PHYS 4520 Physics in Meteorology

Atmospheric Thermodynamics

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Ideal gas law

$$n = \frac{m}{M}$$

$$p = \frac{n}{V} R^* T$$

$$p = \rho R T$$

$$p = \frac{N}{V} k_B T$$

n : no of moles

m : total mass

M : molecular mass

N : no of molecules

N_A : Avogadro's number

R^* : universal gas constant

$R = R^* / M$: gas constant

$k_B = R^* / N_A$: Boltzmann's constant

Dry air: a multiple-component system

total mass = $\sum m_i$

total no. of moles = $\sum \frac{m_i}{M_i}$

Apparent molecular mass of dry air: $M_d = \frac{\sum m_i}{\sum \frac{m_i}{M_i}}$

Partial pressure

$p_i =$ pressure that would be exerted by the i-th constituent gas if it alone was to occupy the same volume at the same temperature as the whole system

Dry air: a multiple-component system

Each constituent gas obeys the ideal gas law:

$$p_i = \rho_i R_i T$$

In particular, for water vapor,

vapor pressure: $e = \rho_v R_v T$

Dalton's law of partial pressure

total pressure : $p = \sum_i p_i$

Moist air = dry air + water vapor

Ideal gas law: $p = \rho_{\mathrm{moist}} R_{\mathrm{moist}} T$

$$\because \quad M_{\mathrm{moist}} < M_d$$

$$\Rightarrow \quad R_{\mathrm{moist}} = \frac{R^*}{M_{\mathrm{moist}}} > \frac{R^*}{M_d} = R_d$$

Virtual temperature

$$p = \rho_{\mathrm{moist}} R_d T_{\mathrm{virt}}$$

$$T_{\mathrm{virt}} = \frac{T}{1 - \frac{e}{p}(1-\varepsilon)}, \quad \varepsilon \equiv \frac{R_d}{R_v}$$

Moisture parameters

Mixing ratio (for a given sample of air)

$$w = \frac{m_v}{m_d} = \frac{\text{mass of water vapor}}{\text{mass of dry air}}$$

unit: grams of water vapor per kilogram of dry air

Specific humidity

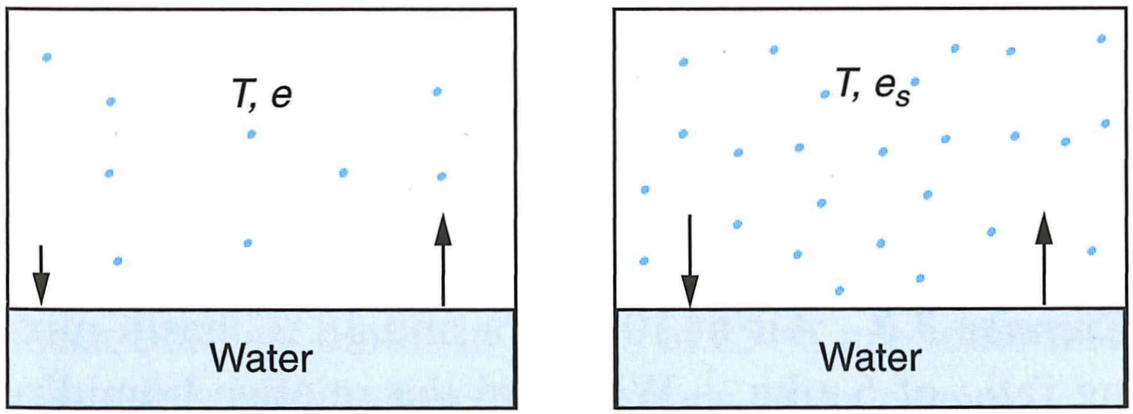
$$q = \frac{\text{mass of water vapor}}{\text{total air mass}}$$

$$= \frac{m_v}{m_v + m_d} = \frac{w}{1+w}$$

w, q are independent of pressure and temperature

Moisture parameters

Saturation vapor pressure $e_s(T)$



(a) Unsaturated (b) Saturated (Wallace & Hobbs 2006)

- assume initially the air is completely dry
- water begins to evaporate
- vapor pressure increases
- rate of condensation increases
- vapor pressure at which rate of evaporation = rate of condensation is called the *saturation vapor pressure over a plane surface of pure water at temperature T*

Moisture parameters

$e_s(T)$ increases with temperature T

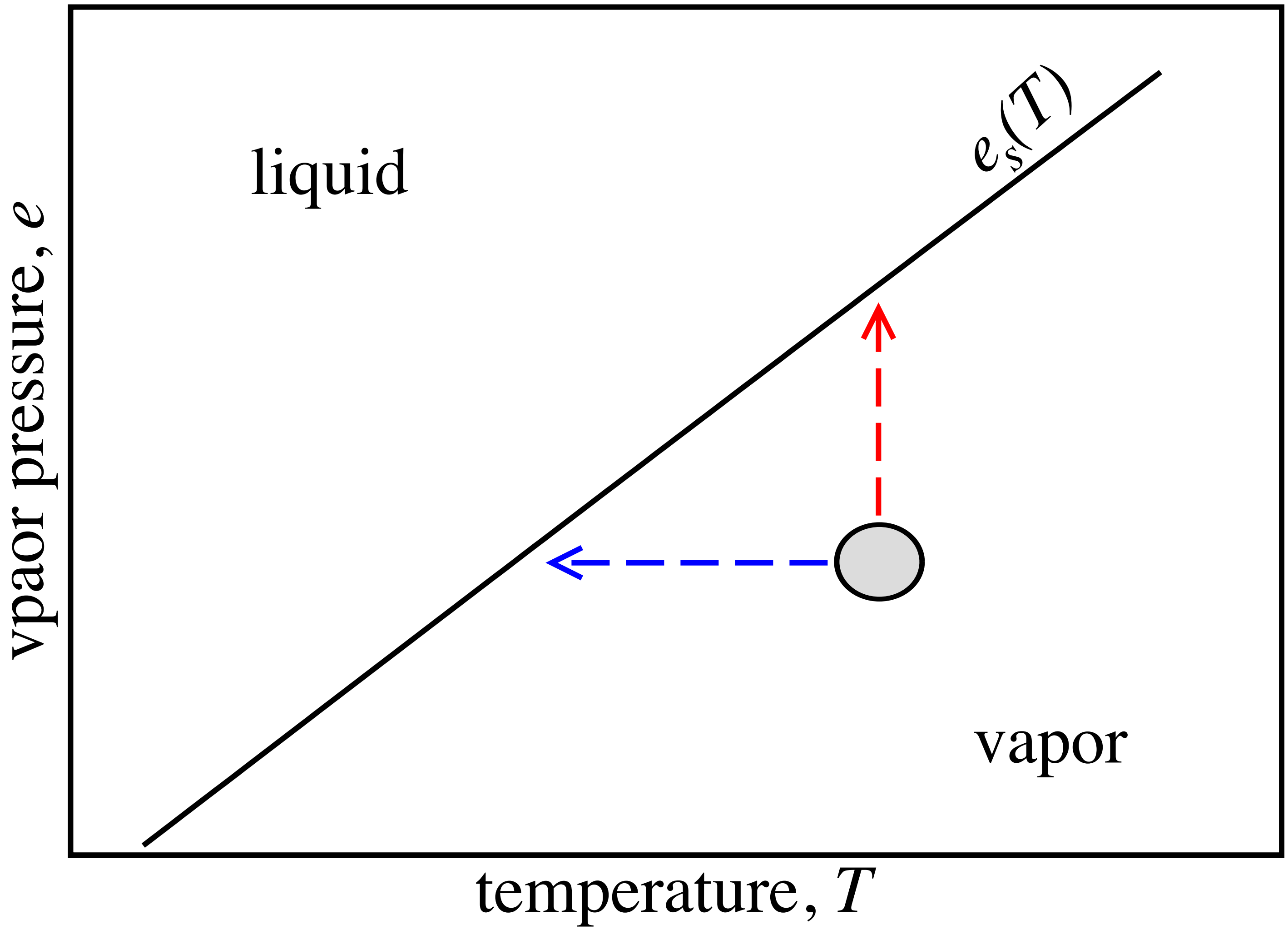
Clausius-Clapeyron equation (for water vapor in air)

$$\frac{de_s}{dT} = \frac{Le_s}{R_v T^2}$$

$$e_s(T) = e_s(T_0)\exp\left[\frac{L}{R_v}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right]$$

L = specific latent heat of vaporization for water

T_0 is some reference temperature



Condensation can occur in a sample of unsaturated air if

- more water vapor is added to the sample (e increases)
- temperature of the sample decreases (T decreases)

Moisture parameters

Saturation mixing ratio

$$w_s \equiv \frac{m_{vs}}{m_d} = \frac{\rho_{vs}}{\rho_d} = \varepsilon \frac{e_s}{p - e_s}$$

Relative humidity

$$r = \frac{w}{w_s}$$

- r indicates how close the air is to saturation, not the actual amount of water vapor in the air
- at a given pressure, r varies with (i) the actual amount water vapor in air m_v and (ii) the air temperature T
- if m_v remains constant, r increases when T decreases and decreases when T increases

Moisture parameters

$$\text{Since } p \gg e_s, \quad w_s \approx \varepsilon \frac{e_s}{p}$$

$$\Rightarrow \quad r = \frac{w}{w_s} \approx \frac{e}{e_s}$$

Dew point temperature

$T_d =$ the temperature to which air must be cooled at constant pressure for it to become saturated

- T_d measures the actual moisture content in the air
- moist air: high T_d
- dry air: low T_d

An example

$$L = 2.5 \times 10^6 \text{J kg}^{-1}$$

$$R_v = 461.5 \text{ J kg}^{-1}\text{K}^{-1}$$

$$T_0 = 300\text{K}$$

$$e_s(T_0) = 36 \text{ mbar}$$

Take surface pressure to be $p = 1 \text{ bar}$

Location A: cold and "wet"

$$T = -10°\text{C}$$

$$r = 100\%$$

which gives

$$e_s(T) = 28.38 \text{ mbar}$$

$$w_s = 2 \text{ g/kg}$$

$$w = 2 \text{ g/kg}$$

Location B: hot and "dry"

$$T = 25°\text{C}$$

$$r = 20\%$$

which gives

$$e_s(T) = 31.89 \text{ mbar}$$

$$w_s = 20 \text{ g/kg}$$

$$w = 4 \text{ g/kg}$$

The moisture content at the "dry" location B is higher than at the "wet" location A!

Estimation of T_d in terms of T and r

For an air sample at temperature T with vapor pressure e, the dew point temperature T_d by definition satisfies:

$$e_s(T_d) = e$$

Integrating the Clausius-Claperyon equation from T_d to T, we obtain

$$e_s(T) = e_s(T_d) \exp\left[\frac{L}{R_v}\left(\frac{1}{T_d} - \frac{1}{T}\right)\right]$$

$$\Rightarrow r \approx \frac{e_s(T)}{e} = \exp\left[\frac{L}{R_v}\left(\frac{1}{T_d} - \frac{1}{T}\right)\right]$$

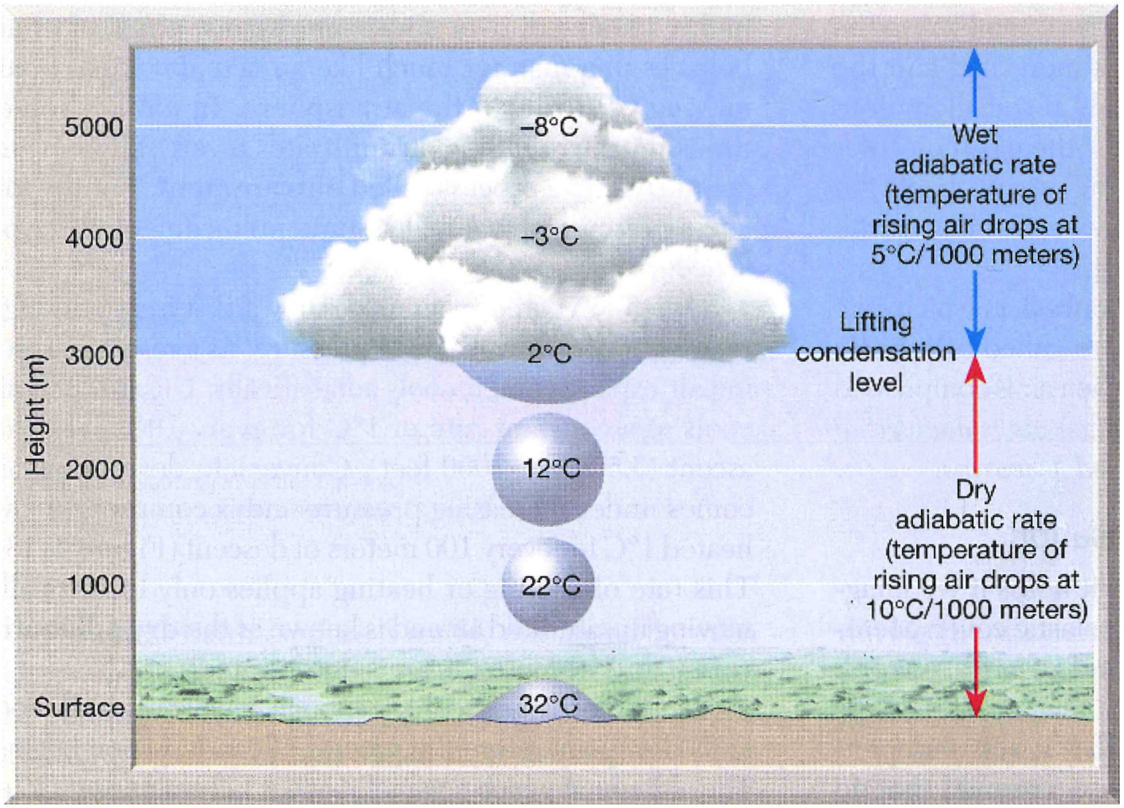
therefore, $T_d = \left(\frac{1}{T} - \frac{R_v}{L}\ln r\right)^{-1}$

Condensation by cooling

- We have seen that when the temperature of an air sample decreases, $e_s(T)$ decreases
- Condensation occurs at a temperature T when $e = e_s(T)$, or equivalently, $T = T_d$
- Air temperature may decreases because heat is lost, e.g. radiation cooling of the ground lowers the air temperature near the surface and dew or fog is formed
- Air temperature can also change without heat exchange: adiabatic temperature change, e.g. formation of cloud
- Concept of air parcel
  1. thermally insulated from its environment so that temperature changes adiabatically
  2. always remain at exactly the same pressure as its environment

Adiabatic cooling

- air parcel forced to rise, its pressure decreases (since it is assumed to be the same as the environmental pressure)
- air parcel *expands* and cools adiabatically
- rate of cooling for unsaturated air --- dry adiabatic lapse rate Γ_a
- at the lift condensation level (LCL) --- parcel temperature reaches T_d , air is saturated and *condensation occurs*
- above LCL, latent heat of condensation absorbed by air parcel, air parcel continues to rise and cool at a lower rate: (wet) saturated adiabatic lapse rate Γ_s



(Lutgens & Tarbuck 2001)

Dry adiabatic lapse rate (DALR), Γ_a

Consider a *unit mass* of unsaturated (not necessarily dry) air, i.e., no condensation occurs.

First law of thermodynamics: $dU = dQ - dW$

$$= dQ - p\,d\alpha \qquad (V/m = 1/\rho \equiv \alpha)$$

enthalpy: $H = U + p\,\alpha$

$$\Rightarrow \quad dH = dQ + \alpha dp$$

Hence, for a general thermodynamics process,

$$c_p dT = dQ + \alpha\, dp$$

where the specific heat capacity at constant pressure, $c_p = \left(\dfrac{\partial H}{\partial T}\right)_p$

For an adiabatic process, $dQ = 0$

$$\therefore \quad c_p \frac{dT}{dz} = \frac{1}{\rho}\frac{dp}{dz}$$

Using the hydrostatic balance, $\dfrac{dp}{dz} = -\rho g$

$$\Rightarrow \quad \Gamma_a \equiv -\frac{dT}{dz} = \frac{g}{c_p} \approx 10 \text{ K km}^{-1}$$

Saturated adiabatic lapse rate (SALR), Γ_s

As a *unit mass* of saturated air parcel cools, condensation occurs. Assume all latent heat released is given to the air parcel and all condensed liquid water falls out of the parcel.

Latent heat given to the air parcel: $dQ = -L\,dw_s \quad (dw_s < 0)$

$$w_s \approx \varepsilon \frac{e_s}{p} \quad \Rightarrow \quad \frac{dw_s}{w_s} = \frac{1}{e_s}\frac{de_s}{dT}\,dT - \frac{dp}{p}$$

$$= \frac{L}{R_v T^2}\,dT - \frac{dp}{p} \qquad \text{(\textit{Clausius-Clapeyron equation})}$$

therefore, $$dQ = \frac{-w_s L^2}{R_v T^2}\,dT + w_s L \frac{dp}{p}$$

Thermodynamic relation: $c_p dT = dQ + \alpha\,dp$

By the *ideal gas law* and *hydrostatic balance*, eliminate dp in favor of dz:

$$c_p dT = \frac{-w_s L^2}{R_v T^2}\,dT - \frac{w_s L}{R_m T} g dz - g dz$$

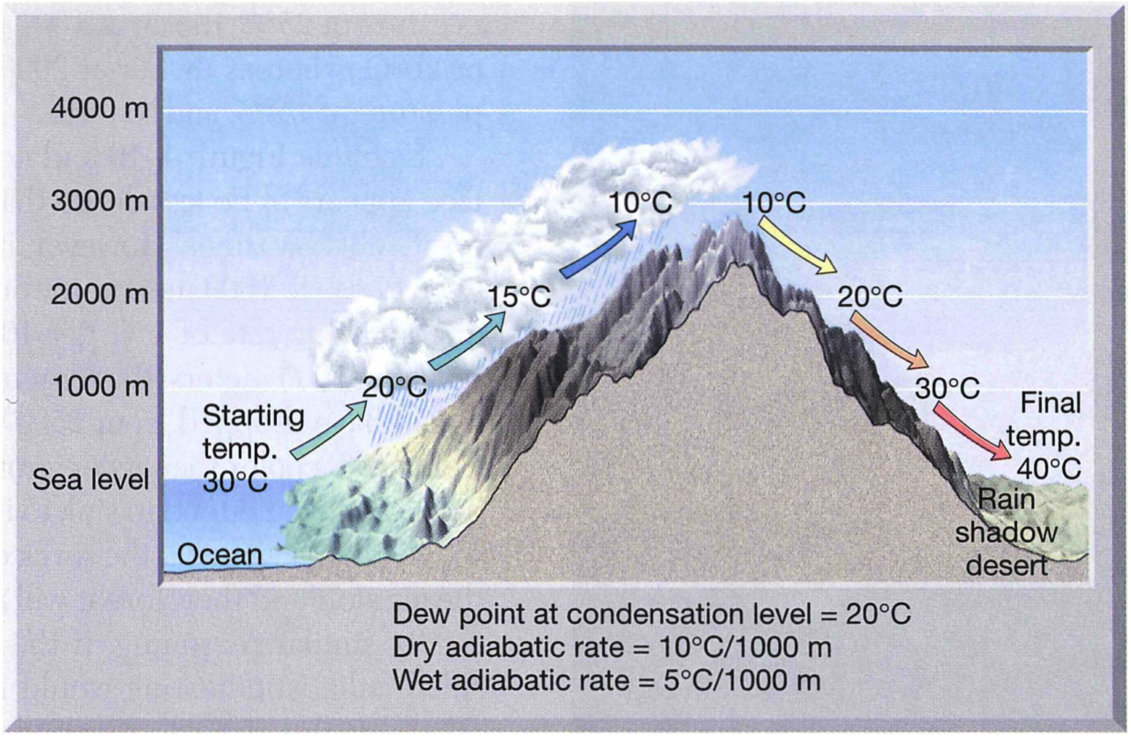
R_m : gas constant for moist air

$$\Rightarrow \quad \Gamma_s \equiv -\frac{dT}{dz} = \frac{g}{c_p} \frac{1 + \dfrac{w_s L}{R_m T}}{1 + \dfrac{w_s L^2}{c_p R_v T^2}}$$

Note that $\Gamma_s < \Gamma_a$

Orographic lifting and the Chinook ("snow-eater") wind

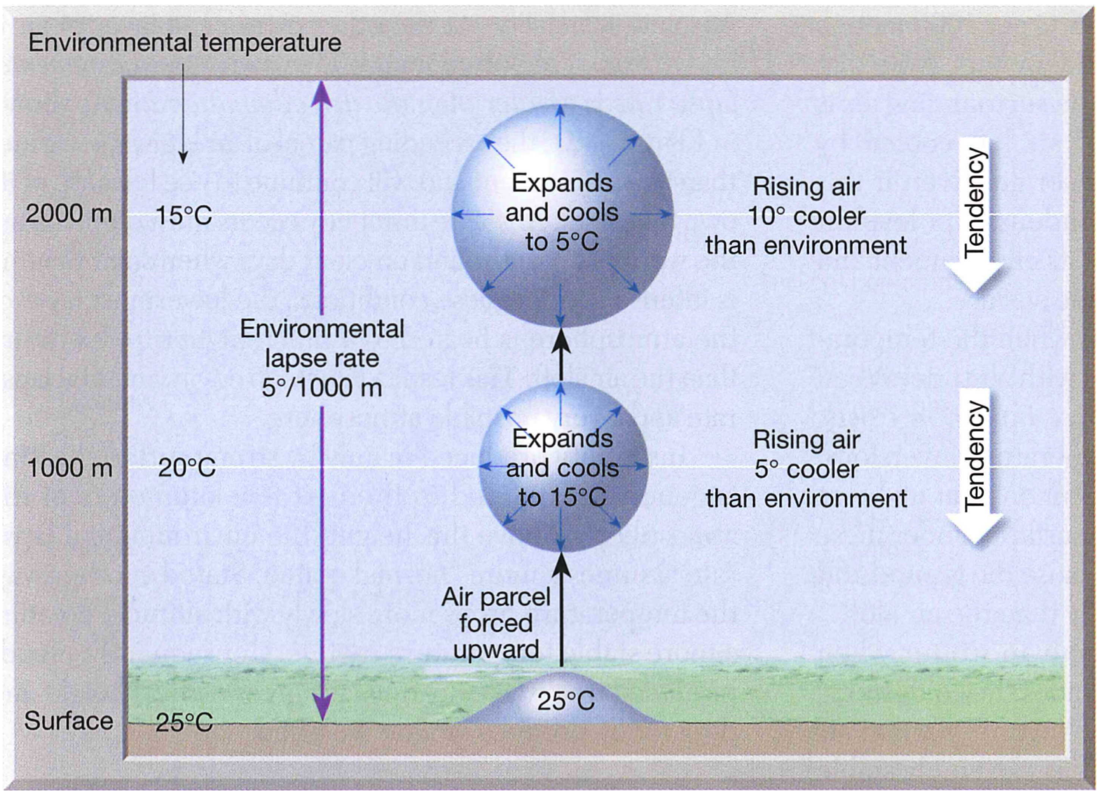
- *unsaturated* air *forced* up a mountain range *cools at the* DALR
- at the LCL, the dew point temperature T_d is reached, air becomes *saturated*
- *condensation* occurs as the air continues to rise, remains saturated and *cools at the* SALR
- assume all water vapor that condensed falls as *precipitation*
- suppose the air at the top of the mountain is cooler and denser than the surrounding air so it starts to flow down the leeward side of the mountain
- as the air travels *downslope*, it is compressed and *heated*, hence its saturated vapor pressure e_s increases and its temperature becomes higher than T_d
- the air becomes *unsaturated* and its temperature *increases at the* DALR
- the air at the mountain base on the *leeside* is warmer with *lower relative humidity*



(Lutgens & Tarbuck 2001)

Atmospheric stability

- if an air parcel is carried up from z_0 to z_1 , its pressure decreases and its volume increases, the parcel temperature T_{parcel} decreases adiabatically
- if the temperature of the surrounding air at z_1 is $T_{environ}$ and $T_{parcel} < T_{environ}$, the parcel is denser than its environment, it will tend to fall back to z_0 , i.e., it resists vertical movement, so the atmosphere is *statically stable* near height z_0
- if $T_{parcel} > T_{environ}$, the parcel is less dense than its environment, it will continues to rise, then the atmosphere is *statically unstable* near height z_0
- depending on whether the air is saturated or not, the stability of the atmosphere can be investigated by comparing the environmental lapse rate Γ with Γ_a or Γ_s

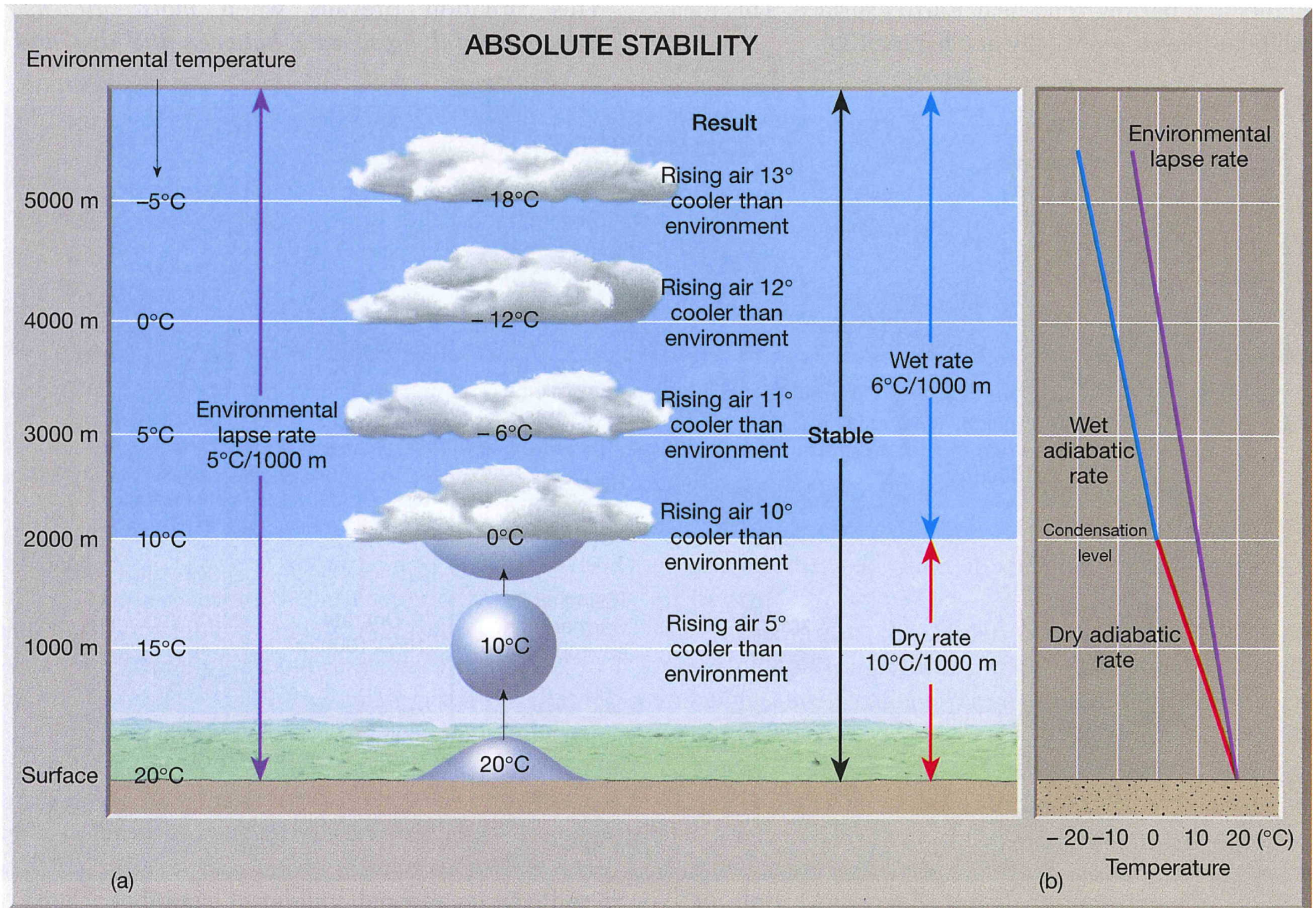


(Lutgens & Tarbuck 2001)

Absolute stability

$$\Gamma < \Gamma_s \; (< \Gamma_a)$$

the air parcel, saturated or not, is always denser than its surrounding

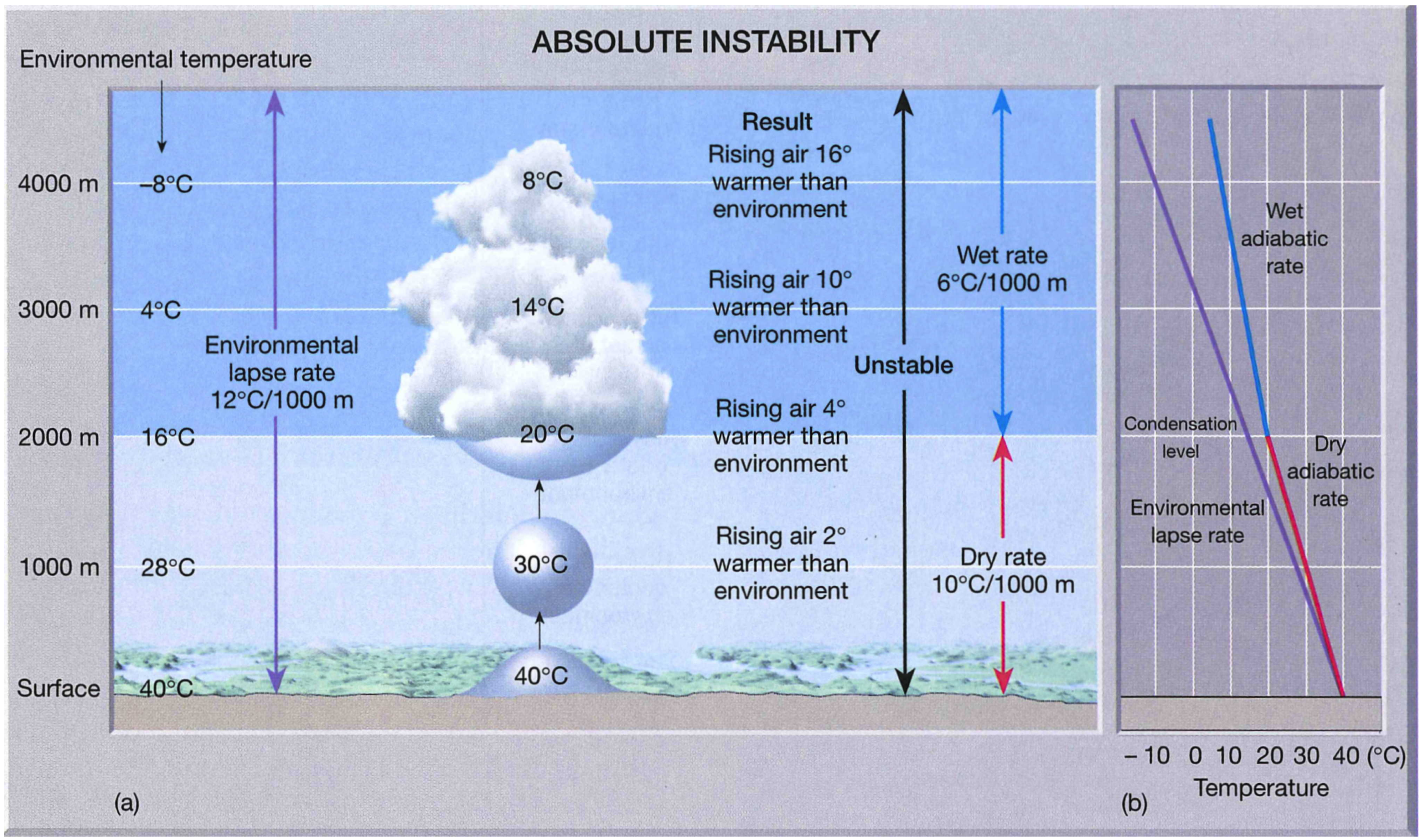


(Lutgens & Tarbuck 2001)

Absolute instability

$$\Gamma > \Gamma_a \ (> \Gamma_s)$$

- the air parcel, saturated or not, is always lighter than its surrounding
- such instability generally do not persist in the free atmosphere except in the lowermost layer above the ground when it is strongly heated from below on hot days

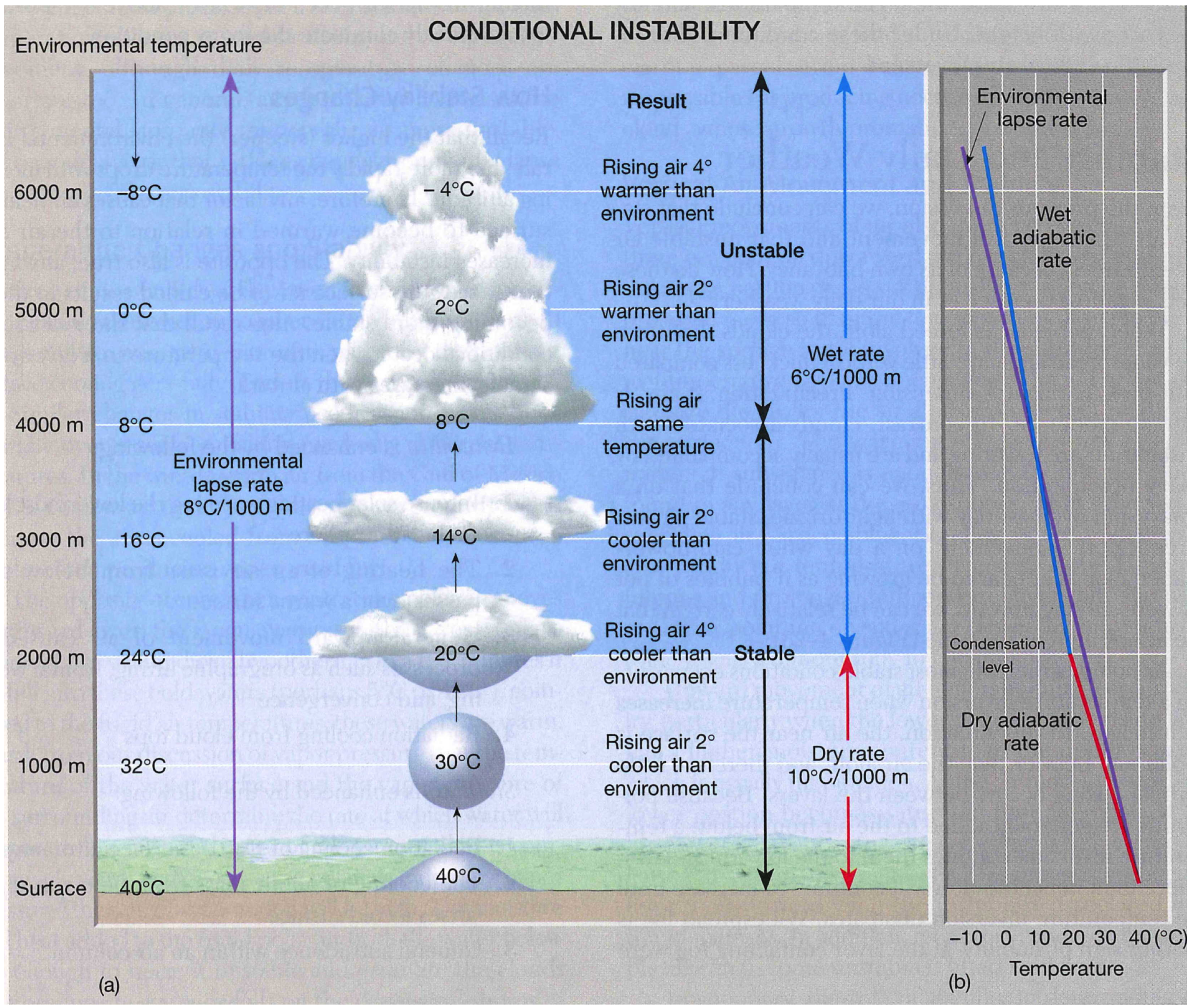


(Lutgens & Tarbuck 2001)

Conditional instability

$$\Gamma_a > \Gamma > \Gamma_s$$

a saturated parcel is unstable but an unsaturated parcel is stable



(Lutgens & Tarbuck 2001)