1. (SB3) Given: $f = 3.00\ s^{-1}$, $A = 5.00$ cm

(a) total distance moved $= 4A = 20.0$ cm #

(b) max. speed $= \omega A = (2\pi f)A = 94.2$ cm/s $= 0.942$ m/s #
This occurs when the particle passes through the equilibrium position.

(c) max. acceleration $= \omega^2 A = (2\pi f)^2 A = 17.8\ ms^{-2}$ #
This occurs at maximum excursion from the equilibrium position.

2. (SB11) From $\omega = \sqrt{\frac{k}{m}}$, we have

$$k = m\omega^2$$

$$= m\left(\frac{2\pi}{T}\right)^2$$

$$= (7.00\ kg)\left(\frac{2\pi}{2.60s}\right)^2$$

$$= 40.9\ Nm^{-1}$$ #

3. (SB30) The Quick and Dirty way :

period of simple pendulum : $T = 2\pi\sqrt{\frac{L}{g}}$

We shall substitute g in the above formula by the "effective" gravity "feels" by the pendulum, hence

let a be the magnitude of the acceleration of the elevator

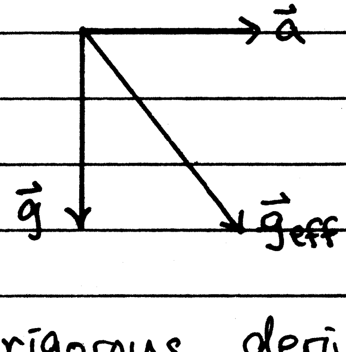
(a) $$T = 2\pi\sqrt{\frac{L}{g+a}} \quad \#$$

(b) $$T = 2\pi\sqrt{\frac{L}{g-a}} \quad \#$$

(c) let a be the magnitude of the acceleration of the truck

$$T = 2\pi\sqrt{\frac{L}{(g^2+a^2)^{1/2}}}$$

Note :



P.S. For a rigorous derivation of the answer in (c), see the Appendix.

4.(SB 41) The motion of the pendulum is given by

$$\theta(t) = A e^{-\frac{b}{2m}t} \cos(\omega t + \phi), \quad \omega = \sqrt{\frac{g}{L} - \left(\frac{b}{2m}\right)^2}$$

We define the amplitude of a damped harmonic oscillator $\theta_m(t)$ as

$$\theta_m(t) = A e^{-\frac{b}{2m}t}$$

It is given that $\theta_m(t=1000\,s) = 5.50°$.

Rigorously speaking, we are also given $\theta(0) = 15.0°$ (NOT $\theta_m(0)$), $\therefore$ $A\cos\phi = 15.0°$. This makes the problem very difficult to solve, so we make the following approximation:

$$\dot{\theta}(t) = -A e^{-\frac{b}{2m}t}\left[\omega \sin(\omega t + \phi) + \frac{b}{2m}\cos(\omega t + \phi)\right]$$

$$\dot{\theta}(0) = 0 \Rightarrow \tan\phi = -\frac{b}{2m\omega} \text{ and } \frac{b}{2m} \ll \omega \Rightarrow \phi \text{ is small}$$

$$\Rightarrow |\cos\phi| \approx 1$$

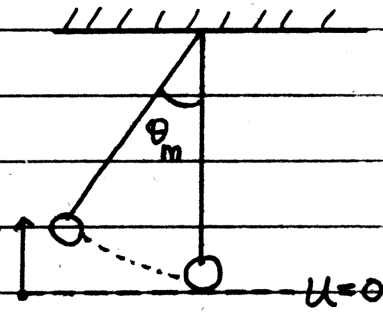
$$\therefore \quad \theta_m(0) \approx \theta(0)$$

Then, $$\frac{\theta_m(0)}{\theta_m(1000)} = e^{\frac{b}{2m}(1000)} = \frac{15.0°}{5.50°} \Rightarrow \frac{b}{2m} = 1 \times 10^{-3}\,s^{-1}$$ #

<u>NOTE</u>: I think the question should be re-phrased as: the <u>amplitude</u> at $t = 0\,s$ is $15.0°$ and then it is reduced to $5.50°$ at $t = 1000\,s$.

5. (SB 63) (a) $T = 2\pi\sqrt{\dfrac{L}{g}} = 3.00\ s$ #

(b) ∵ the total energy is constant
we consider the pendulum
at $t=0$,



let the potential energy $U=0$
when the pendulum is in its
equilibrium position

$$\therefore\ E = \frac{1}{2}mv_i^2 \qquad (v_i = \text{initial speed})$$

$$= \frac{1}{2}(6.74\,kg)(2.06\,ms^{-1})^2$$

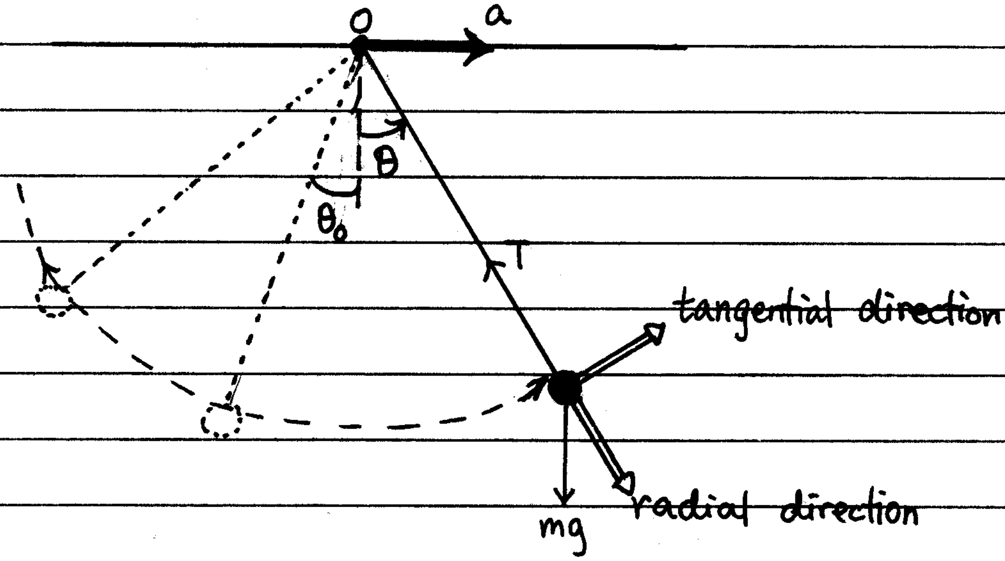
$$= 14.3\ J\ \#$$

NOTE: asking for the total energy without defining
the reference level for the gravitational potential
energy is in fact ambigious, anyway, we
define it ourselves.

(c) at maximum angular displacement, $KE=0$

$$\therefore\ \frac{1}{2}mv_i^2 = mgh = mgL(1-\cos\theta_m)$$

$$\cos\theta_m = 1 - \frac{v_i^2}{2gL} \Rightarrow \theta_m = 25.5^\circ\ \#$$



Appendix

- the coordinate system we use are shown above, it is attached to the pendulum, thus accelerating to left as the pendulum does
- Newton's Laws only apply to inertial frame. Accelerating coordinate systems are not inertial frames $\Rightarrow$ need a "modified" Newton's 2nd law
- the equilibrium position of the pendulum is NOT $\theta = 0°$, but $\theta = \theta_0$, so when the pendulum is not swinging, it is tilted at an angle θ_0 from the vertical, when it swings, it oscillates about $\theta = \theta_0$ instead of the vertical ($\theta = 0°$)

let us first determine the equilibrium position θ_0 with pendulum at rest. resolve forces into vertical and horizontal direction, in a fixed frame:

$$-T\sin\theta_0 = ma$$

$$T\cos\theta_0 = mg$$

$$\theta_0 = -\tan^{-1}\left(\frac{a}{g}\right)$$

now we turn to our moving coordinate system, in this system, we observe that the tangential force acting on the pendulum is $-mg\sin\theta$ and the tangential acceleration is $L\ddot{\theta}$, if we apply Newton's 2nd law naively, we have

$$-mg\sin\theta = mL\ddot{\theta}$$

which is not right. The correct equation of motion in a moving coordinate system is:

$$F' - f = ma'$$

where F' and a' are forces and acceleration observed from the moving frame and f, called a pseudoforce, equals ma where a is the acceleration of the moving frame with respect to a fixed frame.

Therefore, $$-mg\sin\theta - \underbrace{\underline{ma\cos\theta}}_{\text{pseudoforce}} = mL\ddot{\theta} \quad \text{(tangential)}$$

$$L\ddot{\theta} = -g\sin\theta - a\cos\theta$$

Next, assuming amplitude of oscillation is small i.e. $(\theta - \theta_0)$ is small, we expand $\sin\theta$ and $\cos\theta$ about $\theta = \theta_0$ (NOT $\theta = 0°$):

$$\left.\begin{aligned} \sin\theta &\approx \sin\theta_0 + (\theta-\theta_0)\cos\theta_0 \\ \cos\theta &\approx \cos\theta_0 - (\theta-\theta_0)\sin\theta_0 \end{aligned}\right\} \text{Taylor series expansion}$$

Using the above approximations together with the relation : $a = -g\tan\theta_0$, we get

$$\ddot{\theta} = -\frac{g}{L\cos\theta_0}(\theta-\theta_0)$$

$$\tan\theta_0 = -\frac{a}{g} \Rightarrow \cos\theta_0 = \frac{g}{\sqrt{a^2+g^2}}$$

Hence, $$\ddot{\theta} = -\frac{\sqrt{a^2+g^2}}{L}(\theta-\theta_0)$$

To make it looks more like the equation for simple harmonic motion, define

$$\theta' = \theta - \theta_0$$

$$\ddot{\theta}' = -\frac{\sqrt{a^2+g^2}}{L}\theta'$$

$$\ddot{\theta}' = -\omega^2\theta' \quad \text{with} \quad \omega^2 = \frac{\sqrt{a^2+g^2}}{L}$$

Therefore, $\theta(t)$ oscillate about θ_0 with period

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{\sqrt{a^2+g^2}}}$$

#