j) \\ b^*(i) &= B^*(i) - B^*(i-1)\end{aligned}$$

Structure function for $\Delta_n b^*(i)$

$$\Delta_n b^*(i) = \Delta_n b(i) - [\Delta_n p_d^{(n)}(i) - \Delta_n p_d^{(n)}(i-1)]$$



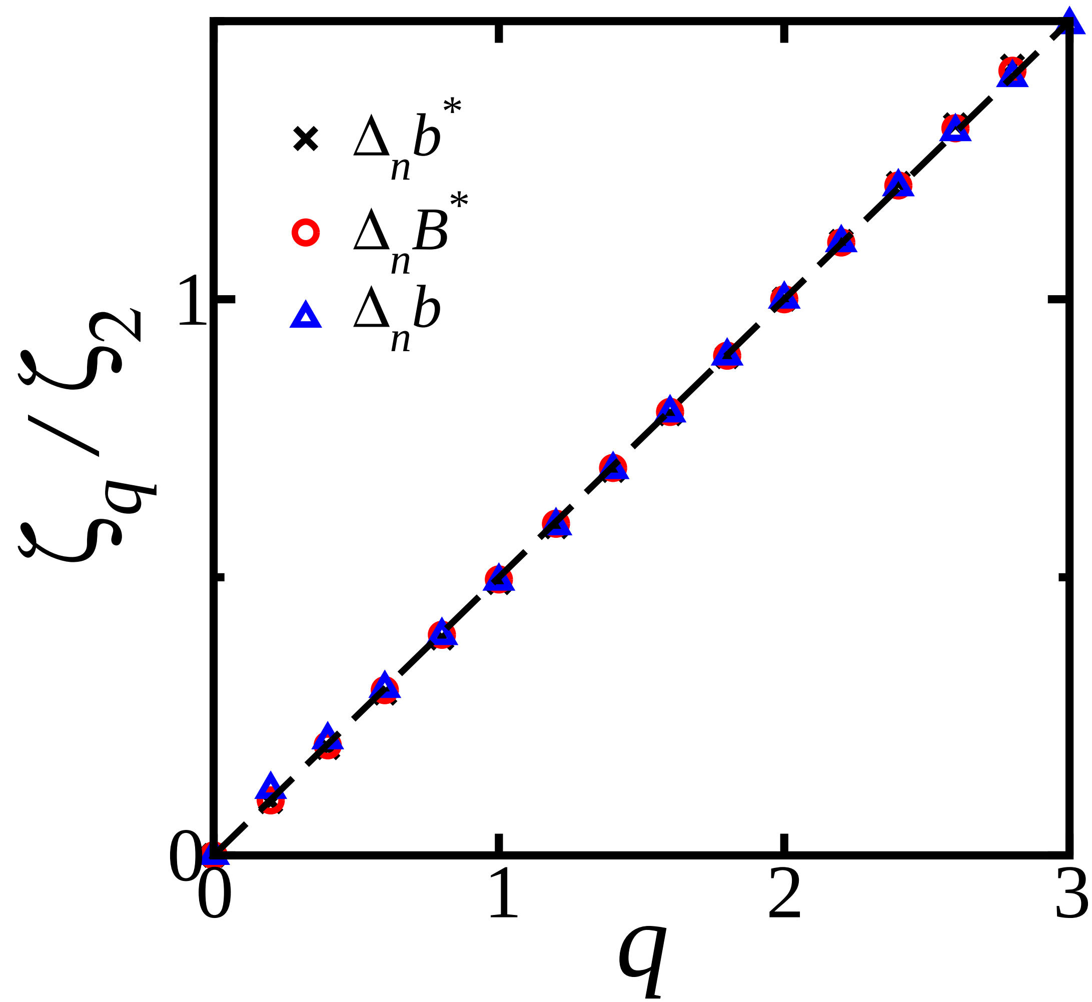
ζ_q for Healthy Heart Rate



The detrended healthy heart rate $b^*(i)$ is indeed multifractal

ζ_q for Pathological Heart Rate

Data from congested heart failure patients:



Scale-invariance of $P_n(\Delta_n B^*)$ is a characteristic independent of the multifractality of HRV

Detrend Analysis in Turbulence

Follow the ideas of detrended analysis of HRV:

$\theta(t_i) =$ temperature measurement from thermal convective experiments:

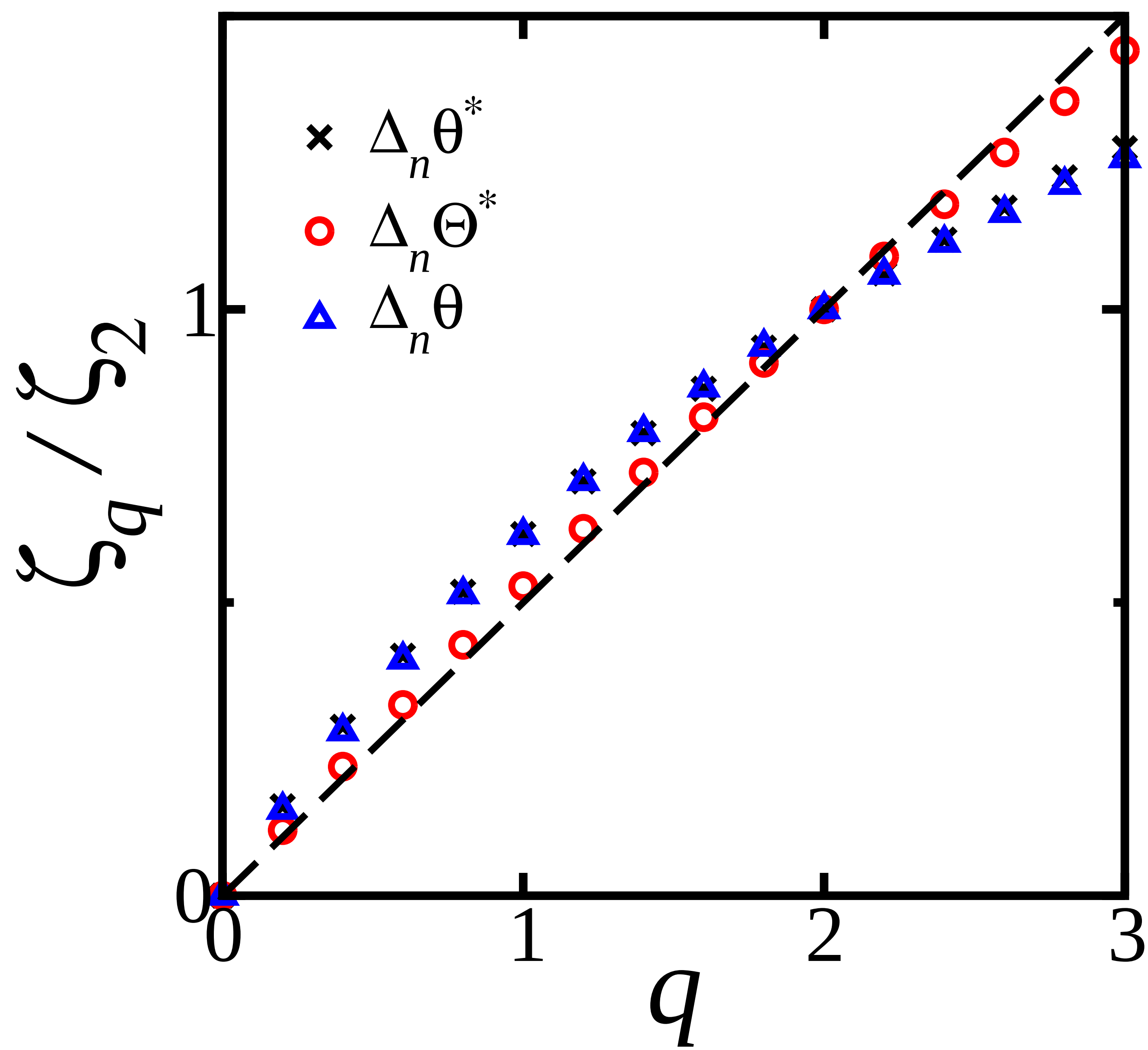
$$\Theta(t_i) \equiv \sum_{j=1}^{i} \theta(t_j)$$

$$\Theta^*(t_i) \equiv \Theta(t_i) - p_d^{(n)}(t_i)$$

$$\Delta_n \Theta^*(t_i) = \Theta^*(t_{i+n}) - \Theta^*(t_i)$$

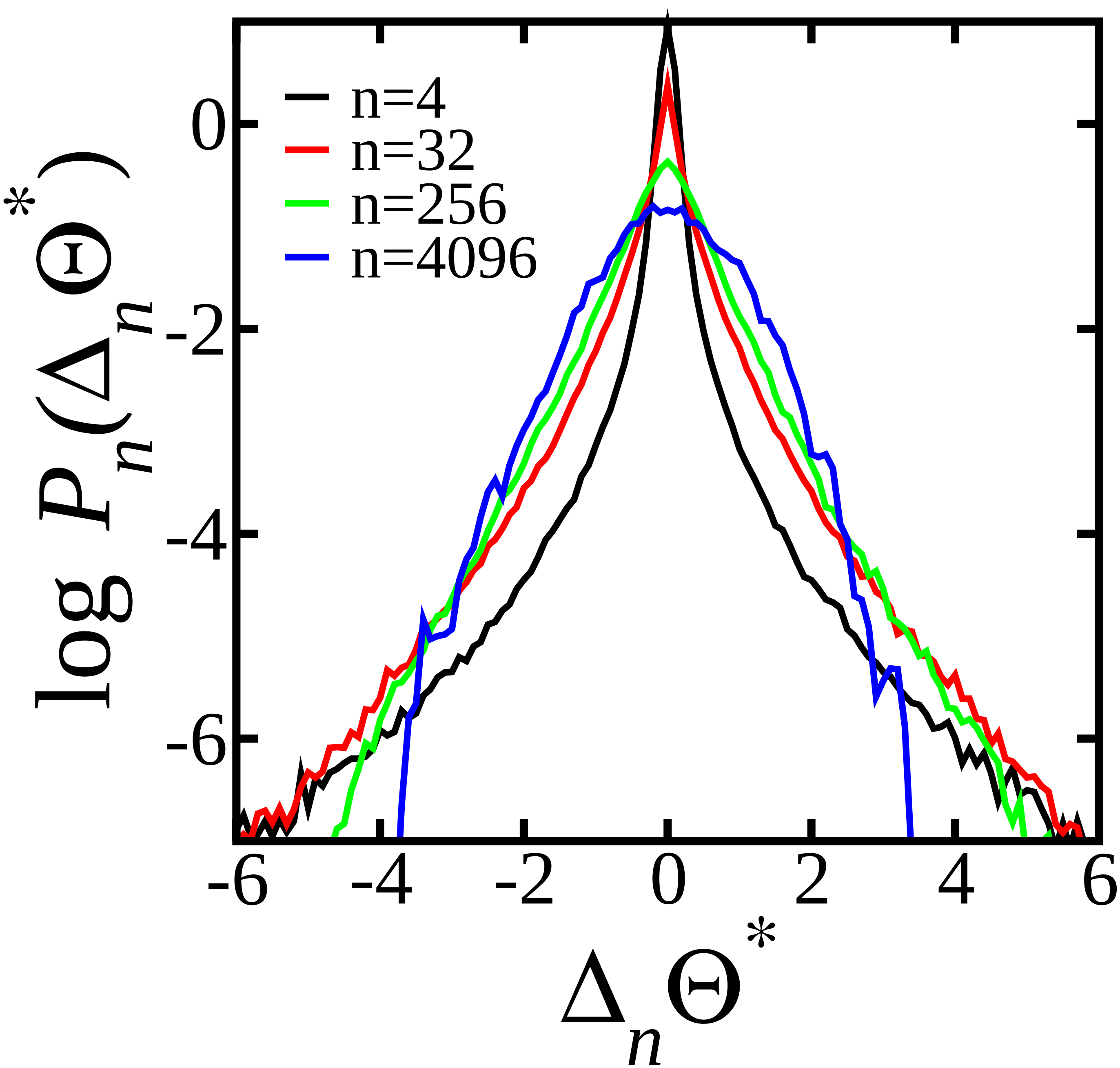
$$\Theta^*(t_i) \equiv \sum_{j=1}^{i} \theta^*(t_j)$$

Detrend Analysis in Turbulence



PDF of $\Delta_n\Theta^*$ is not scale-invariant.

Detrend Analysis in Turbulence



Detrend Analysis in Turbulence

Recall :
$$B(i) \equiv \sum_{j=1}^{i} b(j)$$

$$\Delta_n B^*(i) = \sum_{j=i+1}^{i+n} b(j) - \Delta_n p_d^{(n)}(i)$$

$$\Longrightarrow \quad \Delta_n B^*(i) = \Delta_n B(i) - \Delta_n p_d^{(n)}(i)$$

$$\Delta_n \Theta^*(i) = \Delta_n \Theta(i) - \Delta_n p_d^{(n)}(i)$$

Now,
$$\langle|\Delta_n B(i)|^q\rangle \sim n^q$$
$$\langle|\Delta_n \Theta(i)|^q\rangle \sim n^q$$

So $p_d^{(n)}(i)$ is responsible for the different scaling behavior in $\langle|\Delta_n B^*(i)|^q\rangle$ and $\langle|\Delta_n \Theta^*(i)|^q\rangle$

Summary

- We clarify that the scale-invariant of $P_n(\Delta_n B^*)$ in healthy heart rate is for the sum of detrended heart rate b^*
- $P_n(\Delta_n b^*)$ for healthy heart rate increments is indeed scale dependent, as expected from the multifractality of b.
- $P_n(\Delta_n B^*)$ is scale-invariant in pathological heart rate in patients suffering from congestive heart failure
- $P_n(\Delta_n \Theta^*)$ is scale dependent in the multifractal temperature measurements in turbulent thermal convective flows.