PHYS4520 Physics in Meteorology

Problem Set 4

1. *Dimensional analysis.* Consider a sphere of diameter d moving at speed u_0 through a fluid otherwise at rest. The density of the fluid ρ_0 is taken to be constant.The sphere experiences a drag force F_D due to the viscosity μ of the fluid. It is convenient to quantify such drag force by a dimensionless drag coefficient C_D defined as

$$C_D = \frac{F_D}{\frac{1}{2}\rho_0 u_0^2 d^2} \,.$$

In general, F_D and C_D depend on all the parameters of the problem: d, u_0, ρ_0 and μ.

   (a) As discussed in class, for low Reynolds number steady flows, the density becomes unimportant. Hence, we can write

$$F_D = K d^\alpha u_0^\beta \mu^\gamma \,,$$

   where K is a dimensionless constant. By matching the dimensions on both sides of the above relation, determine the exponents α, β and γ. Also determine the dependence of C_D on the Reynolds number.

   (b) Repeat the analysis for high Reynolds number flows in which the viscosity becomes unimportant and determine how C_D depends on the Reynolds number.

2. The acceleration due to gravity $\vec{g}_{\text{grav}}$ can be expressed in terms of a potential ϕ,

$$\vec{g}_{\text{grav}} = -\nabla\phi \,.$$

Similarly, the centrifugal acceleration is expressed in terms of ϕ_c,

$$-\vec{\Omega} \times (\Omega \times \vec{r}) = -\nabla\phi_c \,.$$

We can define an effective gravity $\vec{g}_{\text{eff}}$ using the geopotential $\Phi \equiv \phi + \phi_c$,

$$\vec{g}_{\text{eff}} \equiv -\nabla\Phi = \vec{g}_{\text{grav}} - \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) \,.$$

As discussed in class, we use $\vec{g}_{\text{eff}}$ instead of $\vec{g}_{\text{grav}}$ to define the vertical direction in the tangent plane approximation. Let ψ be the angle between $\vec{g}_{\text{grav}}$ and $\vec{g}_{\text{eff}}$. At what latitudes is $\psi = 0$? Estimate the value of ψ at Hong Kong?

3. For a two-dimensional flow $\vec{u} = (u, v, 0)$, u and v are independent of the vertical coordinate z and the vorticity has only one component $\vec{\omega} = \zeta(x, y)\hat{k}$. Assume the flow is incompressible, inviscid and there is no body force, derive the equation of motion for ζ in a rotating frame under the β-plane approximation