Parametrization of stochastic effects in an advection–condensation model

Yue-Kin Tsang

Centre for Geophysical and Astrophysical Fluid Dynamics, Mathematics, University of Exeter

Jacques Vanneste (Edinburgh), Geoff Vallis (Exeter)

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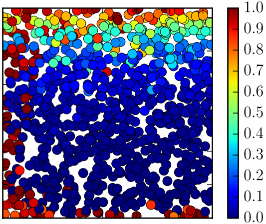
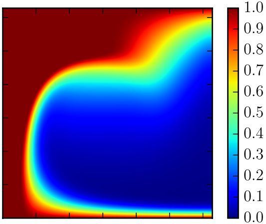




Atmospheric moisture and climate

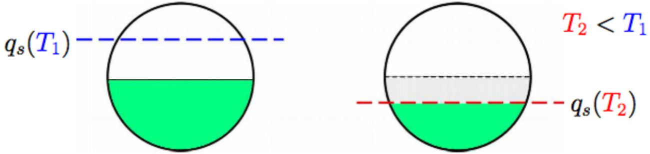
- Earth's radiation budget:
  - absorption of incoming short-wave radiation generates heat
  - heat carried away by outgoing long-wave radiation (OLR)
- water vapour is a greenhouse gas that traps OLR
- $\mathrm{OLR} \sim -\langle \log[\langle q \rangle + q'] \rangle \approx -\log \langle q \rangle + \frac{1}{2\langle q \rangle^2} \langle q'^2 \rangle$
- how fluctuation q' is generated?
- what is the probability distribution of water vapour in the atmosphere?

Condensation of water vapour

- specific humidity of an air parcel:

$$q = \frac{\text{mass of water vapour}}{\text{total air mass}}$$

- saturation specific humidity, $q_s(T)$
  - when $q > q_s$, condensation occurs
  - excessive moisture precipitates out, $q \to q_s$
  - $q_s(T)$ decreases with temperature T
  - $q_s(y)$ as $T = T(y)$, y = latitude (advection on a mid-latitude isentropic surface) or altitude (vertical convection in troposphere)

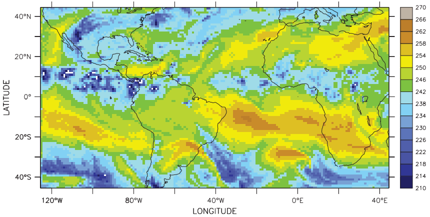


Advection–condensation paradigm

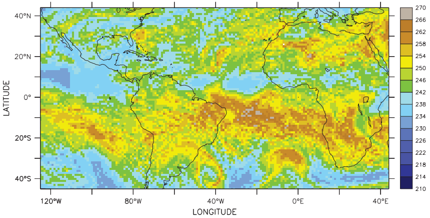
Large-scale advection + condensation

→ reproduce (leading-order) observed humidity distribution

observation



simulation



- velocity and q_s field from observation
- trace parcel trajectories backward to the lower boundary layer (source)
- track the minimum q_s encountered along the way
- ignore: cloud-scale microphysics, molecular diffusion, ...**etc**

(Pierrehumbert & Roca, GRL, 1998)

Advection–condensation model

- Particle formulation:

$$\mathrm{d}\vec{X}(t) = \vec{u}\,\mathrm{d}t\,, \quad \mathrm{d}Q(t) = (S - C)\mathrm{d}t$$

air parcel at location $\vec{X}(t)$ carrying specific humidity $Q(t)$

  - $S =$ moistures source (evaporation)
  - $C =$ condensation sink, in the rapid condensation limit

$$C : Q \mapsto \min\left[\,Q\,,\; q_s(\vec{X})\,\right]$$

  - saturation profile: $q_s(y) = q_0 \exp(-\alpha y)$

- Mean-field formulation:

$$\frac{\partial \bar{q}}{\partial t} + \vec{u} \cdot \nabla \bar{q} = S - C$$

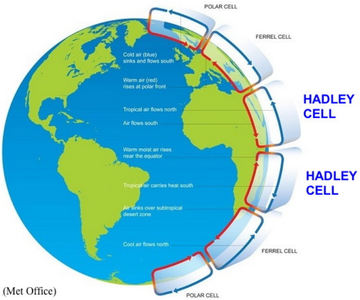
$\bar{q}(\vec{x}, t)$ is treated as a passive scalar field advected by $\vec{u}$

Particle models: previous analytical results

1-D stochastic models: $u \sim$ spatially uncorrelated random process

- **Pierrehumbert, Brogniez & Roca 2007**: white noise, $S = 0$
- **O'Gorman & Schneider 2006**: Ornstein–Uhlenbeck process, $S = 0$
- **Sukhatme & Young 2011**: white noise with a boundary source

Coherent circulation in the atmosphere

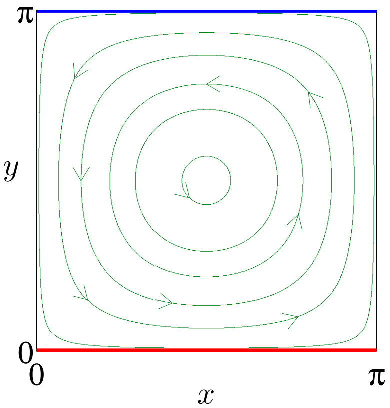


(Met Office)

Q: response of rainfall patterns to changes in the Hadley cells?

Advection–condensation in cellular flows

- bounded domain: $[0,\pi] \times [0,\pi]$, reflective boundaries
- $q_s(y) = q_{\max} \exp(-\alpha y)$: $q_s(0) = q_{\max}$ and $q_s(\pi) = q_{\min}$
- resetting source: $Q = q_{\max}$ if particle hits $y = 0$

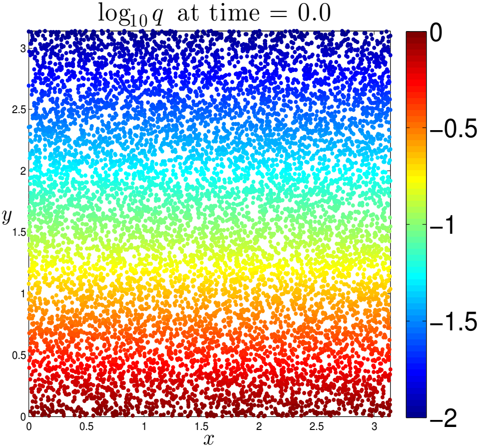


cellular flow: $\psi = -U \sin(x) \sin(y)$; $(u, v) = (-\psi_y, \psi_x)$

Particle formulation

$$\mathrm{d}X(t) = u(X,Y)\,\mathrm{d}t + \sqrt{2\kappa}\,\mathrm{d}W_1(t)$$
$$\mathrm{d}Y(t) = v(X,Y)\,\mathrm{d}t + \sqrt{2\kappa}\,\mathrm{d}W_2(t)$$
$$\mathrm{d}Q(t) = [S(Y) - C(Q,Y)]\mathrm{d}t$$

$$\psi = -U\sin x \sin y$$
$$u = -\psi_y$$
$$v = \psi_x$$



$U = 1$

$\kappa = 10^{-2}$

PDF of specific humidity – a dry spike

$$\mathrm{d}X(t) = u(X,Y)\,\mathrm{d}t + \sqrt{2\kappa}\,\mathrm{d}W_1(t)$$
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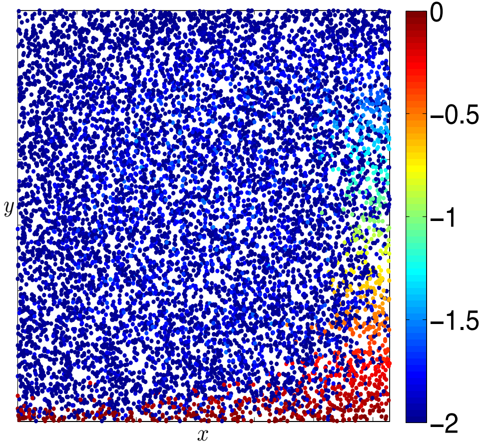
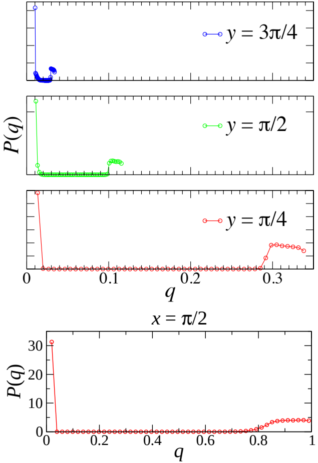


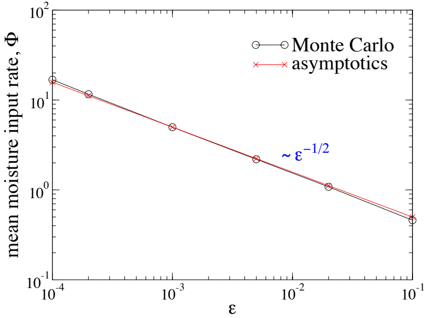


Fokker-Planck equation: solution and diagnostics

- Steady-state Fokker-Planck equation for $P(x, y, q)$:

$$\epsilon^{-1}\vec{u} \cdot \nabla P = \nabla^2 P\,, \quad \epsilon = \kappa/(UL) \ll 1$$

- solve for $P(x, y, q)$ by matched asymptotics as $\epsilon \to 0$
- dry spike: $P(x, y, q) = \delta(q - q_{\min})\beta(x, y)/\pi^2 + F(x, y, q)$
- mean moisture input rate: $\Phi = \epsilon^{-1/2}\kappa\sqrt{8/\pi}(q_{\max} - q_{\min})$



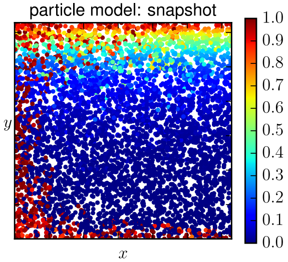
Other diagnostics: horizontal rainfall profile, moisture flux, …etc, see *"Advection–condensation of water vapour in a model of coherent stirring", Yue-Kin Tsang & Jacques Vanneste (2016)*

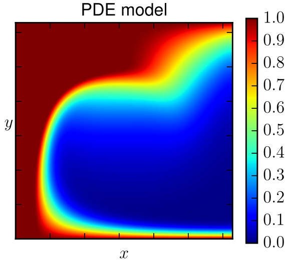
Mean-field PDE model

- Weather/climate models represent atmospheric moisture as a coarse-grained field $\bar{q}(\vec{x}, t)$ governed by deterministic PDE
- Advection–condensation–diffusion:

$$\frac{\partial \bar{q}}{\partial t} + \vec{u} \cdot \nabla \bar{q} = \kappa_q \nabla^2 \bar{q} - C + S$$

  - κ_q: eddy diffusivity representing un-resolved processes
  - boundary source: $\bar{q}(x, y = 0, t) = q_{\max}$
  - rapid condensation $C : \bar{q}(\vec{x}, t) \to \min[\bar{q}(\vec{x}, t), q_s(y)]$





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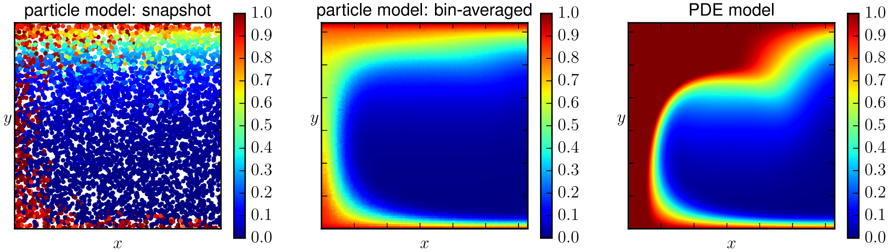
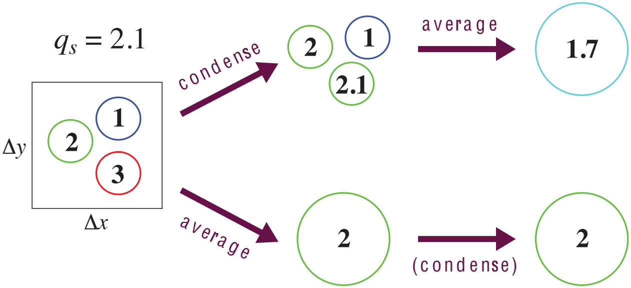


Why PDE models saturate the domain?

The coarse-graining process and the condensation process do not commute:



Parametrization of condensation

$$\frac{\partial \bar{q}}{\partial t} + \vec{u} \cdot \nabla \bar{q} = \kappa_q \nabla^2 \bar{q}, \quad \bar{q} \to C(\bar{q}, q_s)$$

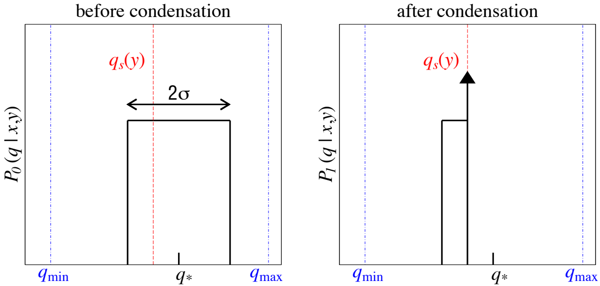
- at a grid point (x, y) and time t, after advection and diffusion steps

$$\text{let's say} \quad \bar{q}(x, y, t) = q_*$$

- imagine there is a distribution $P_0(q|x, y)$ such that

$$q_* = \int q' P_0(q'|x, y)\, \mathrm{d}q'$$

$$\text{then,} \;\; \bar{q}(x, y, t + \Delta t) = \int q' P_1(q'|x, y)\, \mathrm{d}q'$$

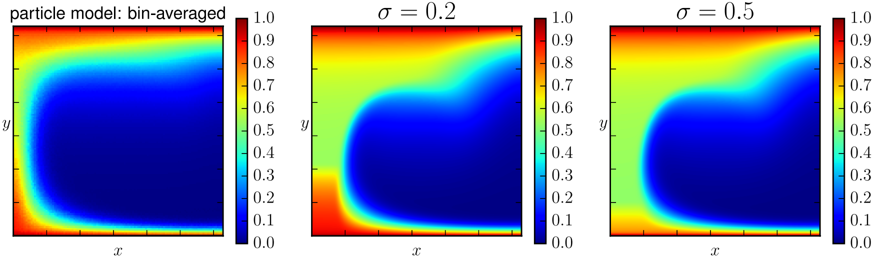


Test results

- $P_0(q|x,y)$: a top hat distribution of width 2σ
- as a test, prescribe a constant σ
- for $\bar{q} - \sigma < q_s < \bar{q} + \sigma$, condensation occurs as:

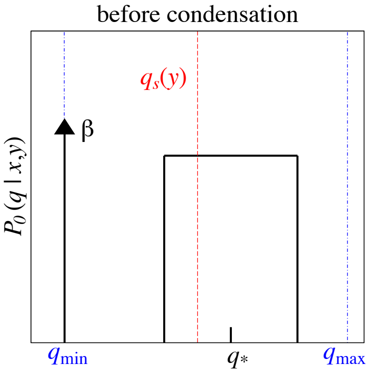
$$\bar{q} \to \bar{q} - \frac{[\bar{q} + \sigma - q_s]^2}{4\sigma}$$

$$\underline{\kappa_q = 0.01}$$



Parametrization with dry spike

- subsidence of dry air parcels is important
- include a dry spike of amplitude β in $P_0(q|x, y)$

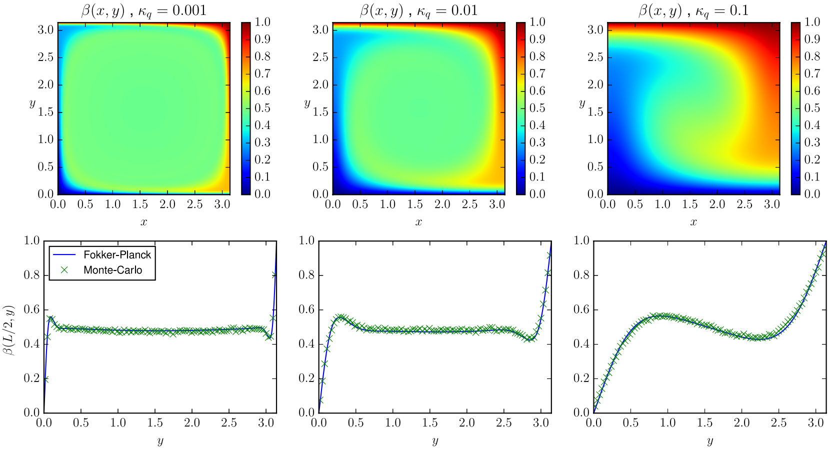


Amplitude of dry spike

$$P(q_{\min}, x, y, t) = \pi^{-2}\beta(x,y)\delta(q - q_{\min})$$

$$\frac{\partial \beta}{\partial t} + \vec{u}\cdot\nabla\beta = \kappa_q \nabla^2 \beta$$

$$\beta(x,0,t) = 0\,, \quad \beta(x,\pi,t) = 1$$



Results with dry spike