PHYS4520 Physics in Meteorology

Problem Set 10

1. We have derived the quasi-geostrophic (QG) approximation for the shallow-water system in class. In the limit of infinite deformation radius $L_d \to \infty$, the QG equation simplified to,

$$\frac{\partial q}{\partial t} + J(\psi_g, q) = 0\,,$$
$$\nabla^2 \psi_g + f = q\,.$$

Let ψ_g' be a small perturbation about the rest state: $\psi_g = \psi_g^{(0)} + \psi_g'$.

   (a) Linearize the above simplified QG equation about the rest state and obtain the time evolution equation for ψ_g'.

   (b) Derive the dispersion relation in this limit and check it is consistent with the more general case considered in class.

   (c) Using the relation between ψ_g and (u_g, v_g), write down the corresponding expression for the perturbation velocities, u_g' and v_g'.

2. Let R_d, R_v and R_m be the gas constants of dry air, water vapor and moist air respectively. The ideal gas law for moist air is,

$$p = \rho R_m T\,.$$

If the mixing ratio is w, derive an expression for R_m in terms of w, R_v and R_d.