Physics 262 First Term Exam (Spring 2003)
Solutions

1. Let l_0 be the natural length of the spring. Given m=0.25 kg, k=58 Nm^{-1} and x=0.15 m, by the principle of conservation of energy,

$$\begin{aligned} \frac{1}{2}mv^2 + mgl_0 &= \frac{1}{2}kx^2 + mg(l_0 - x) \\ v &= \sqrt{\frac{k}{m}x^2 - 2gx} \\ &= 1.5 \text{ ms}^{-1} \end{aligned}$$

2. Let N be the number of whole log required and ρ_w be the density of water. Given $r = 0.0800$ m, $l = 3.00$ m and $m = 80$ kg. So, the volume of one log $V = \pi r^2 l$

$$\text{buoyant force} = \text{weight of four people} + \text{weight of raft}$$

Assuming the raft is completely submerged and using Archimedes' principle for the buoyant force,

$$\begin{aligned} \rho_w NVg &= 4mg + \rho NVg \\ N &= \frac{4m}{(\pi r^2 l)(\rho_w - \rho)} \\ &= 19.3 \end{aligned}$$

Therefore, the smallest number of whole logs needed is 20.

3. Let s be the required distance, v be the speed of the water, A and r be the cross-sectional area and radius of the stream respectively. The subscripts 1 and 2 denote the initial and final values of the corresponding quantities. Using the continuity equation,

$$\begin{aligned} v_1 A_1 &= v_2 A_2 \\ v_2 = \frac{A_1}{A_2}v_1 = \frac{{r_1}^2}{{r_2}^2}v_1 &= 4v_1 \quad (\text{since } r_2 = 0.5r_1) \\ 2gs &= v_2^2 - v_1^2 \\ s &= \frac{15v_1^2}{2g} \\ &= 0.28 \text{ m} \end{aligned}$$

4. Let F be the tension in the string and μ be its linear density. It is given that

$$y = (0.022\text{m})\sin(2t - 2.0x) \ .$$

Thus, the wavenumber $k = 2.0$ m^{-1} and the angular frequency $\omega = 2$ s^{-1} and wave speed $v = \dfrac{\omega}{k}$.

$$\begin{aligned} v &= \sqrt{\frac{F}{\mu}} \\ F = \mu v^2 &= \frac{\mu\omega^2}{k^2} \\ &= 1.6 \times 10^{-2} \text{ N} \end{aligned}$$

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