Probabilistic parametrization of condensation in coarse-grained moisture transport models

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Funded by the EPSRC network
Research on Changes of Variability and Environmental Risk (ReCoVER)
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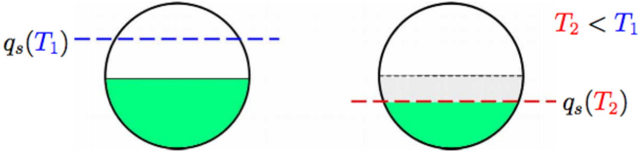
Condensation of water vapour

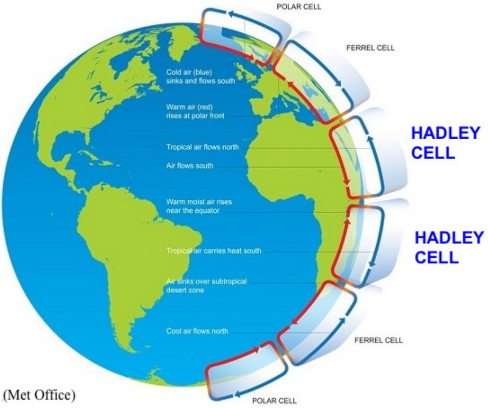
- specific humidity of an air parcel:

$$Q = \frac{\text{mass of water vapor}}{\text{total air mass}}$$

- saturation specific humidity, $q_s(T)$
  - $Q = \begin{cases} q_s & \text{if} \quad Q > q_s \quad \text{(excessive moisture condensed)} \\ Q & \text{otherwise} \end{cases}$
  - $q_s(T)$ decreases with temperature T, hence generally decreases with height



Moisture transport in the atmosphere

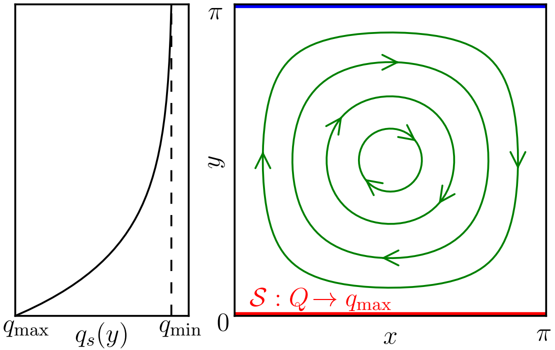


(Met Office)

- water vapour enters the atmosphere by evaporation from surface
- large-scale circulation, e.g. the Hadley cell:
  - moisture converges toward the Tropics, then rises and condensed
  - dry air moves poleward and subside near the sub-tropics
- small-scale flow: mixing between moist and dry air

Advection–condensation model

- bounded domain: $[0, \pi] \times [0, \pi]$, reflective boundaries
- $q_s(y) = q_{\max} \exp(-\alpha y)$: $q_s(0) = q_{\max}$ and $q_s(\pi) = q_{\min}$
- resetting source: $Q = q_{\max}$ if parcel hits $y = 0$
- large-scale cellular flow: $\psi = \sin x \sin y$; $\quad (u, v) = (-\psi_y, \psi_x)$
- small-scale turbulence: Brownian motion



Lagrangian versus Eulerian formulation

- Stochastic Lagrangian approach
  - consider an ensemble of moist parcels
  - position and specific humidity of individual parcel described by the random variables (Q, X, Y)
  - stochastic differential equations governing the evolution of (Q, X, Y)
- Deterministic Eulerian approach
  - specific humidity described by a coarse-grained field $q(x, y, t)$
  - evolution of $q(x, y, t)$ governed by a partial differential equation

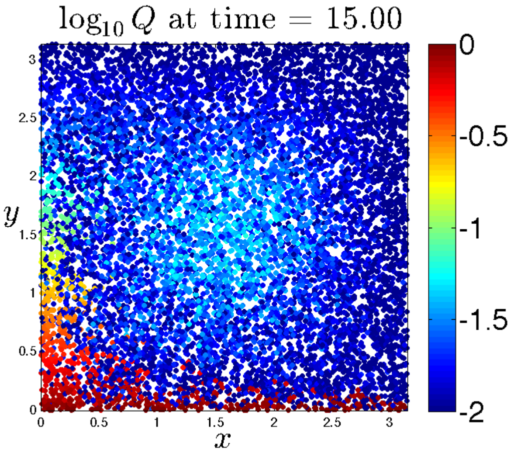
These two approaches give very different results!

Lagrangian particle formulation

$$\mathrm{d}X(t) = u(X,Y)\,\mathrm{d}t + \sqrt{2\kappa}\,\mathrm{d}W_1(t)$$
$$\mathrm{d}Y(t) = v(X,Y)\,\mathrm{d}t + \sqrt{2\kappa}\,\mathrm{d}W_2(t)$$
$$\mathrm{d}Q(t) = [S(Y) - C(Q,Y)]\mathrm{d}t$$

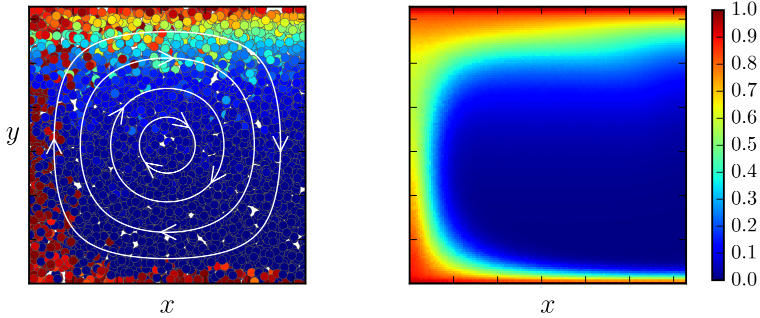
fast condensation $C : Q \to \min[\,Q\,, q_s(Y)\,]$

$$\psi = -\sin x \sin y$$
$$u = -\psi_y$$
$$v = \psi_x$$
$$\kappa = 10^{-2}$$



Lagrangian model: relative humidity

snapshot of parcels   "Observed" $q_{\text{bin}}(x, y)$



$$\text{Relative humidity} = \frac{Q}{q_s(Y)}$$

- Observation: divide the domain into small bins and average over parcels in each bin to get the field $q_{\text{bin}}(x, y)$

Eulerian field formulation

Weather/climate models: moisture field $q(x, y, t)$ governed by a PDE, the advection–condensation–diffusion equation:

$$\frac{\partial q}{\partial t} + \vec{u} \cdot \nabla q = \kappa \nabla^2 q + S - C$$

- κ: eddy diffusivity representing subgrid-scale turbulence (molecular diffusion neglected)

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- boundary source: $q(x, y = 0, t) = q_{\max}$

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$$\frac{\partial q_*}{\partial t} + \vec{u} \cdot \nabla q_* = \kappa \nabla^2 q_*$$

$$C : q(x, y, t) \rightarrow \min[q_*(x, y, t), q_s(y)]$$

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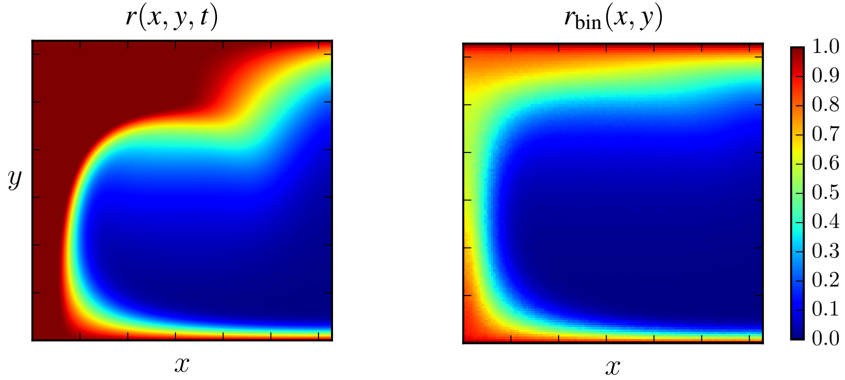
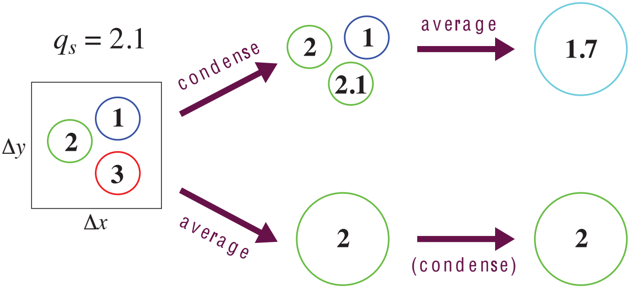


Why are PDE models so wet?

The coarse-graining process and the condensation process do not commute:



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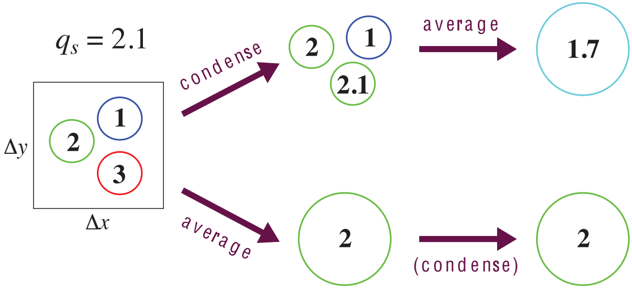


Question: How do we improve a PDE model? Can we put back some subgrid-scale fluctuation in the condensation term?

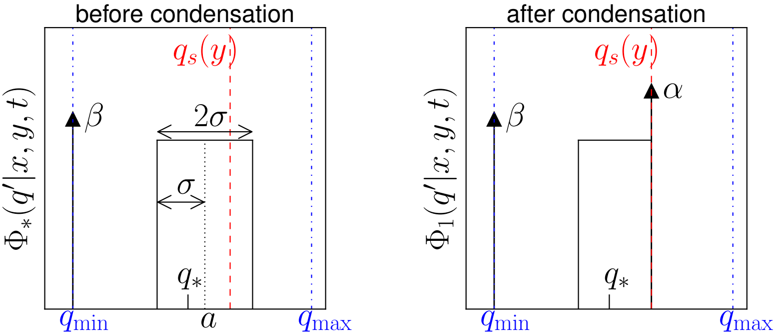
Condensation parametrization

$$\frac{\partial q_*}{\partial t} + \vec{u} \cdot \nabla q_* = \kappa \nabla^2 q_*$$

$$C : q(x,y,t) \to \min[q_*(x,y,t), q_s(y)]$$

- after advection–diffusion steps, imagine there is a distribution $\Phi_*(q'|x,y,t)$:

$$q_* = \int q' \Phi_*(q'|x,y,t) \, \mathrm{d}q'$$



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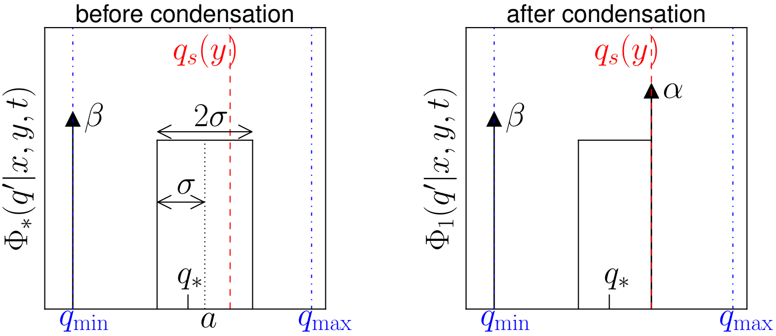
- fast condensation over the distribution Φ_* to gives $\Phi_1(q'|x, y; t)$



Condensation parametrization

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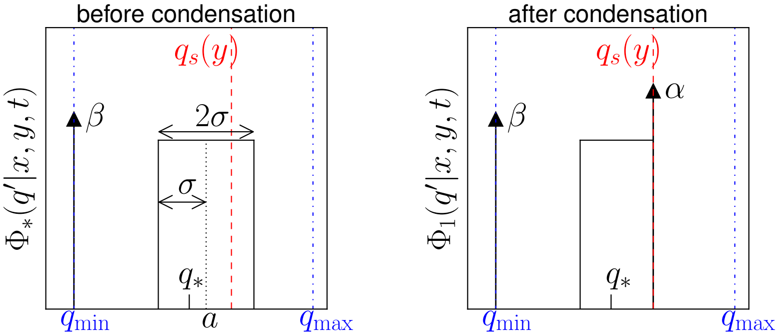
$$q_* = \int q' \Phi_*(q'|x, y, t) \, \mathrm{d}q'$$

- fast condensation over the distribution Φ_* to gives $\Phi_1(q'|x, y; t)$
- $q(x, y, t + \Delta t) = \int q' \Phi_1(q'|x, y, t) \, \mathrm{d}q'$

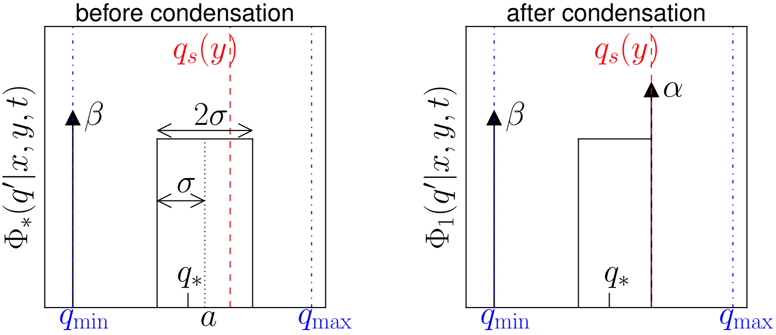


Method of assumed PDF: shape of Φ_*

- To implement the parametrization, need to fix the shape of Φ_*
- For our idealised overturning cell:

$$\Phi_*(q'|x,y,t) = \beta(x,y,t)\delta(q - q_{\min}) + \tilde{\Phi}_*(q'|x,y,t)$$

with three parameters (β, a, σ)



Determination of the PDF parameters

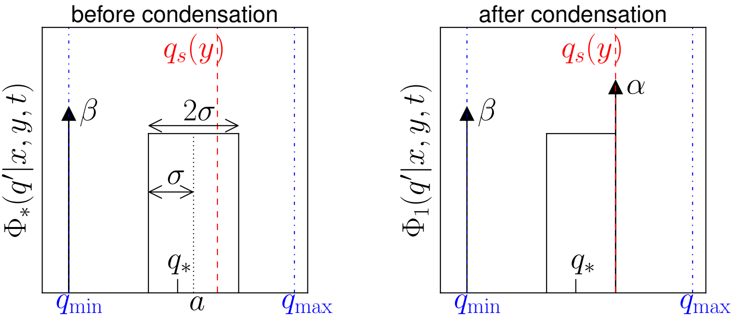
- The dry spike, β

$$\partial_t \beta + \vec{u} \cdot \nabla \beta = \kappa \nabla^2 \beta, \quad \beta(x, 0, t) = 0, \quad \beta(x, \pi, t) = 1$$

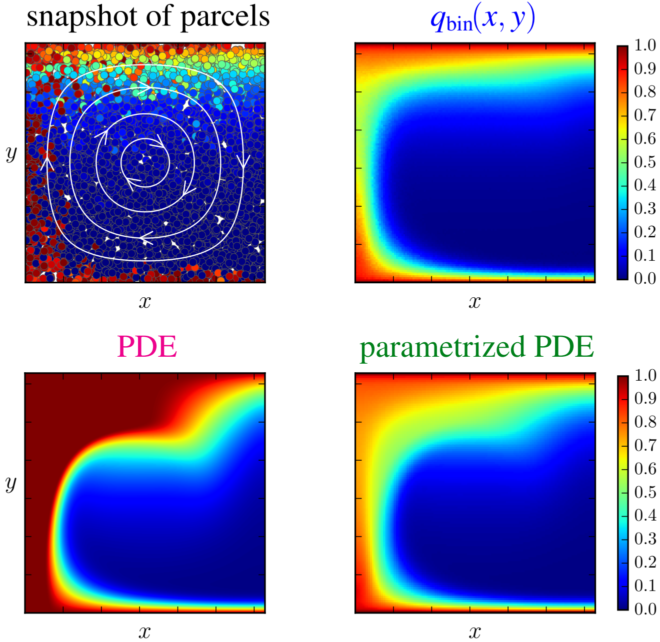
- The location, a

$$q_* = \int q' \Phi_*(q'|x, y, t)\, \mathrm{d}q' \quad \Rightarrow \quad a = \frac{q_* - \beta q_{\min}}{1 - \beta}$$

- The width, σ
  - fixed empirically "by hand"
  - considering the second moment of q' (additional computation)



Results: relative humidity



Mathematics of the parametrization scheme

- Recall (Q, X, Y) are governed by a set of SDEs
  $\Rightarrow$ the joint PDF $P(q, x, y; t)$ satisfies the Fokker-Planck equation:

$$\frac{\partial P}{\partial t} + \vec{u} \cdot \nabla P - \frac{\partial}{\partial q}(CP) = \kappa \nabla^2 P$$

- $q_{\mathrm{bin}}(x, y, t) \approx \overline{q}(x, y, t) = \int_{q_{\min}}^{q_{\max}} q' \, \hat{P}(q'|x, y; t) \, \mathrm{d}q'$

$$\frac{\partial \overline{q}}{\partial t} + \vec{u} \cdot \nabla \overline{q} = \kappa \nabla^2 \overline{q} - \int_{q_{\min}}^{q_{\max}} C(q', q_s) \hat{P}(q'|x, y; t) \, \mathrm{d}q'$$

- The coarse-grained field

$$\frac{\partial q}{\partial t} + \vec{u} \cdot \nabla q = \kappa \nabla^2 q - C(q, q_s)$$

- Parametrized condensation

$$\frac{\partial q}{\partial t} + \vec{u} \cdot \nabla q = \kappa \nabla^2 q - \int_{q_{\min}}^{q_{\max}} C(q', q_s) \Phi_*(q'|x, y; t) \, \mathrm{d}q'$$