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MMath Report

New Higgs Inflation for a Massive Scalar Field

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Abstract

This project assess the level in which New Higgs Inflation can be used to accurately model inflation. Beginning with a brief introduction on inflation theory while detailing aims and objectives of the project, followed by notation implemented and observations used. Derivation of density and pressure in terms of scale factor is calculated to form a basis of the slow-roll equations and slow-roll parameters, parameters are then restricted for the end of inflation. The number of e-folds is calculated for the cases of a massive scalar field and a self-interacting scalar field, tensor-scalar ratio and spectral index are then written in terms of the number of e-folds. This procedure is repeated for New Higgs Inflation and finally a graph of the model is analysed and further work suggested, this is concluded by a summary of the model.

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1 Introduction

Since it's creation in the 1980s cosmological inflation has been used to account for the several discrepancies found in standard big bang cosmology, while also providing a basis to generate inflationary models for the approximation of the universe. These models have a wide range of complexity, from the most basic using just simple approximations, to more complicated models involving several of the observed parameters. In this project I will be examining some of these models to try and asses their validity, while also casting out any discredited or those less useful, hopefully leaving a small group of potentially viable options.

Inflation is the expansion of the universe following the big bang, it is believed to explain why the universe is isotropic and why the cosmic microwave background (CMB) is evenly distributed throughout it, see figure 3. A relatively modern concept, the hypothesis was put forward in the early 1980s and has solved several of the "unwanted relics" of cosmology such as the horizon, smoothness, and entropy problems as well as the presence of magnetic monopoles detailed in [1], see figure 1. Currently the wait for confirmation that inflationary gravitational waves have been detected is still ongoing, this would in effect cement the theory of inflation with strong experimental evidence.

1.1 Aims

At the end of this project I aim to have analysed the observational consequences of a specific model of inflation, by using both experimental and observed data. The most important thing is first to state the governing equations as well as defining it's parameters such as; the tensor-scalar ratio, spectral index and number of e-folds all of which will be examined later. For a given value of potential the number of e-folds can be written purely in terms of the slow-roll parameters, in this project I will be concentrating on the massive scalar field and the self-interacting scalar field, or ϕ^2 and ϕ^4 cases respectively as explained by [2]. From here both the tensor-scalar ratio and spectral index can be written as functions of one variable; the number of e-folds and a graph can be plotted. Finally I must repeat this process using the more general dynamical slow-roll approximations in order to assess New Higgs Inflation, for completeness I will also show how the slow-roll approximations can be derived from a Lagrangian.

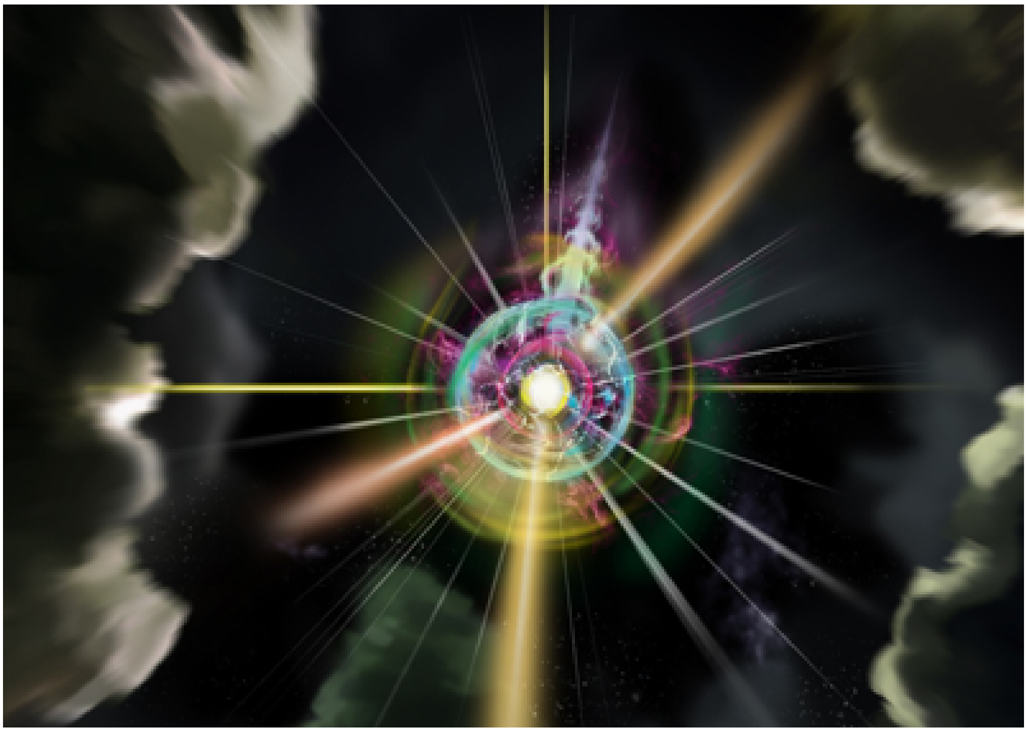

Figure 1: Artistic impression of a magnetic monopole, the magnetic monopole problem claims that in the very hot early universe a large number of magnetic monopoles were produced, however none have been discovered to this day, inflation theory accounts for this by claiming that inflation occurs after these monoploes were produced, and in effect they are destroyed by expansion (image courtesy: Heikka Valja).

1.2 Observations

Our understanding of the universe is dependent on the observations taken, the accuracy of the models is judged within their relation to experimental observations from satellites such as COBE [3] and Planck [1] , the closer the model is to the observed values the better it is at modelling the universe. These satellites allow the observation of:

- Expansion rate $H(t)$, known as the Hubble parameter this measures the rate at which the universe is expanding, Hubble expansion is caused by repulsive gravity caused by a false vacuum, see [4] for details.

- Temperature $T(t)$, at large scales the temperature of the universe may seem uniform, however with increased accuracy it is possible to see infinitesimally small fluctuations in CMB temperature. In an expanding universe temperature is inversely proportional to the scale factor, see notation section for details.

- Fluctuations in the cosmic microwave background, the CMB is the radiation remaining after the big bang, temperature of the CMB is uniformly distributed, yet infinitesimal fluctuations in temperature can yield vast amounts of information regarding the composition, growth and origin of the universe. Fluctuations in the CMB are a result of the primordial spectrum, which is made up of scalar and tensor perturbations as will be examined later.

The scientific community are constantly trying to improve their measuring techniques to gain greater detail of observations, the more accurate the measurements the smaller the range of possible theories needed to be considered, due to the tight constraints placed upon them.

1.3 Notation

Other parameters within the inflationary model are as follows:

- Scale Factor $a(t)$ is a function of time, this is the relative expansion rate of the universe, or the proper distance of two objects changing over time due to the expanding universe, as the expansion of the universe is exponential so is the scale factor.

- Density of the Universe $\rho(t)$ is a function of time, taking both matter and energy density of the universe into account. In general the energy density is approximately equal to the rest mass density, due to the speed of light being set equal to one as explained below.

- Pressure $p(t)$ is a function of time, the pressure of the universe may be positive or negative, positive pressures induce gravitational attraction which reduces the expansion rate of the universe, while negative pressures cause gravitational repulsion leading to an increase in expansion.
- The idea that the universe is homogeneous and isotropic comes from the cosmological principle, the basis for the big bang theory, it states that the universe should look the same for all observers.

As is the standard convention in cosmology, constants such as the speed of light c are set to be equal to one.

These parameters give the properties of the materials that make up the universe, which in turn govern its expansion, hence can be used to form analysing conditions on the slow-roll equations.

2 Cosmology

Theories can be tested by comparing their predictions to the actual state of the universe known from observations. Cosmology aims to better understand the origin and expansion of the universe through pragmatic use of various hypotheses and models, the closer the predictions of the observations the closer a model is to being complete. By being able to trace a path for the universe, it is hoped that its future couse can be predicted and ultimate destiny of it.

As the universe is expanding the first law of thermodynamics is applicable, from [4],

$$0 = dU + pdV, \tag{1}$$

where U is the energy of matter and radiation of the universe, p is the pressure, and V is the volume. As the speed of light has been set equal to one the mass density is equal to that of the energy density which is,

$$\rho = \frac{U}{V}, \tag{2}$$

hence by differentiation,

$$\begin{aligned} d\rho &= \frac{dU}{V} - \frac{UdV}{V^2}, \\ &= \frac{dV}{V}\left(\frac{dU}{dV} - \frac{U}{V}\right), \end{aligned} \tag{3}$$

and using (1) and (2) in (3) gives,

$$d\rho = -(p+\rho)\frac{dV}{V}. \tag{4}$$

Setting the volume of the universe to be the cube of the scale factor,

$$V = a^3, \tag{5}$$

then,

$$dV = 3a^2 da, \tag{6}$$

subbing back into (4),

$$d\rho = -3(p+\rho)\frac{da}{a}. \tag{7}$$

Using $dp = (dp/dt)dt$,

$$\frac{d\rho}{dt} = -3(p+\rho)\frac{da}{dt}\frac{1}{a}, \tag{8}$$

hence,

$$\dot{\rho} = -3(p+\rho)\frac{\dot{a}}{a}. \tag{9}$$

According to [4] this is one of the Friedmann equations, another being stated as,

$$\ddot{a} = -\frac{4\pi}{3}G(\rho+3p)a, \tag{10}$$

Where G is the gravitational constant, from this equation it can be seen that a positive pressure will cause $\ddot{a}$ to be negative and thus causing the deceleration of the universe, so for the universe to be accelerating negative pressures will be required. This naturally leads to a scalar field ϕ to be chosen as our inflaton, as they can quite easily construct negative pressures, and in effect the repulsion force needed.

For the radiation of the universe set $p = \rho/3$ (recall $c = 1$), as a result (9) becomes,

$$\dot{\rho} = -4\rho\frac{\dot{a}}{a}, \tag{11}$$

rearranging,

$$\frac{\dot{\rho}}{\rho} = -4\frac{\dot{a}}{a}, \tag{12}$$

hence by integration,

$$\ln\rho = \ln a^{-4} + b, \tag{13}$$

where b is a constant, finally taking the exponential leaves

$$\rho = Aa^{-4}. \tag{14}$$

This can be rationalised; as the energy density for the radiation is proportional to temperature as well as volume, thus $\rho \propto Ta^{-3}$, however as the universe cools due to expansion, temperature must be inversely proportional to the expansion rate and as a result $T \propto a^{-1}$, leaving density proportional to a^{-4}.

Expansion can also be related to density using the equation,

$$3H^2 = 8\pi G\rho, \tag{15}$$

which is given in [5]. Using a given function for density, it is possible to relate the Hubble parameter to the potential of the scale factor, in effect formulating a chief equation in the field of inflation.

2.1 Inflation

As first postulated by Guth in [6] the main restriction placed on inflation to occur in physical space, is that a state can exists with negative pressure. Consider $p = -\rho$ in (9),

$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho - \rho), \tag{16}$$

this leaves,

$$\dot{\rho} = 0, \tag{17}$$

as with most inflationary models ρ is approximately constant which leads to exponential expansion of the scale factor. Setting $\rho = \rho_0$ in the equation,

$$3H^2 = 8\pi G\rho, \tag{18}$$

implies that H is a constant and therefore,

$$a(t) \propto exp(Ht). \tag{19}$$

This becomes the first key equation for studying cosmological inflation.

From [2] density and pressure can be given as functions of the parameters scalar field and its potential; $\phi(t)$ and $V(\phi)$ respectively,

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{20}$$

similarly for pressure,

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi), \tag{21}$$

the $\dot{\phi}^2/2$ term in the equations can be viewed as kinetic energy, while the V_ϕ term is the potential energy. From here sub (20) into (18) to yield,

$$3H^2 = 8\pi G\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right), \tag{22}$$

as given in [5]. The next key equation is derived by first differentiating equation (20) in terms of t,

$$\dot{\rho} = \dot{\phi}\ddot{\phi} + \frac{dV}{d\phi}\dot{\phi}. \tag{23}$$

Rearranging (16) gives,

$$a\dot{\rho} + 3\dot{a}(\rho + p) = 0, \tag{24}$$

substituting (20), (21) and (23) into equation (24) results in,

$$a\left(\dot{\phi}\ddot{\phi} + \frac{dV}{d\phi}\dot{\phi}\right)+3\dot{a}\left(\frac{1}{2}\dot{\phi}^2 + V + \frac{1}{2}\dot{\phi}^2 - V\right)= 0, \tag{25}$$

working through the bracket,

$$a\dot{\phi}\left(\ddot{\phi} + \frac{dV}{d\phi}\right)+3\dot{a}\dot{\phi}^2 = 0, \tag{26}$$

dividing by $a\dot{\phi}$,

$$\ddot{\phi} + \frac{dV}{d\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = 0, \tag{27}$$

substituting $H = \dot{a}/a$ leaves the equation of motion given in [7],

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0. \tag{28}$$

This will be the basis for the first slow-roll approximation, as will be shown in the next section.

3 Slow-Roll Approximation

The slow-roll model suggests that inflation was caused by a scalar field rolling down a potential energy hill, as seen in figure 2. When the gradient is small, the field rolls very slowly in comparison to the expansion of the universe and inflation occurs. Yet if the gradient of the hill is large, the field rolls more quickly causing inflation to end and the universe to reheat. When $\ddot{\phi}$ is small equation (28) can be approximated as the first slow-roll approximation,

$$3H\dot{\phi} + \frac{dV}{d\phi} = 0, \tag{29}$$

and $3H^2 = 8\pi G\rho$ becomes the second slow-roll approximation

$$3H^2 = 8\pi GV. \tag{30}$$

Now ϕ is the inflaton field and V is it's potential.

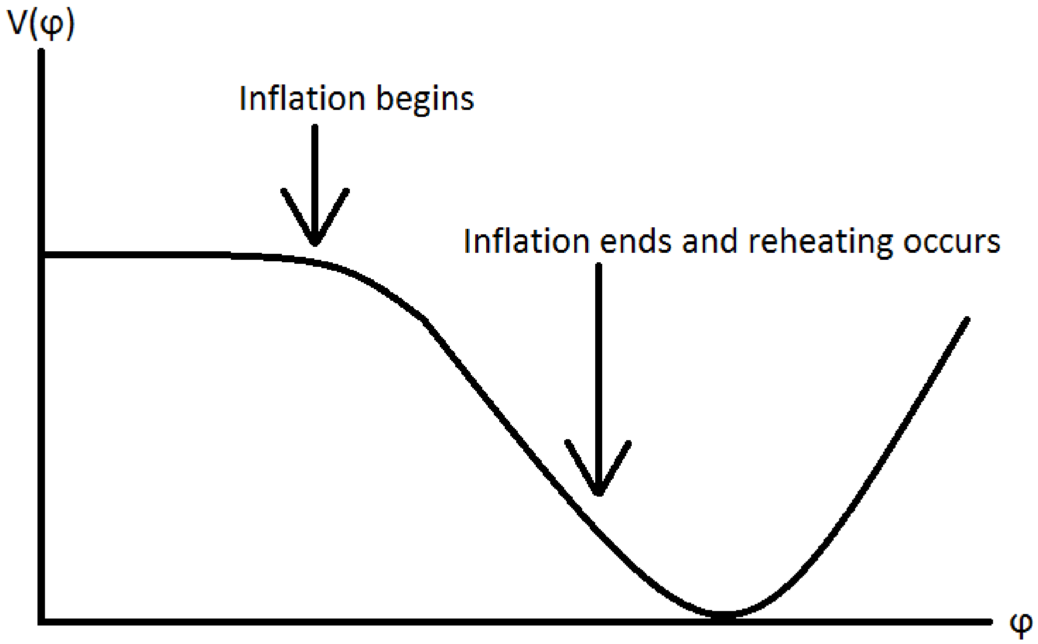

Figure 2: Scalar field ϕ rolling down a potential energy hill, the gradient of the hill must be approximately flat so that the inflaton rolls down very slowly, at the bottom of the image inflation ends and reheating occurs due to the steep gradient.

3.1 Dynamical slow-roll parameters

The dynamical slow-roll parameters are defined in this form in [8],

$$\epsilon_x = \frac{\dot{x}}{Hx}. \tag{31}$$

Therefore taking x to equal to H,

$$\epsilon_H = \frac{\dot{H}}{H^2}, \tag{32}$$

and x equal to $\dot{\phi}$,

$$\epsilon_{\dot{\phi}} = \frac{\ddot{\phi}}{H\dot{\phi}}. \tag{33}$$

As ϵ_H and $\epsilon_{\dot{\phi}}$ are small then ϵ_v and η_v the slope and curvature of the potential respectively must also be small. Therefore inflation occurs when the slow-roll conditions are satisfied, ie when $\epsilon_v < 1$, with prolonged inflation when $\eta_v < 1$.

As shown in [7] it is often convenient for the parameters of inflation to be written as,

$$\epsilon_v = \frac{1}{16\pi G}\left(\frac{1}{V}\frac{dV}{d\phi}\right)^2, \tag{34}$$

and,

$$\eta_v = \frac{1}{8\pi G}\frac{1}{V}\frac{d^2V}{d\phi^2}. \tag{35}$$

Using slow-roll approximations (see section A.1),

$$\epsilon_H = -\epsilon_v. \tag{36}$$

Working out $\epsilon_{\dot{\phi}}$ similarly (see section A.2),

$$\epsilon_{\dot{\phi}} = \epsilon_v - \eta_v. \tag{37}$$

Later it will be required that $\dot{H}/H^2$ is related to ϵ_v, this is done by simply combining (32) and (36) thus,

$$\frac{\dot{H}}{H^2} = -\epsilon_v. \tag{38}$$

3.2 End of inflation

The period following inflation is known as reheating as described in [9], after supercooled expansion(inflation) ends the temperature of the universe begins to rise back to the pre-inflation value. This is due to the energy density of the inflaton returning to conventional matter as prescribed by the standard big bang theory given by [2], the inflaton field itself breaks down into electromagnetic radiation and thus begins the radiation dominated phase of the universe. However due to quantum fluctuations within the scalar field, the field will not roll at a constant rate down the energy hill, but will speed up and slow down at certain sections as claimed by [4], therefore inflation will not end in all locations at the same time, when it finishes is in fact dependent on position.

As the rate of expansion of the universe is accelerating, then the second derivative of the scale factor must greater than zero, as a result conditions can be placed on the inflationary parameters by deriving $\ddot{a}$ in terms of ϵ_v. Starting with,

$$\dot{a} = aH, \tag{39}$$

and differentiating in t gives,

$$\ddot{a} = \dot{a}H + a\dot{H}. \tag{40}$$

Using relations (38) and (39),

$$\begin{aligned} \ddot{a} &= aH^2 - aH^2\epsilon_v, \\ &= aH^2(1 - \epsilon_v). \end{aligned} \tag{41}$$

As $\ddot{a} > 0$ then $\epsilon_v < 1$, as shown by [10] inflation ends when slow-roll is no longer valid this occurs when $\epsilon_v = 1$. As inflation is present when the slope of the potential is less than one, constraints can be formed on the expansion rate and as a result the scale factor a.

4 e-folds

As stated in [4] the universe contains around 10^{90} particles, neglecting the possibility of expansion would mean that these 10^{90} particles would have needed to be present at the birth of the universe. This itself seems quite hard to believe; that the entire observable universe sprang into life all at once, and as a result would be difficult to explain. A much simpler account of all these particles is that the universe is modelled on an exponential, the exponential expansion of inflation simplifies the origin of theses 10^{90} particles to just the $50 - 60$ e-foldings required to produce them, note that 54 e-foldings is sufficient to satisfy all constraints. Using this exponential term then in effect drastically reduces the task of explaining all the matter of the universe, and as a result seems far more viable.

To measure inflation use $N(t_*)$ the number of e-folds, which is the logarithm of the amount of expansion, N is measured from $t = t_*$ the beginning of the inflationary era to the end $t = t_f$. Data from Planck 2013 [7] suggests that $50 < N < 60$, so calculations will be concentrated on this range.From [2] the formula for N is,

$$N(t_*) = \ln\left(\frac{a(t_f)}{a(t_*)}\right). \tag{42}$$

following on express N as a function of $\phi_* = \phi(t_*)$.

$$\begin{aligned} N(t_*) &= \int_{a(t_*)}^{a(t_f)} \frac{da}{a} = \int_{t_*}^{t_f} \frac{1}{a}\frac{da}{dt}dt = \int_{t_*}^{t_f} H dt, \\ &= \int_{\phi_*}^{\phi_f} H\frac{dt}{d\phi}d\phi = \int_{\phi_*}^{\phi_f} H\frac{d\phi}{\dot{\phi}}. \end{aligned} \tag{43}$$

Where ϕ_* and ϕ_f mark the initial and final value of the inflaton during inflation as seen in [11]. Finally using equations (29) and (30) to derive the formula below in accordance with [7],

$$N(t_*) = -\int_{\phi_*}^{\phi_f} 3H^2\frac{d\phi}{V_\phi} = -8\pi G\int_{\phi_*}^{\phi_f} \frac{V}{V_\phi}d\phi. \tag{44}$$

From this general case continue to use different values of the potential V to derive formulas for ϵ_v in N. Later this will help to relate both tensor modes and spectral index to N, allowing a graph of the models to be plotted and their usefulness assessed in comparison to observed data.

4.1 Examples

For the massive scalar field; $V(\phi) = m^2\phi^2/2$ implies $V'(\phi) = m^2\phi$ where m is the particle mass, when these are subbed into (34) it results in,

$$\epsilon_v(\phi) = \frac{1}{4\pi G\phi^2}. \tag{45}$$

Now also work out $N(\phi_*)$ for this value of V from (44),

$$\begin{aligned} N(\phi_*) &= -8\pi G\int_{\phi_*}^{\phi_f} \frac{m^2\phi^2}{2m^2\phi}d\phi, \\ &= -4\pi G\int_{\phi_*}^{\phi_f} \phi d\phi, \\ &= -2\pi G(\phi_f^2 - \phi_*^2). \end{aligned} \tag{46}$$

From equation (45) take $\phi^2 = 1/4\pi G\epsilon_v$ substituting this into (46) gives,

$$N(\phi_*) = -2\pi G\left(\frac{1}{4\pi G\epsilon_f} - \frac{1}{4\pi G\epsilon_*}\right) = \frac{1}{2\epsilon_*} - \frac{1}{2\epsilon_f}. \tag{47}$$

For the purpose of simplifying notation take $\epsilon_* = \epsilon_v(t_*)$ and $\epsilon_f = \epsilon_v(t_f)$. In this case ϵ_f occurs at the end of inflation and is equal to 1, therefore,

$$N(\phi_*) = \frac{1}{2\epsilon_*} - \frac{1}{2}. \tag{48}$$

As the model must be parametrised in terms of N, rearrange to find ϵ_* as a function of $N(\phi_*)$,

$$\epsilon_* = \frac{1}{2N(\phi_*) + 1}. \tag{49}$$

Similarly for the case of a self-interacting scalar field take $V(\phi) = \lambda\phi^4/4$ and thus $V'(\phi) = \lambda\phi^3$ where λ is known as the Higgs self-coupling term, later stringent limits will be placed on λ, but for now calculate ϵ_* by the same method,

$$\epsilon_* = \frac{1}{N(\phi_*) + 1}. \tag{50}$$

5 Fluctuations in the cosmic microwave background

The photons which the cosmic microwave background is made up of are moving throughout the universe as it expands and cools, from [4] it is believed that they have always been moving in straight lines, so they can be traced back to form an image of what the universe looked like 400,000 years after the big bang when it's radiation was emitted . These photons were originally closely connected to matter in the early hot dense universe, but were released as the temperature began to fall, before this the universe was believed to have been uniform in temperature due to the observed uniformity of its radiation. Both the temperature and polarization give us vast amounts of information of the physical composition of the universe.

As stated in [6] the universe was thought to have initially been made up of massless particles forming a gas with thermal equilibrium. Observing the CMB seems to corroborate that it is homogeneous and isotropic to better than 1% accuracy, implying that it came from a simpler time, when the universe was not cluttered by planets and stars, the theory accounts for this uniformity by claiming that the thermal-equilibrium found at microscopic scales was expanded by inflation to cover the whole universe. Due to the simplicity of this very early universe, predictions of the CMB properties are highly accurate. This gives a base which to compare the theoretical models with experimental data and the probability of future CMB polarization, to determine how applicable the inflationary model is. Fluctuations within the CMB are in part made up of scalar and tensor perturbations which can be described using a power spectrum.

The presence of such fluctuations have often been a source of contention within inflation theory, as inflation homogenises the universe completely removing all differences. However discrepancies in the density of the universe are thought to have been the result of quantum fluctuations in the inflaton field itself after the end of inflation, as explained in [4].

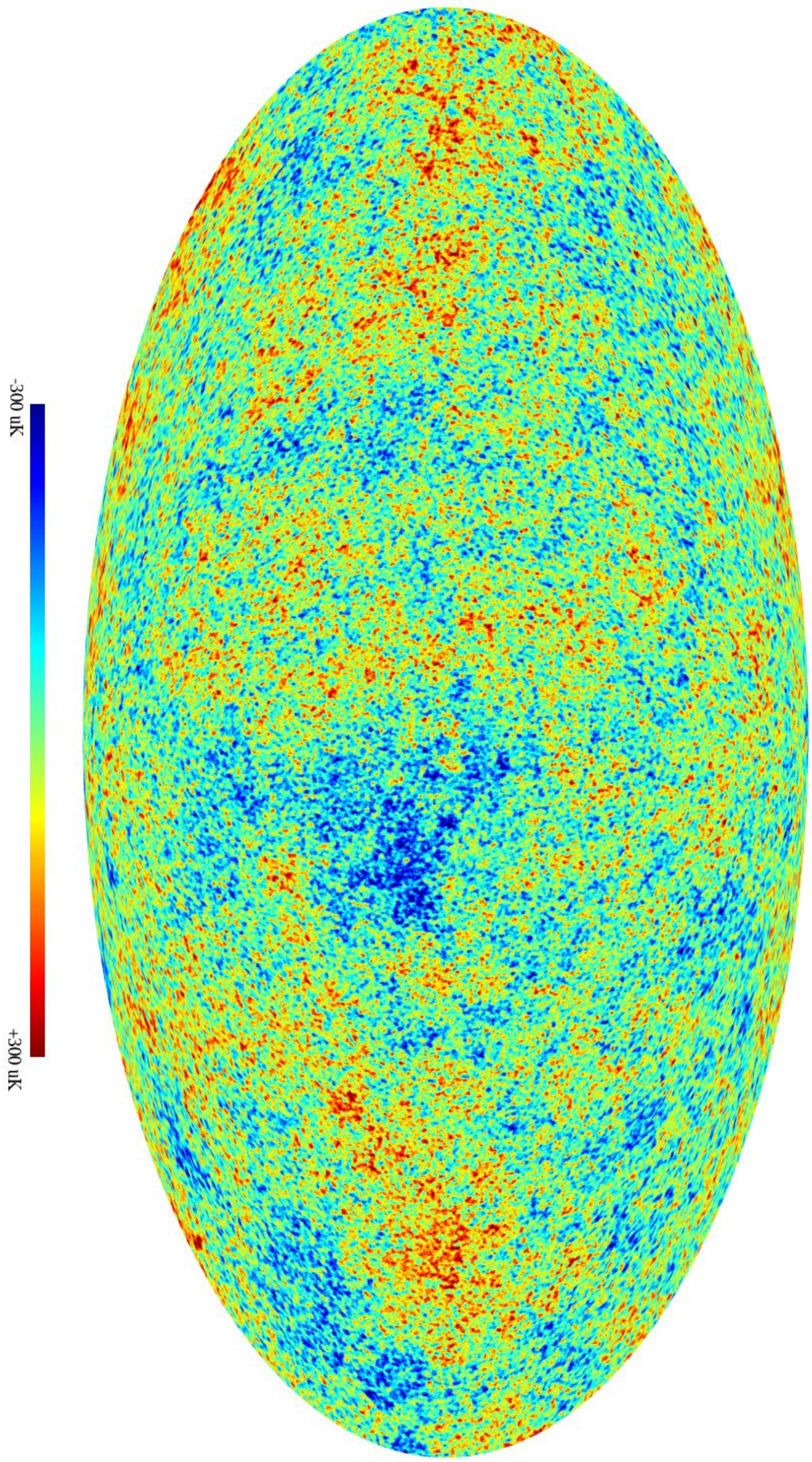

Figure 3: Cosmic microwave background of the night sky taken by satellite (WMAP Science Team / NASA [12]).

5.1 Scalar and tensor perturbations

At large scales it may seem that the CMB is uniform, however with increased accuracy fluctuations in temperature throughout are visible, this is analogous to viewing the earth's surface as covered in water at a macroscopic level, but land mass being visible at a microscopic one.

Scalar modes are perturbations in the energy density of the CMB and are the most common of all fluctuations. Unlike any other fluctuations scalar modes use gravitational instability to form structure in the universe. Use the following formula from [8] to describe scalar perturbations,

$$P_s = \frac{1}{4\pi^2}\frac{H^4}{\dot{\phi}^2}. \tag{51}$$

Tensor modes are plane gravitational wave perturbations that stretch out space in 4 dimensions, these gravity waves distort space in the perturbation plane. Early tensor fluctuations came in the form of gravitational waves which added to both the temperature and polarization of the CMB. Tensor perturbations are described in [8] as follows,

$$P_t = \frac{16GH^2}{\pi}. \tag{52}$$

5.2 Density fluctuation

Due to inflation there are fluctuations in the density of the very early universe. By observing the cosmic microwave background numerical data on the fluctuations which outline the structure of the universe is gained. As the fluctuations have been caused by inflation they can be used to place restrictions on the inflationary models. At large scales the density fluctuations of the CMB seem uniform, however as observations became more detailed dipole anisotropy can be noticed, the variation of temperature over angle. Later still, in 1992, the COBE [12] satellite detected the first cosmological temperature fluctuations in the CMB. It may seem that many models are a good fit at large scales, yet the more accurate observations become the less models hold up to the scrutiny of observed data, this allows a large portion of potential inflationary candidates to be discarded, leaving a few strong candidates remaining.

5.3 Tensor-scalar ratio

The tensor-scalar ratio r is the ratio of tensor to scalar amplitudes, where tensor fluctuations are related to gravity waves, and scalar fluctuations arise from the density fluctuations, these are the two components that make up the primordial spectrum. Theoretically the tensor-scalar ratio is believed to be between 0 and 0.3, the latest observed data now confirms this. From [7] the tensor-scalar ratio is given as (see section A.3),

$$\begin{aligned} r &= \frac{P_t}{P_s}, \\ &= -16\epsilon_H. \end{aligned} \tag{53}$$

Now by (36) write r in terms of ϵ_v ,

$$r = 16\epsilon_v. \tag{54}$$

5.4 Spectral index n_s

As the spectral index can be used to measure the dependence of the radiative flux density on frequency of any given source, the measure of spectral index often indicates the type of source that is being analysed, and it's associated properties According to [4] n_s will in general deviate from 1 by a value of order 0.1. From [5],

$$P_s = Ak^{n_s-1}, \tag{55}$$

where (see section A.4),

$$n_s = 1 + 2\eta_v - 6\epsilon_v. \tag{56}$$

Now that the equations for tensor-scalar ratio and the spectral index have been parametrised plot r over n_s, to asses the validity of the model in comparison to observed information.

6 The parameters of inflation

The finals aim is to write the tensor-scalar ratio r and the spectral index n_s in terms of N the number of e-folds. First derive equations for ϵ_v and η_v in terms of N so that they can substituted back into the original equations for r and n_s. Of course the values of r and n_s differ depending on what value V is taken to be equal to, hence here are two examples.

6.1 More examples

This time examining the self-interacting scalar field in more detail, $V = \lambda\phi^4/4$ derive $V' = \lambda\phi^3$ and $V'' = 3\lambda\phi^2$.

$$N(\phi_*) = \frac{1}{\epsilon_*} - 1. \tag{57}$$

From (35),

$$\eta_v = \frac{3}{2\pi G\phi^2} = \frac{3}{2}\epsilon_*, \tag{58}$$

rearranging and using (57),

$$\eta_* = \frac{3}{2(N_* + 1)}. \tag{59}$$

As before,

$$r = 16\epsilon_v. \tag{60}$$

Equation (57) shows that $\epsilon_v = 1/(N+1)$ and so,

$$r = \frac{16}{1+N}. \tag{61}$$

For n_s in terms of N start with,

$$n_s = 1 + 2\eta_v - 6\epsilon_v, \tag{62}$$

substitute in (57) and (59),

$$\begin{aligned} n_s - 1 &= \frac{3}{1+N} - \frac{6}{1+N} \\ &= -\frac{3}{1+N}. \end{aligned} \tag{63}$$

Similarly for the massive scalar field, $V = m^2\phi^2/2$ rearranging (48) gives,

$$\epsilon_* = \frac{1}{1+2N}. \tag{64}$$

From (35),

$$\eta_v = \frac{1}{4\pi G\phi^2} = \epsilon_v, \tag{65}$$

thus,

$$\eta_* = \frac{1}{1+2N}. \tag{66}$$

By a similar procedure,

$$r = \frac{16}{1+2N}, \tag{67}$$

and,

$$n_s - 1 = -\frac{4}{1+2N}. \tag{68}$$

Later this equation will be used examine the model, see plots.

7 New Higgs Inflation

Though previously the Standard Model Higgs boson has been used as the principle candidate for inflation, [13] states that no slow-rolling inflation can occur for the Higgs boson when it is minimally coupled with gravity. To remedy this a non-minimal coupling of the Einstein tensor with the Higgs boson must be made, so that inflation may occur without any negative consequences. The approximation for the New Higgs model is derived from a Lagrangian, the derivation of the Lagrangian itself can be found in [11], but takes the final form,

$$L = \left\{ \frac{R}{16\pi G} - \frac{1}{2}\left(g^{\mu\nu} - w^2 G^{\mu\nu} \right)\phi_\mu \phi_\nu - V(\phi) \right\} \sqrt{|g|}. \tag{69}$$

Where $\sqrt{|g|} = a^3$, the metric tensor $g^{\mu\nu}$ has components,

$$g^{tt} = -1, \tag{70}$$

and,

$$g^{xx} = g^{yy} = g^{zz} = a^{-2}, \tag{71}$$

the Einstein tensor $G^{\mu\nu}$ has components,

$$G^{tt} = 3H^2, \tag{72}$$

and,

$$G^{xx} = G^{yy} = G^{zz} = -2a^{-2}\dot{H} - a^{-2}H^2. \tag{73}$$

Using summation convention on,

$$\frac{\partial}{\partial \mu}\left(\frac{\partial L}{\partial \phi_\mu} \right) - \frac{\partial L}{\partial \phi} = 0, \tag{74}$$

results in,

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \phi_t} \right) + \frac{\partial}{\partial x}\left(\frac{\partial L}{\partial \phi_x} \right) + \frac{\partial}{\partial y}\left(\frac{\partial L}{\partial \phi_y} \right) + \frac{\partial}{\partial z}\left(\frac{\partial L}{\partial \phi_z} \right) - \frac{\partial L}{\partial \phi} = 0. \tag{75}$$

To formulate the approximation neglect spatial terms and use,

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \phi_t} \right) - \frac{\partial L}{\partial \phi} = 0, \tag{76}$$

from the Lagrangian, $\partial L / \partial \phi_t$ can be written as,

$$\frac{\partial L}{\partial \phi_t} = -(g^{tt} - w^2 G^{tt}) a^3 \dot{\phi}. \tag{77}$$

Inserting g^{tt} and G^{tt} gives,

$$\frac{\partial L}{\partial \phi_t} = a^3\dot{\phi}(1 + 3w^2H^2). \tag{78}$$

Differentiating the Lagrangian in terms of ϕ equates as,

$$\frac{\partial L}{\partial \phi} = -a^3 V_\phi, \tag{79}$$

now using equations (78) and (79) in (76) results in,

$$[a^3\dot{\phi}(1 + 3w^2H^2)]' = -a^3V_\phi. \tag{80}$$

This is the basis for one of the slow-roll approximations, and therefore all the equations previously taken for granted can be derived from this point.

7.1 Deriving the slow-roll equations

Differentiating the left hand side of equation (80) in terms of t gives,

$$3a^2\dot{a}\dot{\phi}(1 + 3w^2H^2) + a^3\ddot{\phi}(1 + 3w^2H^2) + 6w^2H\dot{H}a^3\dot{\phi} = -a^3V_\phi, \tag{81}$$

dividing through by a^3,

$$3(\frac{\dot{a}}{a})\dot{\phi}(1 + 3w^2H^2) + \ddot{\phi}(1 + 3w^2H^2) + 6w^2H\dot{H}\dot{\phi} + V_\phi = 0, \tag{82}$$

expanding the brackets and using $H = \dot{a}/a$,

$$3H\dot{\phi} + \ddot{\phi} + 9w^2H^3\dot{\phi} + 6w^2H\dot{H}\dot{\phi} + 3w^2H^2\ddot{\phi} + V_\phi = 0, \tag{83}$$

finally,

$$(\ddot{\phi} + 3H\dot{\phi})[1 + 3w^2H^2] + 6w^2H\dot{H}\dot{\phi} + V_\phi = 0. \tag{84}$$

Approximating the slow-roll equation by eliminating the small terms; $\ddot{\phi}$ and $\dot{H}\dot{\phi}$ leaves,

$$3H\dot{\phi}[1 + 3w^2H^2] = -V_\phi. \tag{85}$$

Taking $w = 0$ will in effect return the slow-roll approximation for the equation of motion, as previously stated.

7.2 Spatial terms and scalar perturbations

When including the spatial terms of (75) the following identities are required,

$$
\begin{aligned}
\frac{\partial L}{\partial \phi_x} &= -(g^{xx} - w^2 G^{xx}) a^3 \phi_x, \\
&= -a\phi_x - 2aw^2 \dot{H} \phi_x - aw^2 H^2 \phi_x.
\end{aligned} \tag{86}
$$

Differentiating in terms of x,

$$
\begin{aligned}
\frac{\partial}{\partial x}\left(\frac{\partial L}{\partial \phi_x}\right) &= -a\phi_{xx} - 2aw^2 \dot{H} \phi_{xx} - aw^2 H^2 \phi_{xx} \\
&= -a(1 + 2w^2 \dot{H} + w^2 H^2)\phi_{xx}.
\end{aligned} \tag{87}
$$

Similarly for the y and z components simply replace x with y and z respectively, subbing everything back into (75),

$$
[a^3 \dot{\phi}(1 + 3w^2 H^2)]' - a(1 + 2w^2 \dot{H} + w^2 H^2)(\phi_{xx} + \phi_{yy} + \phi_{zz}) = -a^3 V_\phi, \tag{88}
$$

dividing by a^3 and using $\phi_{xx} + \phi_{yy} + \phi_{zz} = \nabla^2 \phi$ leaves,

$$
a^{-3}[a^3 \dot{\phi}(1 + 3w^2 H^2)]' - a^{-2}(1 + 2w^2 \dot{H} + w^2 H^2)\nabla^2 \phi + V_\phi = 0. \tag{89}
$$

Compare this equation to its value when $w = 0$, as is the case in the original slow-roll approximations,

$$
a^{-3}[a^3 \dot{\phi}]' - a^{-2}\nabla^2 \phi + V_\phi = 0. \tag{90}
$$

Overall there is a factor of $1 + 3w^2 H^2$ missing, this implies that for scalar perturbations,

$$
P_w = \frac{P_{w=0}}{1 + 3w^2 H^2}. \tag{91}
$$

7.3 Heat equation

To produce the heat equation take the density of the Lagrangian as shown in [11],

$$
\rho = -\frac{1}{2}(g^{\mu\nu} - 3w^2 G^{\mu\nu})\phi_\mu \phi_\nu + V. \tag{92}
$$

Taking μ and ν equal to t,

$$
\rho = -\frac{1}{2}(g^{tt} - 3w^2 G^{tt})\dot{\phi}^2 + V, \tag{93}
$$

and subbing in g^{tt} and G^{tt},

$$\rho = \frac{1}{2}\dot{\phi}^2(1 + 9w^2H^2) + V. \tag{94}$$

As from (15),

$$3H^2 = 8\pi G\rho, \tag{95}$$

now replace ρ with (94),

$$3H^2 = 8\pi G\left(\frac{1}{2}\dot{\phi}^2(1 + 9w^2H^2) + V\right). \tag{96}$$

As $\dot{\phi}$ is small, neglect $\dot{\phi}^2$ leaving the first slow-roll approximation,

$$3H^2 = 8\pi GV. \tag{97}$$

As it was found that the original Higgs model failed for both the $\lambda\phi^4$ and the $m^2\phi^2/2$ cases a new model for inflation must be formulated. From [11] λ is believed to be in the range $0.11 < \lambda < 0.27$ as discovered by direct Higgs boson searches, this means that λ is too large to fit within the parameters of observed inflation, even when taking a non-minimal coupling λ is lower than the minimal bound as shown by [11], therefore this model cannot be counted. For the massive scalar field it was found that ϕ^2 agreed with expected results at large scales, but not at higher levels of accuracy. Now as a result the gravitational parameter w must be included in the model at a value not equal to zero, so as to correct for these anomalies.

One of the most promising candidates for inflation is the Standard Model Higgs Boson. Whereas before slow-roll inflation used a specific form of the initial equations, now a more general case will be taken using the explicit form of the slow-roll approximations by introducing gravity w, as previously stated.

To construct Higgs Inflation a new set of slow-roll equations are required, as previously formulated,

$$3H^2 = 8\pi GV, \tag{98}$$

and,

$$3H\dot{\phi}[1 + 3w^2H^2] = -V_\phi. \tag{99}$$

Take,

$$\epsilon_H = \frac{\dot{H}}{H^2} = -\frac{\epsilon_v}{1+Q}. \tag{100}$$

Where,

$$Q = 8\pi Gw^2V, \tag{101}$$

and (see section B.1),

$$\epsilon_{\dot{\phi}} = \frac{(1+3Q)\epsilon_v}{(1+Q)^2} - \frac{\eta_v}{1+Q}. \tag{102}$$

Now substituting these equations in the tensor-scalar ratio and spectral index formula generates them as functions of ϵ_v, η_v and Q only as desired. As satellite data for both tensor-scalar ratio and spectral index is readily available, then their values are most easily constrained, therefore it is best to aim for a plot of these two variables for which to judge the ϕ^2 model.

7.4 Tensor-scalar ratio

For the Higgs Inflation model the previous formula for scalar perturbations has changed due to the generalised slow-roll approximations. Using (91) then,

$$P_s = \frac{H^4}{4\pi^2\dot{\phi}^2(1+Q)}. \tag{103}$$

Though tensor perturbations remain the same, now calculate the new tensor-scalar ratio (see section B.2).

$$r = \frac{P_t}{P_s} = \frac{16\epsilon_v}{(1+Q)}. \tag{104}$$

7.5 Spectral index

Now deriving the spectral index in terms of ϵ_H and $\epsilon_{\dot{\phi}}$,

$$n_s - 1 = 4\epsilon_H - 2\epsilon_{\dot{\phi}} - \epsilon_{1+Q}. \tag{105}$$

Where,

$$\epsilon_{1+Q} = \frac{2Q}{1+Q}\epsilon_H. \tag{106}$$

Thus (see section B.3),

$$n_s - 1 = \frac{2\eta_v}{1+Q} - \frac{(8Q+6)\epsilon_v}{(1+Q)^2}, \tag{107}$$

The aim remains to be able to write r and n_s in terms of N so that they can be plotted simultaneously, but to do this first ϵ_v, η_v and Q must be derived as functions of N.

7.6 Number of e-folds

It is now possible to derive a new formula for N in terms of ϕ using the new slow-roll approximations. As before,

$$N(\phi_*) = \int_{\phi_*}^{\phi_f} H\frac{d\phi}{\dot{\phi}}, \tag{108}$$

from (151) use,

$$\dot{\phi} = \frac{-V_\phi}{3H(1+3w^2H^2)}, \tag{109}$$

thus (108) becomes,

$$N(\phi_*) = -\int_{\phi_*}^{\phi_f} \frac{3H^2(1+3w^2H^2)d\phi}{V_\phi}, \tag{110}$$

now using $H^2 = 8\pi GV/3$ it is possible to eliminate H.

$$N(\phi_*) = -\int_{\phi_*}^{\phi_f} \frac{8\pi GV(1+Q)d\phi}{V_\phi}. \tag{111}$$

7.7 Massive scalar field

Setting $V = m^2\phi^2/2$ and $V_\phi = m^2\phi$ work out $N(\phi_*)$ for this model (see section B.4).

$$N(\phi_*) = -\frac{1}{2(1+Q)} - \frac{w^2m^2}{4(1+Q)^2} + \frac{1}{2\epsilon_*} + \frac{w^2m^2}{4\epsilon_*^2}, \tag{112}$$

Rearranging and using the quadratic equation enables ϵ_* to be written in terms of N,

$$\epsilon_*^2 N(\phi_*) = -\frac{\epsilon_*^2}{2(1+Q)} - \frac{\epsilon_*^2 w^2m^2}{4(1+Q)^2} + \frac{\epsilon_*}{2} + \frac{w^2m^2}{4}. \tag{113}$$

As $N \sim 50-60$ neglect terms less than one, primarily the first two,

$$\epsilon_*^2 N - \frac{\epsilon_*}{2} - \frac{w^2m^2}{4} = 0, \tag{114}$$

using the quadratic formula,

$$\epsilon_* = \frac{\frac{1}{2} \pm \sqrt{\frac{1}{4} + w^2m^2N}}{2N}, \tag{115}$$

$\epsilon_* > 0$ therefore only take the plus sign of the formula so not to generate a negative ϵ_*.

$$\epsilon_* = \frac{\frac{1}{2} + \sqrt{\frac{1}{4} + w^2m^2N}}{2N}. \tag{116}$$

7.8 Spectral index in terms of N

For spectral index to be a function of N first derive Q in terms of N to proceed.

$$Q_* = 4\pi G w^2 m^2 \phi_*^2. \tag{117}$$

Equation (45) implies $\phi^2 = 1/4\pi G\epsilon_v$ therefore,

$$Q_* = \frac{w^2 m^2}{\epsilon_*}. \tag{118}$$

Subbing in (116) to eliminate ϵ_*,

$$Q_* = \frac{2w^2 m^2 N}{\frac{1}{2} + \sqrt{\frac{1}{4} + w^2 m^2 N}}. \tag{119}$$

Using $\epsilon_* = \eta_*$ in (107),

$$\begin{aligned} n_s - 1 &= \frac{-8\epsilon_* Q - 6\epsilon_* + 2\epsilon_*(1+Q)}{(1+Q)^2}, \\ &= \frac{-6\epsilon_* Q - 4\epsilon_*}{(1+Q)^2}. \end{aligned} \tag{120}$$

Substitute (116) and (119) into (120) to write $n_s - 1$ in terms of N.

$$n_s - 1 = \frac{-6w^2 m^2 N - 1 - 2\sqrt{\frac{1}{4} + w^2 m^2 N}}{N\left(1 + \frac{2w^2 m^2 N}{\frac{1}{2} + \sqrt{\frac{1}{4} + w^2 m^2 N}}\right)^2}. \tag{121}$$

Consider two cases $wm = 0$ and $wm >> 1$, firstly $wm = 0$ results in,

$$n_s - 1 = -\frac{2}{N}. \tag{122}$$

If on the other hand wm is very large the equation simplifies to,

$$n_s - 1 - \frac{3}{2N}. \tag{123}$$

7.9 Tensor-scalar ratio in terms of N

Similarly subbing in (116) and (119) into (104) allows r in terms of N.

$$r = \frac{4 + 8\sqrt{\frac{1}{4} + w^2 m^2 N}}{N\left(1 + \frac{2w^2 m^2 N}{\frac{1}{2} \pm \sqrt{\frac{1}{4} + w^2 m^2 N}}\right)}, \tag{124}$$

once again setting $wm = 0$,

$$r = \frac{8}{N}. \tag{125}$$

However when wm is large,

$$r = \frac{4}{N}. \tag{126}$$

Now that the extremal values of $wm = 0$ and $wm >> 1$ have been calculated for tensor-scalar ratio and spectral index, it makes sense to use computing software such as Matlab to calculate intermediate values and generate a plot.

7.10 Plotting r and n_s

As both r and n_s are in terms of N it is possible to plot the results. As the accepted value for N is believed to be between 50 and 60, both of these will be plotted with wm increasing from zero to infinity. From here proceed to compare the model to the observed data in the form of the CL plot in figure 4 to asses its validity, CL plot and statistical constraints taken from [14] and [12]. As explained in [14] the data taken from WMAP (Wilkinson Microwave Anisotropy Probe) provides strict limits which are used to constrain inflationary models. These limits then restrict the properties of such cosmological phenomena such as primordial fluctuations and gravitational waves, furthermore values of tensor-scalar ratio and spectral index have been limited in [14], so that firm criteria can be set for the models, and in effect can be approved or discredited, .

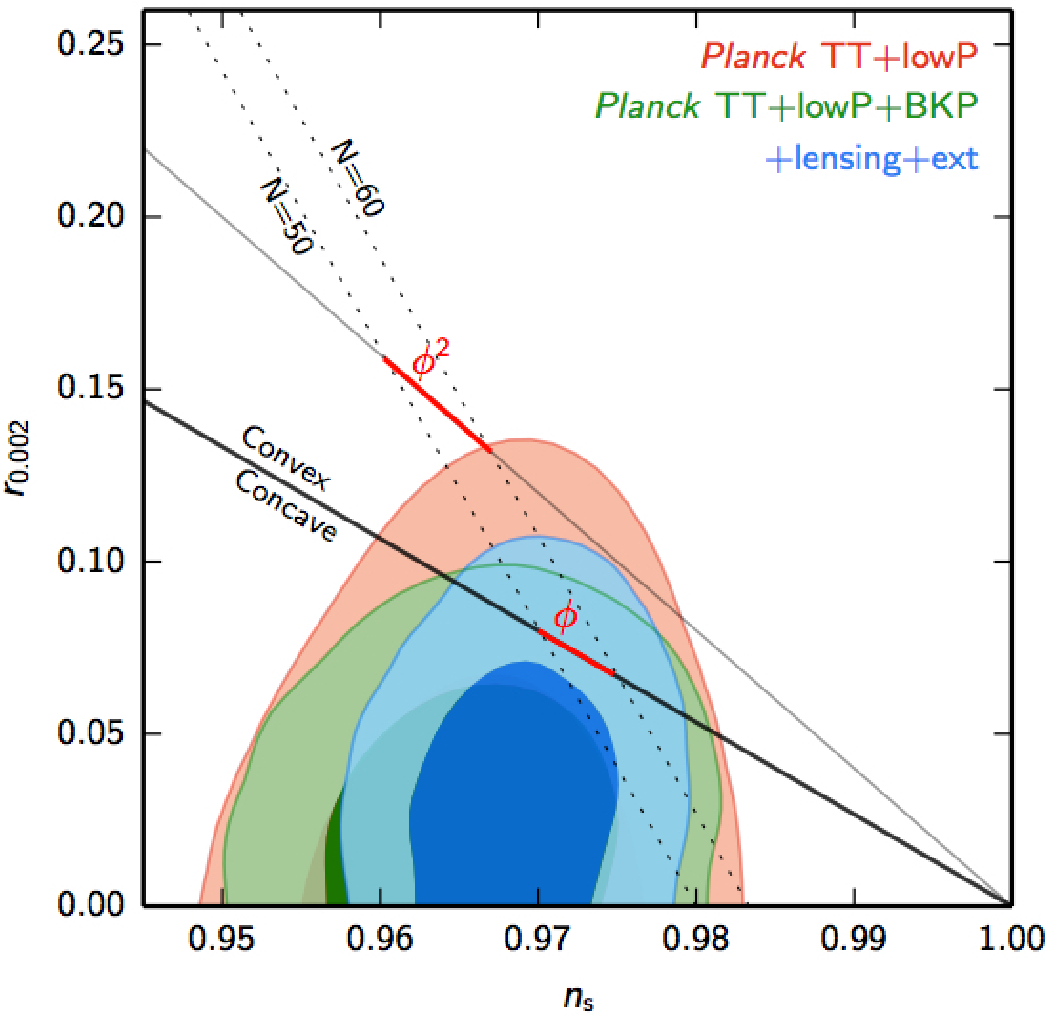

Figure 4: This plot shows the constraints placed on the model, where Planck TT+lowP denotes data from the combination of the TT likelihood at multipoles and a low-l temperature likelihood based on the CMB map, BKP is the constraint placed by the Bicep2/Keck/Planck collaborations 2015 and lensing+ext is the lensing potential power spectrum [15] plus its extensions such as BAO(baryon acoustic oscillation), image taken from [10].

Overlaying the CL plot of figure 4 onto the experimental data generates figure 5, which is in effect the best tool available for assessing the model and its acceptance of the constraints placed upon it.

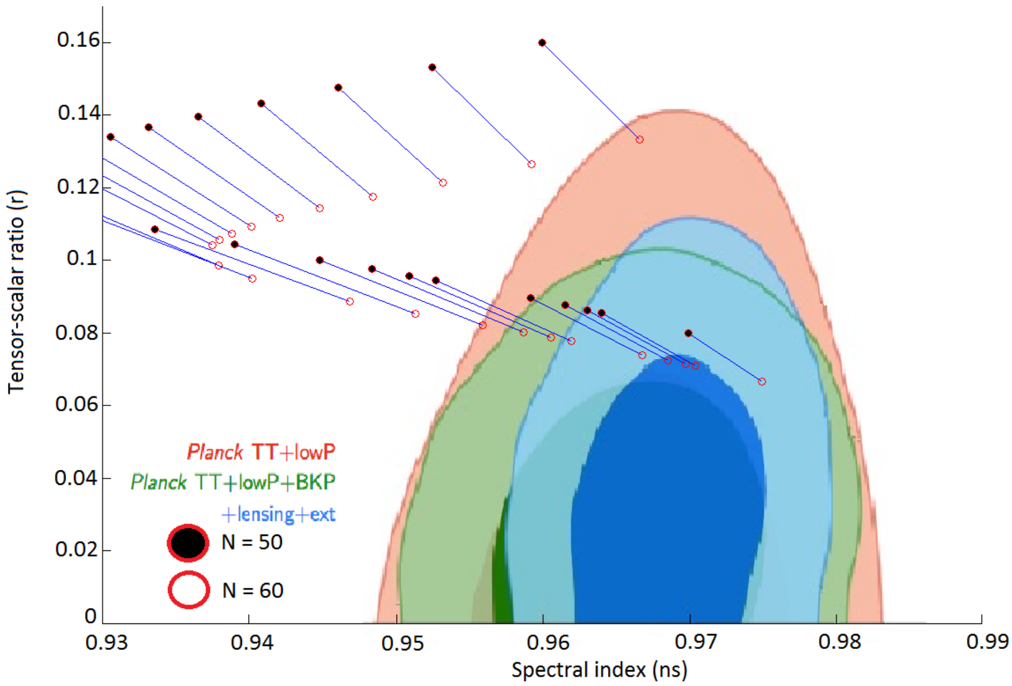

Figure 5: Plot of r vs n_s. The set of approximations form a "C" shape with wm equal to zero at the upper edge of the "C" and wm increasing towards infinity at the lower edge.

From the graph the case in which wm is set to infinity seems to fall within the "Planck TT+lowP" and "+lensing+ext" boundaries, yet it just seems to lie outside of the final constraint. Though the estimate where $wm = 0$ still falls within an accepted range it only satisfies the least rigorous constraint, it seems most models with a smaller value for wm lie outside the range of possible values however in all cases the BKP mode disfavours the ϕ^2 model as described in [10]. It may be a good idea to further restrict the range in which N operates for wm large, as points with $N = 50$ lie just outside the "+lensing+ext" boundary, plus that 54 e-folds are the minimum required to produce the mass of the universe, so anything lower will be redundant. Overall though this plot does not strongly support the belief in the massive scalar field being the elusive inflaton, due to failing to achieve the most rigorous constraints.

8 Planck data and other models

As previously stated due to the fact that very few inflationary models have been discarded, there is currently a libraries worth of candidates to consider at the present time. There are several plots available showing how the most promising models hold up to observed data in the form of confidence intervals produced by the various satellites, such as figure 6. Using these plots it is possible to see how the new Higgs model fares compared to the others, and perhaps determine a frontrunner within the field of inflation, even if it may not be possible to eliminate any more models.

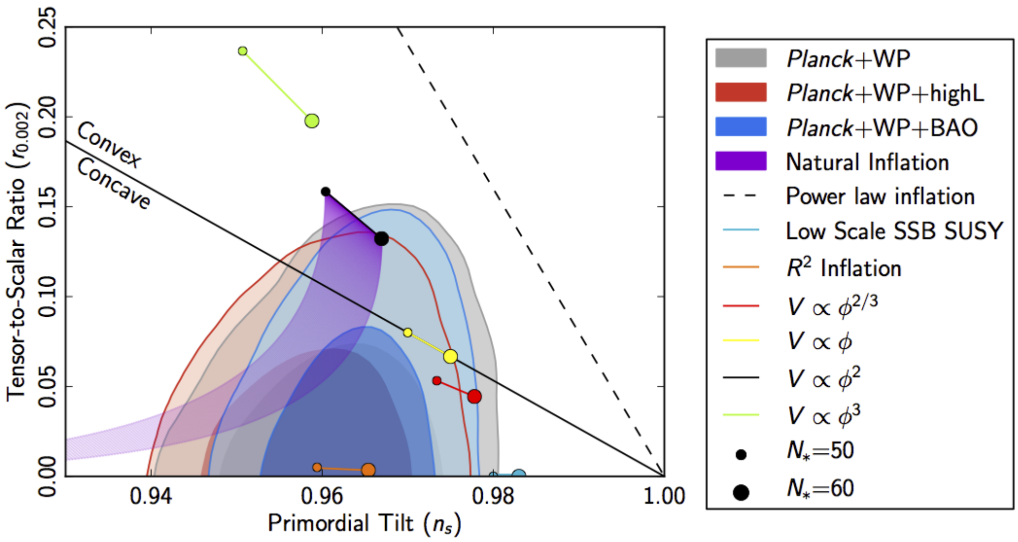

Figure 6: Plot of various inflationary models shown with expected value boundaries taken from given satellites, where WP is the Wilkinson Microwave Anisotropy Probe and highL is the Planck high-l likelihood taken from [7].

Of the different models shown on the plot, the majority involve various orders of the inflaton field ϕ proportional to the potential V, as in the case studied with ϕ being second order. These power-law potentials are within the accepted range when the power is less than or equal to 2, once again the number of e-folds N is believed to be between 50 and 60. What is left are known as hilltop models as inflation occurs at the maximum value of their potential, one example being natural inflation as described in [7].

Now that the method for analysing these models is set out, what remains as further work is to repeat the procedure for other values of the potential V with different estimates for parameters like w. The plots of the CL regions will help to restrict the amount of models needing to be to analysed, making what seems a gargantuan task somewhat less difficult. As there is no one single inflation theory, but rather a set of theorems this project only shows a small glimpse of the inflationary paradigm, naturally it makes sense now to further interpret other inflation theories such as the multi-verse theory to see where they lead.

9 Conclusion

Through examination of the massive scalar field ϕ^2 model it may seem that it presents a reasonable case for being the chief inflationary model due to its close observance of most of the satellite constraints, as can be seen on the graph, as well as remaining within the expected bounds for its parameters. However due to the vast amount of models still not discredited, with many of these fitting into the observed limits as strongly as ϕ^2, (i.e R^2 inflation) it is not possible to predict a front runner in the inflation race. The best course of action remaining is to further study models, during the course of which technology will have hopefully increased the accuracy of satellite observations, so to better extrapolate data from the infinitesimal fluctuations in the CMB. In time it is hoped that a single definitive model may be put forward as the sole inflation theory, yet the idea of inflation itself is currently bound by the limits of technology to reach this point, all that is left to do in the meantime is to analyse any models available and hope to minimise what seems to be a galleries worth of possibilities.

A Slow-roll approximation

A.1 relating ϵ_H to ϵ_v

It is possible to show that ϵ_H is equal to $-\epsilon_V$ as follows, from (32)

$$\epsilon_H = \frac{\dot{H}}{H^2}, \tag{127}$$

using (30) and

$$\dot{H} = \frac{4\pi G V_\phi \dot{\phi}}{3H}, \tag{128}$$

in (32) leaves

$$\epsilon_H = \frac{1}{2H}\frac{V_\phi}{V}\dot{\phi}, \tag{129}$$

replacing $\dot{\phi}$ with $-V_\phi/3H$ gives,

$$\epsilon_H = -\frac{V_\phi^2}{6H^2 V}, \tag{130}$$

and again using (30),

$$\begin{aligned}\epsilon_H &= -\frac{1}{16\pi G}\frac{V_\phi^2}{V^2}, \\ &= -\epsilon_v, \end{aligned} \tag{131}$$

from (34).

A.2 relating $\epsilon_{\dot{\phi}}$ to ϵ_v and η_v

Writing $\epsilon_{\dot{\phi}}$ as a function of ϵ_v and η_v, rearranging (29),

$$\dot{\phi} = -\frac{V_\phi}{3H}, \tag{132}$$

differentiating with respect to t, and using (128) and (34) gives,

$$\begin{aligned}\ddot{\phi} &= \frac{-3HV_{\phi\phi}\dot{\phi} + 3V_\phi \dot{H}}{9H^2}, \\ &= \frac{-3HV_{\phi\phi}\dot{\phi} + \frac{3}{2}H\frac{V_\phi^2}{V}\dot{\phi}}{24\pi G V}, \end{aligned} \tag{133}$$

dividing through by H and $\dot{\phi}$,

$$\frac{\ddot{\phi}}{H\dot{\phi}} = -\frac{V_{\phi\phi}}{8\pi G V} + \frac{V_\phi^2}{16\pi G V^2}. \tag{134}$$

Once again using (34) and (35) gives,

$$\epsilon_{\dot{\phi}} = \epsilon_v - \eta_v. \tag{135}$$

A.3 r in terms of ϵ_v

The ratio between tensor and scalar perturbations is labelled r and given as,

$$r = \frac{P_t}{P_s}, \tag{136}$$

subbing in equations (51) and (52) for P_s and P_t gives,

$$\begin{aligned} r &= \frac{16GH^2}{\pi}\frac{4\pi^2\dot{\phi}^2}{H^4}, \\ &= \frac{64\pi G\dot{\phi}^2}{H^2}. \end{aligned} \tag{137}$$

Using (29) and (30) to eliminate $\dot{\phi}$ and H,

$$r = \frac{1}{\pi G}\frac{V_\phi^2}{V^2}, \tag{138}$$

thus from (131),

$$r = -16\epsilon_H. \tag{139}$$

A.4 n_s in terms of ϵ_v and η_v

From (55),

$$P_s = Ak^{n_s-1}. \tag{140}$$

Where $k = 1, -1, 0$ for a closed, open or flat universe respectively from [6], taking the natural logarithm and differentiating in $\ln k$ results in,

$$n_s = 1 + \frac{d\ln P_s}{d\ln k}, \tag{141}$$

at $t = t_k$ it follows $k = a(t_k)H(t_k)$ and thus $\ln k = \ln a + \ln H$, where,

$$\frac{d\ln P_s}{d\ln k} = \frac{d\ln P_s/dt_k}{d\ln k/dt_k}, \tag{142}$$

changing variables by differentiating $\ln k$ in terms of $\ln t_k$ and then dividing by H leaves,

$$\frac{1}{H}\frac{d\ln k}{d\ln t_k} = \frac{\frac{\dot{a}}{a} + \frac{\dot{H}}{H}}{H}, \tag{143}$$

using $H = \dot{a}/a$ and (32),

$$\begin{aligned} \frac{1}{H}\frac{d\ln k}{d\ln t_k} &= 1 + \frac{\dot{H}}{H^2}, \\ &= 1 + \epsilon_H, \end{aligned} \tag{144}$$

similarly from (51) and using (32) and (33),

$$\frac{1}{H}\frac{d\ln P_s}{dt_k} = \frac{1}{H}\frac{d}{dt_k}(\ln H^4 - \ln\dot{\phi}^2 + const),$$
$$= 4\frac{\dot{H}}{H^2} - 2\frac{\ddot{\phi}}{H\dot{\phi}} = 4\epsilon_H - 2\epsilon_{\dot{\phi}}. \tag{145}$$

Combining these two results and using the fact that $\epsilon_H << 1$,

$$n_s = 1 + \frac{4\epsilon_H - 2\epsilon_{\dot{\phi}}}{1+\epsilon_H},$$
$$= 1 + 4\epsilon_H - 2\epsilon_{\dot{\phi}}. \tag{146}$$

Substituting (36) and (37) gives n_s in terms of ϵ_v and η_v,

$$n_s = 1 - 4\epsilon_v - 2(\epsilon_v - \eta_v),$$
$$= 1 + 2\eta_v - 6\epsilon_v. \tag{147}$$

B Higgs Inflation

B.1 Dynamical slow-roll parameters

Following the same steps as before now generate a general solution for ϵ_v.

$$\epsilon_H = -\frac{1}{2H}\frac{V_\phi}{V}\frac{V_\phi}{3H(1+3w^2H^2)}. \tag{148}$$

Setting $8\pi G w^2 V = Q$ and using (30) the equation becomes,

$$\epsilon_H = -\frac{1}{16\pi G}\frac{V_\phi^2}{V^2}\frac{1}{1+Q}. \tag{149}$$

Using (34),

$$\epsilon_H = -\frac{\epsilon_v}{1+Q}, \tag{150}$$

Working out $\epsilon_{\dot{\phi}}$ similarly, from (99),

$$\dot{\phi} = -\frac{V_\phi}{3H(1+3w^2H^2)}, \tag{151}$$

differentiating with respect to t,

$$\ddot{\phi} = \frac{3H(1+3w^2H^2)(-V_{\phi\phi}\dot{\phi}) + V_\phi(3\dot{H} + 27w^2\dot{H}H^2)}{9H^2(1+3w^2H^2)^2}, \tag{152}$$

dividing through by H and $\dot{\phi}$,

$$\frac{\ddot{\phi}}{H\dot{\phi}} = -\frac{V_{\phi\phi}}{8\pi GV(1+Q)} + \frac{V_\phi^2}{16\pi GV^2(1+Q)^2} + \frac{3w^2V_\phi^2}{2V(1+Q)^2}. \tag{153}$$

Once again using (34) and (35) leaves,

$$\begin{aligned} \epsilon_{\dot{\phi}} &= \frac{\epsilon_v}{(1+Q)^2} - \frac{\eta_v}{1+Q} + \frac{3Q\epsilon_v}{(1+Q)^2}, \\ &= \frac{(1+3Q)\epsilon_v}{(1+Q)^2} - \frac{\eta_v}{1+Q}. \end{aligned} \tag{154}$$

B.2 Tensor-scalar ratio

Inserting (52) and (103) into $r = P_t/P_s$ gives,

$$r = \frac{64\pi G\dot{\phi}^2(1+Q)}{H^2}. \tag{155}$$

Using (98) to replace H,

$$\begin{aligned} r &= \frac{192\pi G\dot{\phi}^2(1+Q)}{8\pi GV}, \\ &= \frac{24\dot{\phi}^2(1+Q)}{V}, \end{aligned} \tag{156}$$

substituting in (99) for $\dot{\phi}$ followed by (98) and (34),

$$\begin{aligned} r &= \frac{24V_\phi^2(1+Q)}{9H^2V(1+Q)^2}, \\ &= \frac{1}{\pi G}\left(\frac{V_\phi}{V}\right)^2\frac{1}{(1+Q)}, \\ &= \frac{16\epsilon_v}{(1+Q)}. \end{aligned} \tag{157}$$

B.3 Spectral index

From (141),

$$\begin{aligned} n_s &= 1 + \frac{d\ln P_s}{d\ln k}, \\ &= 1 + \frac{1}{H}\frac{d\ln P_s}{dt}. \end{aligned} \tag{158}$$

Inserting (103) for P_s,

$$
\begin{aligned}
n_s - 1 &= \frac{1}{H}\frac{d}{dt}\left(\ln H^4 - \ln\dot{\phi}^2 - \ln(1+Q) + const\right), \\
&= \frac{1}{H}\left(\frac{4\dot{H}}{H} - \frac{2\ddot{\phi}}{\dot{\phi}} - \frac{\dot{Q}}{1+Q}\right), \\
&= 4\frac{\dot{H}}{H^2} - 2\frac{\ddot{\phi}}{\dot{\phi}H} - \frac{\dot{Q}}{(1+Q)H}. \qquad (159)
\end{aligned}
$$

Using (31), (32) and (33) then,

$$
n_s - 1 = 4\epsilon_H - 2\epsilon_{\dot{\phi}} - \epsilon_{1+Q}, \qquad (160)
$$

as,

$$
\epsilon_{1+Q} = \frac{\dot{Q}}{(1+Q)H}. \qquad (161)
$$

By the dynamical slow-roll formula (31) when taking $x = Q$,

$$
\begin{aligned}
\epsilon_Q &= \frac{\dot{Q}}{QH}, \\
&= \frac{(1+Q)\epsilon_{1+Q}}{Q}. \qquad (162)
\end{aligned}
$$

Therefore,

$$
n_s - 1 = 4\epsilon_H - 2\epsilon_{\dot{\phi}} - \frac{Q}{1+Q}\epsilon_Q. \qquad (163)
$$

and using (101),

$$
\begin{aligned}
\epsilon_Q &= \frac{6w^2 H\dot{H}}{3w^2H^3}, \\
&= \frac{2\dot{H}}{H^2}, \\
&= 2\epsilon_H. \qquad (164)
\end{aligned}
$$

Finally,

$$
n_s - 1 = 4\epsilon_H - 2\epsilon_{\dot{\phi}} - \frac{2Q}{1+Q}\epsilon_H \qquad (165)
$$

Now substituting in (100) and (102) and writing $n_s - 1$ in terms of ϵ_v, η_v and Q only.

$$\begin{aligned} n_s - 1 &= -\frac{4\epsilon_v}{1+Q} + \frac{2\eta_v}{1+Q} - \frac{2\epsilon_v(1+3Q)}{(1+Q)^2} + \frac{2\epsilon_v Q}{(1+Q)^2}, \\ &= \frac{2\epsilon_v Q - 2\epsilon_v(1+3Q) - 4\epsilon_v(1+Q)}{(1+Q)^2} + \frac{2\eta_v}{1+Q}, \\ &= \frac{2\eta_v}{1+Q} - \frac{(8Q+6)\epsilon_v}{(1+Q)^2}. \end{aligned} \tag{166}$$

B.4 Number of e-folds

Setting $V = \frac{1}{2}m^2\phi^2$ and $V_\phi = m^2\phi$ in (111) with Q replaced by (117), proceed to work out $N(\phi_*)$ for this model.

$$\begin{aligned} N(\phi_*) &= -\int_{\phi_*}^{\phi_f} \frac{4\pi G m^2\phi^2(1+4\pi G m^2\phi^2 w^2)}{m^2\phi} d\phi, \\ &= -4\pi G\left[\frac{1}{2}\phi^2 + \pi G\phi^4 m^2 w^2\right]_{\phi_*}^{\phi_f}, \\ &= -4\pi G\left[\frac{1}{2}\phi_f^2 + \pi G\phi_f^4 m^2 w^2 - \frac{1}{2}\phi_*^2 - \pi G\phi_*^4 m^2 w^2\right]. \end{aligned}$$

Using $\phi^2 = (4\pi G\epsilon_v)^{-1}$ and that $\epsilon_f = 1 + Q$ as $\epsilon_H = -1$.

$$\begin{aligned} N(\phi_*) &= -4\pi G\left(\frac{1}{8\pi G\epsilon_f} + \frac{\pi G w^2 m^2}{16\pi^2 G^2\epsilon_f^2} - \frac{1}{8\pi G\epsilon_*} - \frac{\pi G w^2 m^2}{16\pi^2 G^2\epsilon_*^2}\right), \\ &= -4\pi G\left(\frac{1}{8\pi G(1+Q)} + \frac{w^2 m^2}{16\pi G(1+Q)^2} - \frac{1}{8\pi G\epsilon_*} - \frac{w^2 m^2}{16\pi G\epsilon_*^2}\right), \\ &= -\frac{1}{2(1+Q)} - \frac{w^2 m^2}{4(1+Q)^2} + \frac{1}{2\epsilon_*} + \frac{w^2 m^2}{4\epsilon_*^2}. \end{aligned}$$

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