Physics 262 Final Exam (Spring 2003)
Solutions

1. Let the velocity of the exiting stream be v_x. Since the size of the hole is small compare to the cross-sectional area of the tank, by the continuity equation, we can neglect the velocity of the top surface,

$$v_{surface} = v_x \frac{A_{hole}}{A_{surface}} \approx 0\,.$$

Apply Bernoulli's equation between the top surface and the exiting stream,

$$\begin{aligned}
P_0 + 0 + \rho g h_0 &= P_0 + \frac{1}{2}\rho v_x^2 + \rho g h \\
v_x &= \sqrt{2g(h_0 - h)} \\
x &= v_x t \\
y = h &= \frac{1}{2} g t^2 \\
\Rightarrow \quad x &= 2\sqrt{h(h_0 - h)}
\end{aligned}$$

2. Given $\Delta P = 1.27 \sin(\pi x - 340\pi t)$

(a) The pressure amplitude is : $\Delta P_{max} = 1.27$ Pa.

(b) $\omega = 2\pi f = 340\pi$ s^{-1}, so $f = 170$ Hz.

(c) $k = \dfrac{2\pi}{\lambda} = \pi$ m^{-1}, so $\lambda = 2.00$ m.

(d) $v = f\lambda = 340$ ms^{-1}.

3. (a) For isobaric process, $W = P(V_f - V_i)$. Then by the first law of thermodynamics,

$$\Delta E_{int} = Q - W = 7.50 \text{ kJ}$$

(b) Since pressure is constant, the ideal gas law gives

$$\begin{aligned}
\frac{V_i}{T_i} &= \frac{V_f}{T_f} \\
T_f &= 900 \text{ K}
\end{aligned}$$

4. Given $T_h = 523$ K, $T_c = 323$ K and $Q_h = 1200$ J,

(a) For Carnot cycle, the efficiency e_c is given by

$$\begin{aligned}
e_c = 1 - \frac{T_c}{T_h} &= 1 - \frac{Q_c}{Q_h} \\
\Rightarrow \quad Q_c &= \frac{T_c}{T_h} Q_h \\
&= 741 \text{ J}
\end{aligned}$$

(b) Work done by the gas in one cycle, $W = Q_h - Q_c = 459$ J.

5. Refer to Example 24.7 on P.752 and Example 24.10 on P.756 in the textbook for the details on the arguments used in this problem.

(a) Inside surface: consider a **cylindrical gaussian surface <u>within</u> the conducting cylinder** of length l whose axis coincides with that of the cylinder. Since electric field $\vec{E}$ inside the conducting cylinder is zero, by Gauss's Law, the total charge inside the gaussian surface q_{in} must be zero. The wire has a charge per unit length of λ, therefore

$$\begin{aligned} q_{in} &= 0 \\ \lambda l + (\text{charge/length of inner surface})l &= 0 \\ \text{charge per unit length of inner surface} &= -\lambda \end{aligned}$$

Outside surface: The charge per unit length of the whole conducting cylinder is 2λ, hence

$$\text{charge per unit length of the outer surface } = 2\lambda - (-\lambda) = 3\lambda$$

(b) Consider a **cylindrical gaussian surface that <u>enclosed</u> the conducting cylinder and the wire**, having radius r and length l and its axis coincides with the conducting cylinder, by Guass's Law,

$$\begin{aligned} E\,2\pi rl &= \frac{(\lambda + 2\lambda)l}{\epsilon_0} \\ E &= \frac{3\lambda}{2\pi\epsilon_0 r} \end{aligned}$$

(c) Consider a **cylindrical gaussian surface <u>between</u> the conducting cylinder and the wire**, having radius r and length l and its axis coincides with the conducting cylinder. The gaussian surface encloses only the wire, not the conducting cylinder. By Guass's Law,

$$\begin{aligned} E\,2\pi rl &= \frac{\lambda l}{\epsilon_0} \\ E &= \frac{\lambda}{2\pi\epsilon_0 r} \end{aligned}$$

6. Notice that the electric field $\vec{E}$ inside a material that obeys Ohm's Law is uniform.

(a) For uniform field, $\vec{E} = \frac{V}{L}\hat{i}$

(b) $R = \frac{\rho l}{A} = \frac{4\rho L}{\pi d^2}$

(c) $I = \frac{\Delta V}{R} = \frac{V\pi d^2}{4\rho L}$

(d) $\vec{J} = \frac{I}{A}\hat{i} = \frac{V}{\rho L}\hat{i}$

Hence, $\rho\vec{J} = \rho\frac{V}{\rho L}\hat{i} = \frac{V}{L}\hat{i} = \vec{E}$.

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