A Test of Local Effective Diffusivity Parameterization in a Two-Layer, Wind-Driven Isopycnal Primitive Equation Model

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Means and Eddies

- Low-pass filtering via running average in space and time:

$$\bar{f}(x,y,t) = \frac{1}{\tau\ell^2}\int_{t-\tau/2}^{t+\tau/2} dt' \int_{x-\ell/2}^{x+\ell/2} dx' \int_{y-\ell/2}^{y+\ell/2} dy'\, f(x',y',t')$$

- Eddy components: $f' \equiv f - \bar{f}$
- Averaging the passive tracer advection-diffusion equation

$$c_t + \nabla\cdot(\mathbf{u}c) = \kappa\nabla^2 c + S$$

gives the large-scale equation

$$\boxed{\bar{c}_t + \nabla\cdot(\mathbf{F} + \bar{\mathbf{u}}\bar{c}) = \kappa\nabla^2\bar{c} + \bar{S}}$$

- Eddy tracer flux: $\mathbf{F} = \overline{\mathbf{u}'c'}$

Eddy Diffusivity

- Flux-gradient relation:

$$\mathbf{F} = \overline{\mathbf{u}'c'} = -\mathbf{K}\nabla\bar{c}, \quad \text{where} \quad \mathbf{K} = \begin{pmatrix} K^{\mathrm{xx}} & K^{\mathrm{xy}} \\ K^{\mathrm{yx}} & K^{\mathrm{yy}} \end{pmatrix}$$

- Eddy diffusion for $\bar{c}$,

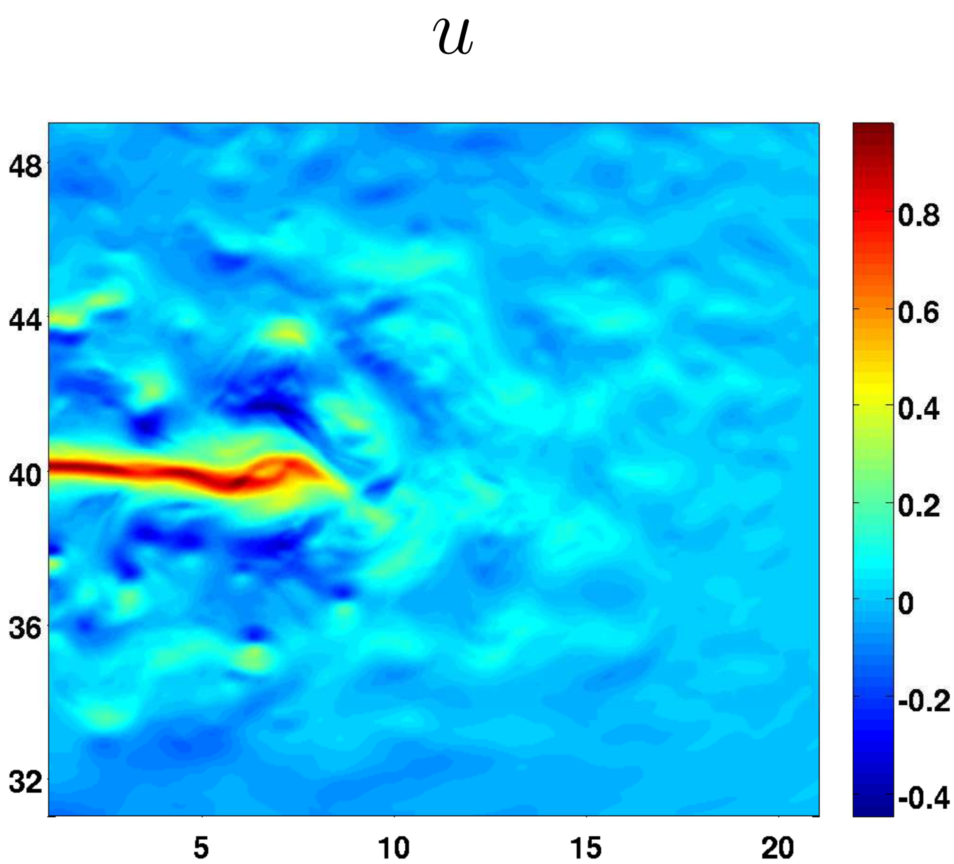
$$\boxed{\bar{c}_t + \nabla \cdot (\bar{\mathbf{u}}\bar{c}) = \nabla \cdot [\,(\kappa\mathbf{I} - \mathbf{K})\nabla\bar{c}\,] + \bar{S}}$$

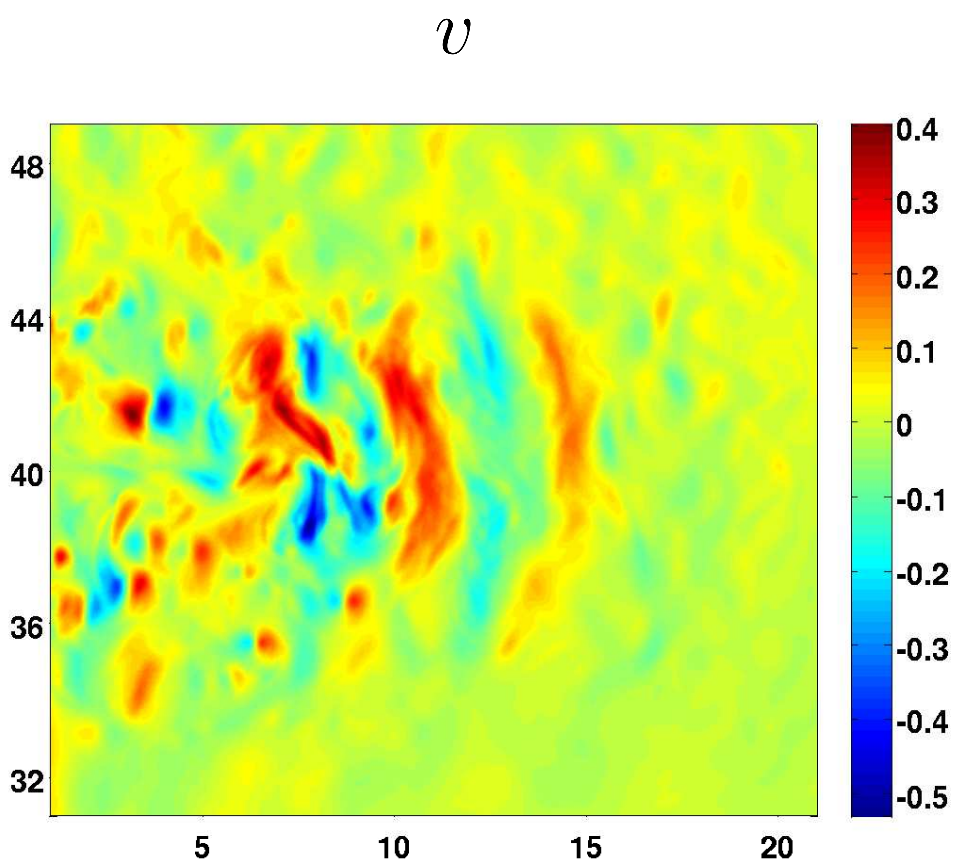
- If the small-scale statistics is inhomogeneous, $\mathbf{K} = \mathbf{K}(\mathbf{x})$
- Try to identify approximately local homogeneous regions (cells) and make estimation to **K** that is constant in a cell

Ocean Circulation Model

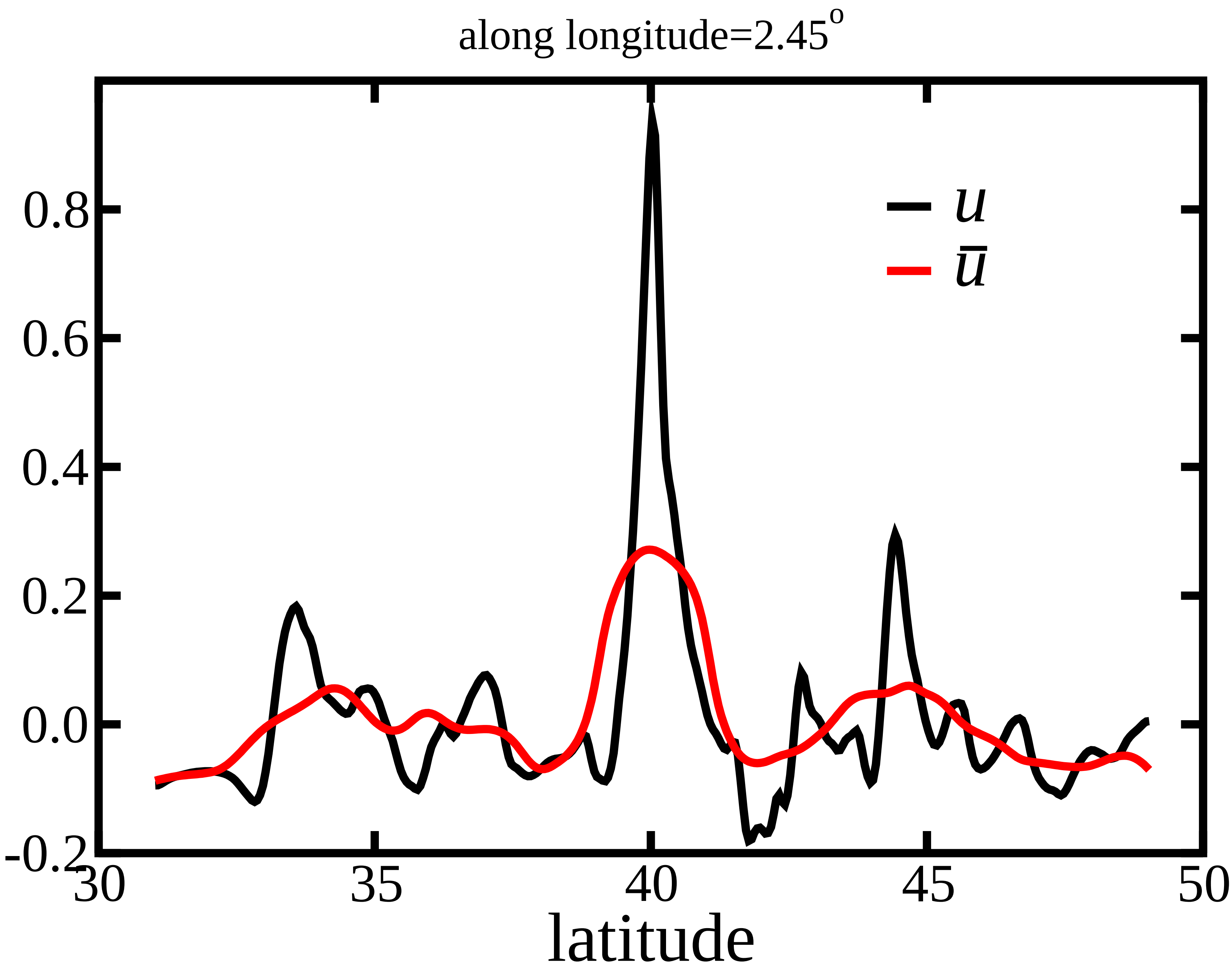
- Two-layer, adiabatic, isopycnal primitive equations model (HIM)
- Mid-latitude, flat bottom basin of size $22^\circ \times 20^\circ$
- Zonal sinusoidal wind-forcing (double-gyre configuration)
- Linear bottom drag and biharmonic viscosity
- Resolution: $1/20$ degree

Snapshots of u and v

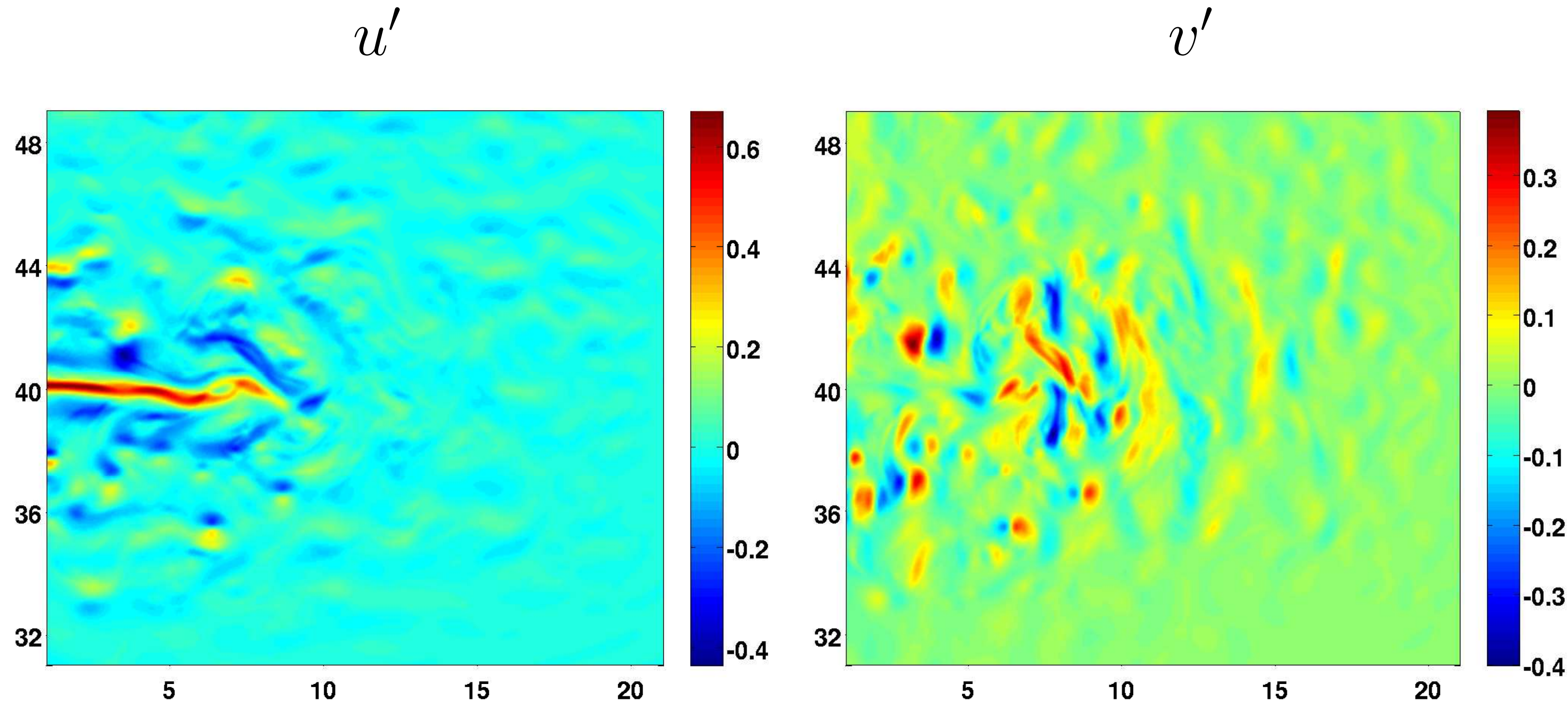




Example of Averaging

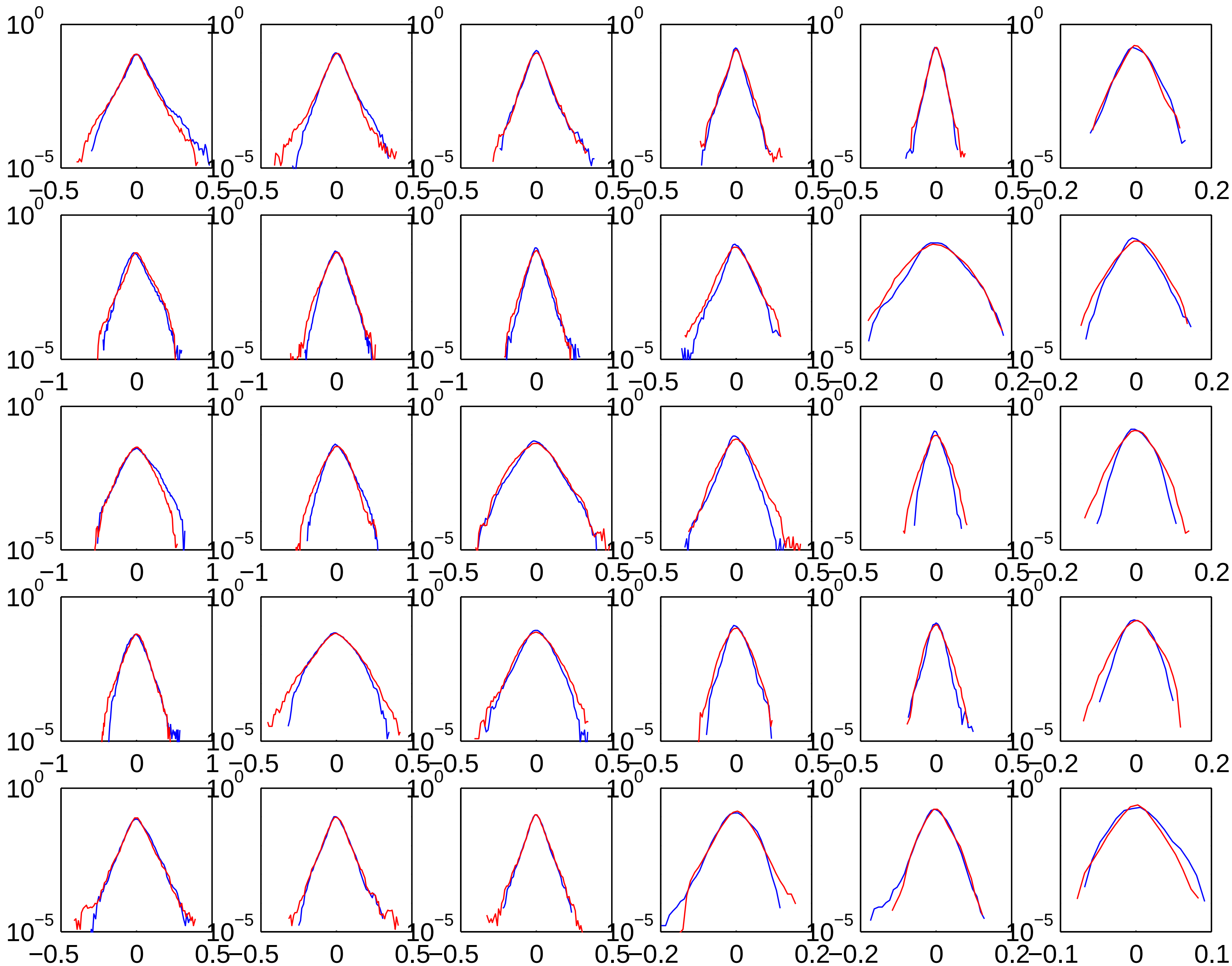


Snapshots of u' and v'



PDF of u' and v'

divide the domain into $2^\circ \times 2^\circ$ cells



Questions

- Can we “measure” **K** directly from high resolution data?
- Can we make theoretical prediction on **K**?

Measuring $\mathbf{K}(\mathbf{x})$

Recall $\overline{\mathbf{u}'c'} = -\mathbf{K}\nabla\bar{c}$, write $\mathbf{G} = \nabla\bar{c}$:

$$\overline{u'c'} = \mathbf{K}^{\mathrm{xx}}\mathbf{G}_x + \mathbf{K}^{\mathrm{xy}}\mathbf{G}_y$$

$$\overline{v'c'} = \mathbf{K}^{\mathrm{yx}}\mathbf{G}_x + \mathbf{K}^{\mathrm{yy}}\mathbf{G}_y$$

$\Rightarrow$ four unknowns and two equations.

$\mathbf{K}$ is property of flow, use two independent tracers a and b forced by different large-scale gradients: $\mathbf{\Gamma} = \nabla\bar{a}$, $\mathbf{\Lambda} = \nabla\bar{b}$,

$$\overline{u'a'} = \mathbf{K}^{\mathrm{xx}}\Gamma_x + \mathbf{K}^{\mathrm{xy}}\Gamma_y$$

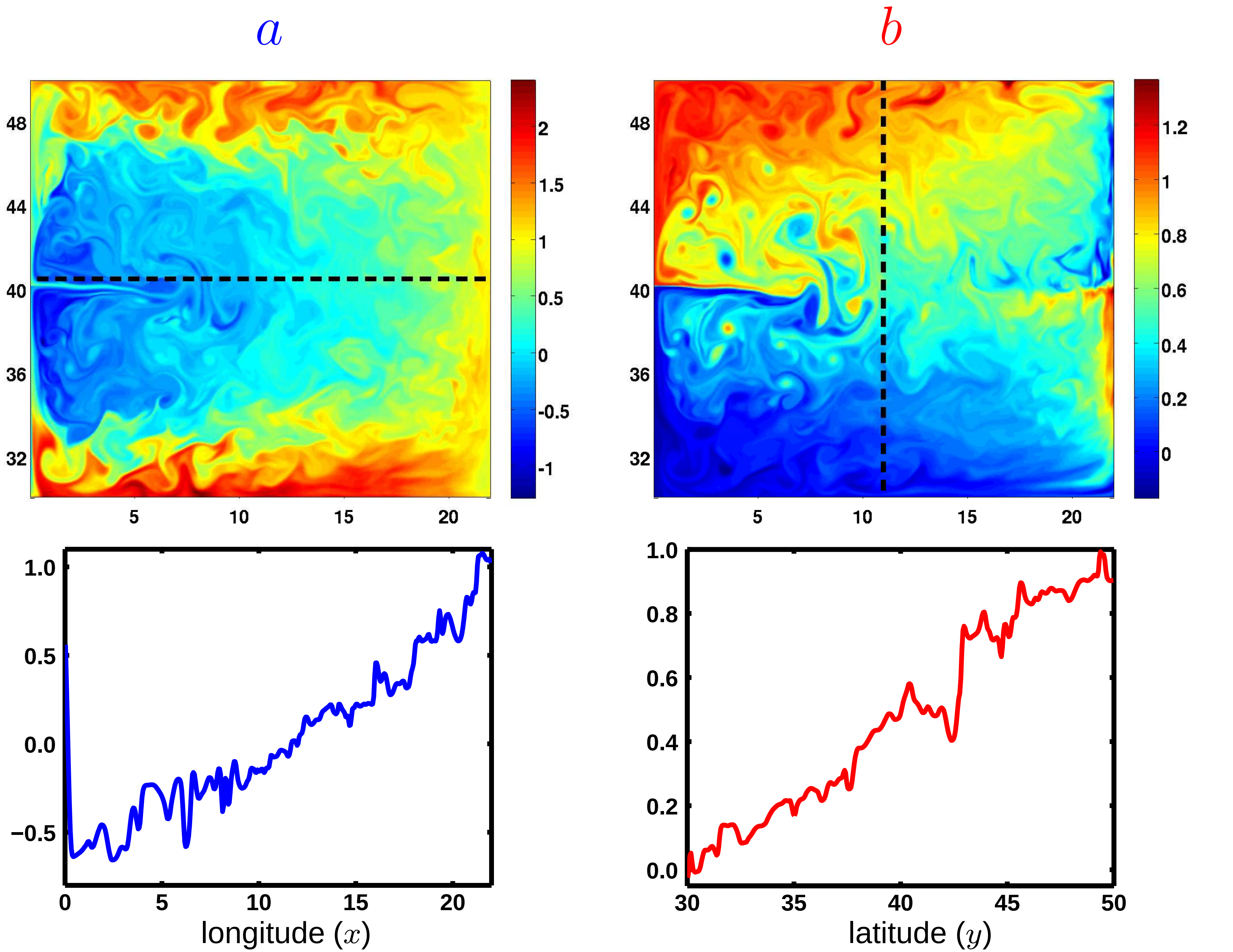
$$\overline{v'a'} = \mathbf{K}^{\mathrm{yx}}\Gamma_x + \mathbf{K}^{\mathrm{yy}}\Gamma_y$$

$$\overline{u'b'} = \mathbf{K}^{\mathrm{xx}}\Lambda_x + \mathbf{K}^{\mathrm{xy}}\Lambda_y$$

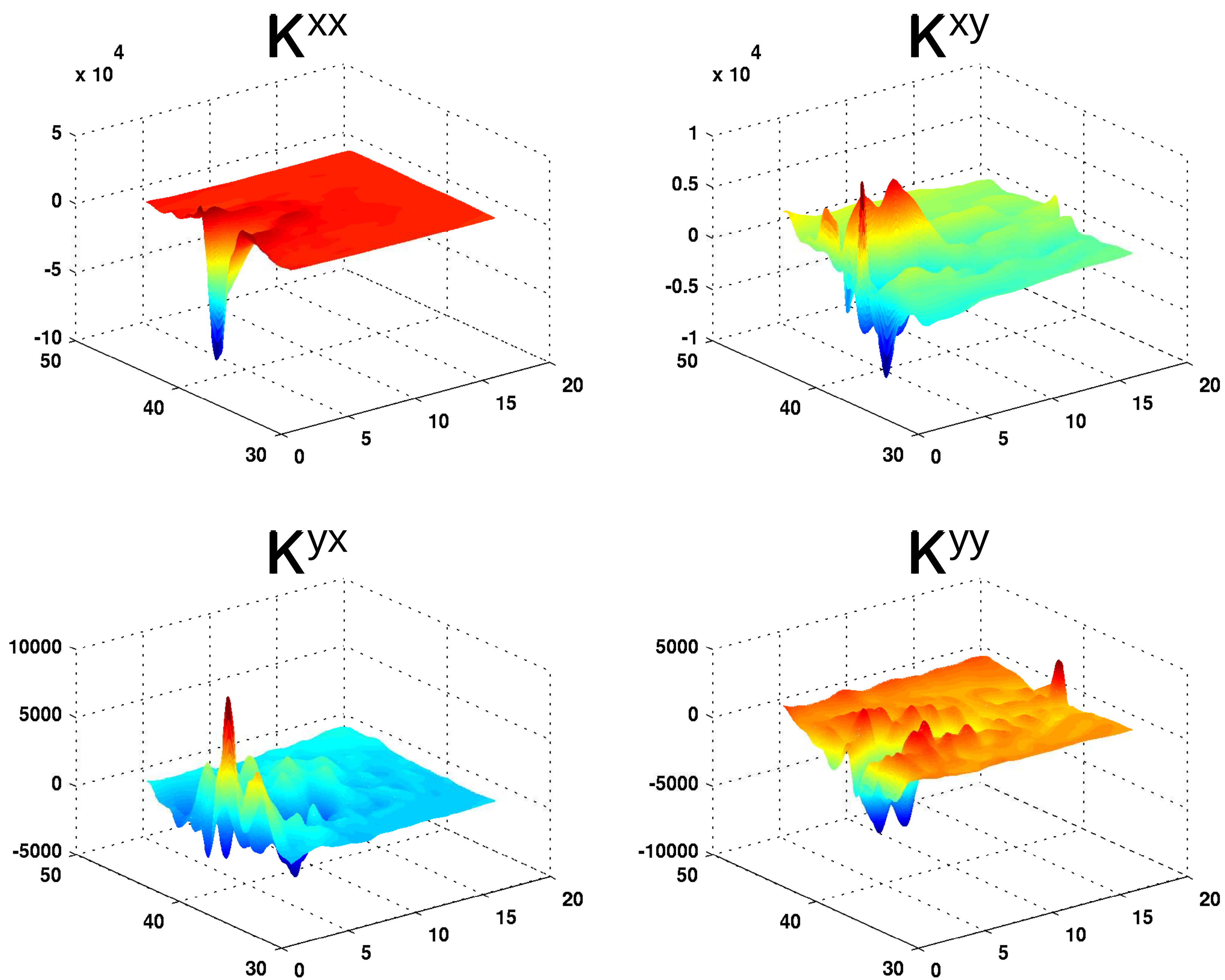
$$\overline{v'b'} = \mathbf{K}^{\mathrm{yx}}\Lambda_x + \mathbf{K}^{\mathrm{yy}}\Lambda_y$$

$\Rightarrow$ four unknowns and four equations.

Tracer a and b



Measured $K(x)$



K^{xx}
x 10^4
5
0
-5
-10
50
40
30
0
5
10
15
20
K^{xy}
x 10^4
1
0.5
0
-0.5
-1
50
40
30
0
5
10
15
20
K^{yx}
10000
5000
0
-5000
50
40
30
0
5
10
15
20
K^{yy}
5000
0
-5000
-10000
50
40
30
0
5
10
15
20

Transport Theory

Isotropic local mixing length estimate

$$V_T = \sqrt{u'^2 + v'^2}$$

$$L_{mix} = V_T / |\nabla V_T|$$

$$\mathbf{K}^{\text{xx}} = \mathbf{K}^{\text{yy}} = c V_T L_{mix}$$

Shear dispersion in x + Mixing length in y

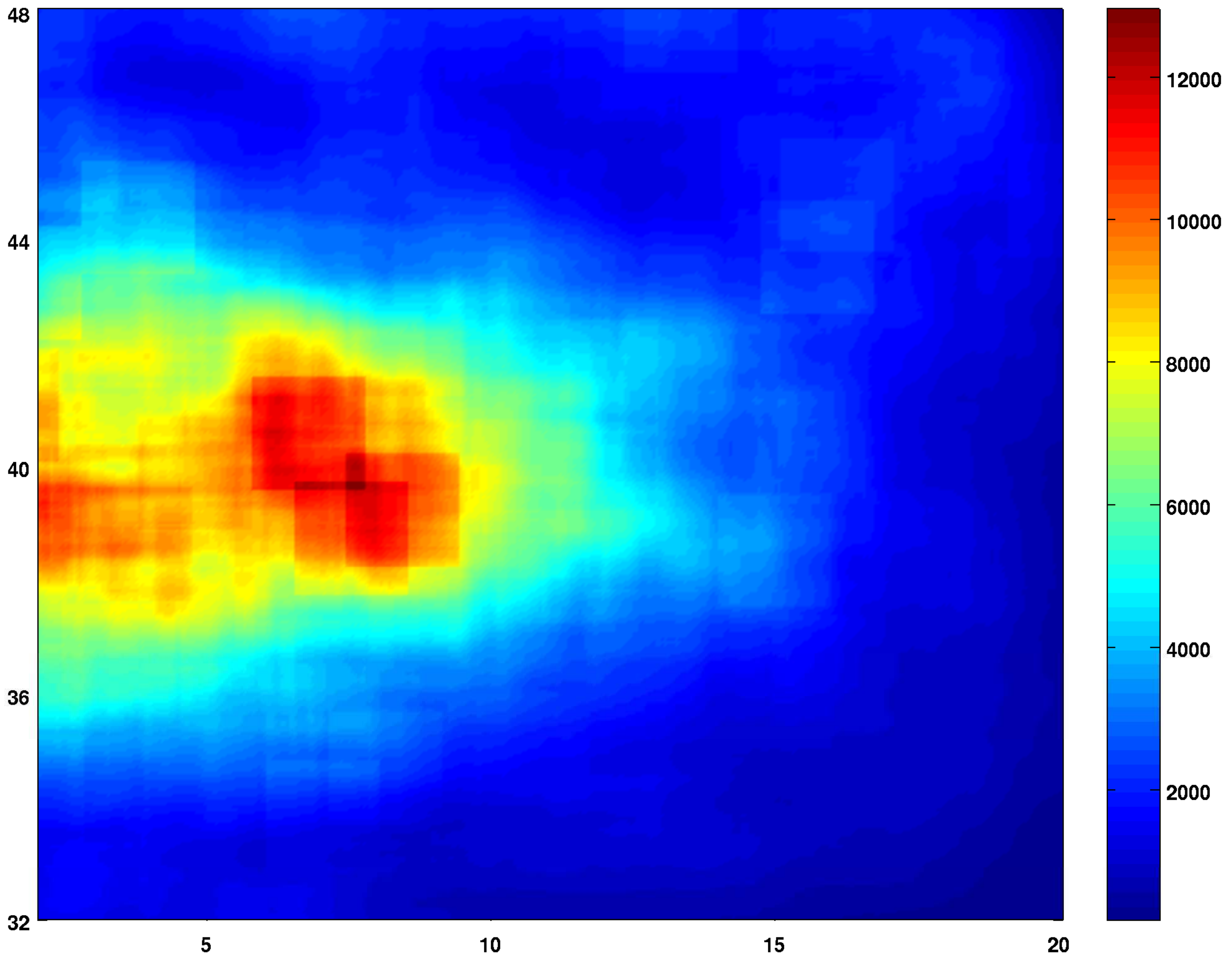
$$\mathbf{K}^{\text{yy}} = c V_T L_{jet} \quad (L_{jet} \approx \text{ jet width})$$

$$\mathbf{K}^{\text{xx}} = \frac{U_{jet}^2 L_{jet}^2}{\mathbf{K}^{\text{yy}}} \quad (U_{jet} \approx \text{jet speed})$$

K. S. Smith, J. Fluid Mech. **544**, 133 (2005)

Isotropic Mixing Length Estimate

K^{xx} (or K^{yy})



Summary

- Study the eddy diffusivity tensor $\mathbf{K}(\mathbf{x})$ in an inhomogeneous system (the double-gyre configuration)
- Measure $\mathbf{K}(\mathbf{x})$ from high resolution simulation data
- Make theoretical estimation on $\mathbf{K}(\mathbf{x})$ in some regions of the domain using mixing length theory
- Work in progress:
  - improve measurement of $\mathbf{K}(\mathbf{x})$
  - use more sophisticated theory to predict $\mathbf{K}(\mathbf{x})$ in the whole domain