PHYS4520 Physics in Meteorology

Problem Set 1

1. Assume the atmosphere is isothermal, at rest and made up of ideal gas,

   (a) show that the mass of a vertical column of air of unit cross-section, extending from the ground to great height, is p_0/g, where p_0 is the surface pressure.

   (b) take the surface temperature $T_0 = 290$K, assuming the top of the troposphere is 20km above the Earth's surface, find the percentage of mass contained in the troposphere.

2. If there is a uniform lapse rate $\Gamma = -dT/dz$, derive the pressure profile $p(z)$ as a function of height z of an atmosphere at rest, assuming the atmosphere behaves as an ideal gas. Where is the upper boundary z_u of the atmosphere where $p(z_u) = 0$.

3. *Parallel flow.* The Navier-Stokes equation simplifies greatly for unidirectional flow: $\vec{u} = u(x, y, z, t)\hat{i}$. By considering the Navier-Stokes equation in its component form, show that

   (a) $\vec{u} = u(y, z, t)\hat{i}$, i.e. the velocity is independent of the streamwise coordinate,

   (b) if there is no body force, the pressure $p = p(x, t)$, and

   (c) the pressure gradient $\partial p/\partial x = G$ where G is a constant.

   (d) Hence, write down the equation of u for a steady unidirectional flow.