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MMath Project Report

Magnetohydrodynamic Galactic Winds

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Abstract

Having introduced the concepts of galactic winds and magnetohydrodynamics, we derive equations that govern the gas density, velocity and magnetic field in the wind flow. The equations are solved and the solutions are graphed in two and three dimensions. Finally the solutions are compared with observational data from the nearby galaxy NGC 253.

All computations were performed with symbolic mathematics program MATLAB. The code can be obtained from my supervisor on request.

Contents

Chapter 1

Introduction

To begin we need a clear definition of magnetohydrodynamic galactic winds by defining the individual concepts and then bringing these definitions together.

1.1 Magnetohydrodynamics

The field of magnetohydrodynamics (or MHD) is concerned with fluid flows in the presence of magnetic fields. The main equations that are involved in MHD are a combination of Navier–Stokes equations from fluid dynamics and Maxwell's equations from electromagnetism. From these equations comes differential equations that have to be solved either analytically or numerically. Maxwell's equations are,

$$\nabla \cdot \vec{E} = \frac{\rho_0}{\epsilon_0}, \tag{1.1a}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \tag{1.1b}$$

$$\nabla \cdot \vec{B} = 0, \tag{1.1c}$$

$$\nabla \times \vec{B} = \mu \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}, \tag{1.1d}$$

where $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields respectively, ρ_0 is the electric charge density, ϵ_0 is the vacuum permittivity, t is time, μ is the vacuum permeability and $\vec{J}$ is the electric current density. Neglecting the displacement current and using Ohm's law, Maxwell's equations yield the induction equation,

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \eta \nabla^2 \vec{B}, \tag{1.2}$$

where η is the magnetic diffusivity and $\vec{v}$ is the velocity. Now that we have established the electromagnetic part of MHD we shall move onto the equations involved in fluid dynamics namely the Navier–Stokes(N–S) equation which is,

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = -\frac{\nabla P}{\rho} - \vec{g}, \tag{1.3}$$

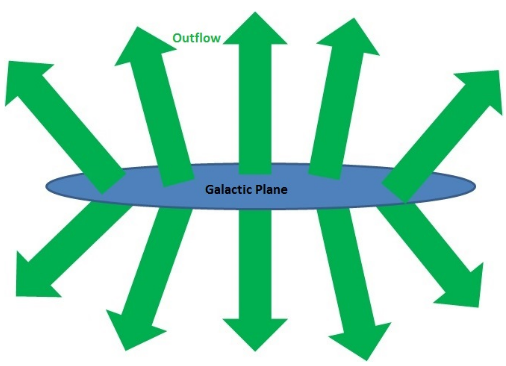
where P is pressure, ρ is density, $\vec{v}$ is the flow velocity and g is a gravity acceleration. We also need an equation for mass conservation,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0, \tag{1.4}$$

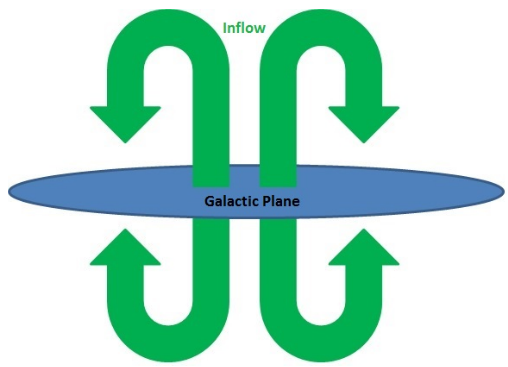
which in our case will involve a cross sectional area A and the z-component of the velocity v_z.

1.2 Galactic Winds and Fountains

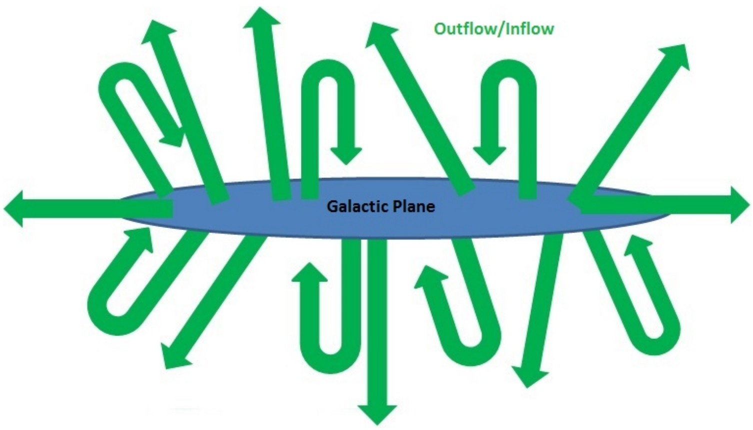
Galactic winds are a natural phenomena in the universe, which put simply, involves streams of ionised gas (outflow) that at high enough speeds can escape the gravitational field of a galaxy into galactic space. A consequence of this is that the outflow will never return and continues on to infinity. This is similar to winds in general as they have a starting point and will continue onwards without ever returning to this start point. An example of this is shown in Fig. 1.1a.



(a) Galactic Wind.



(b) Galactic Fountain.



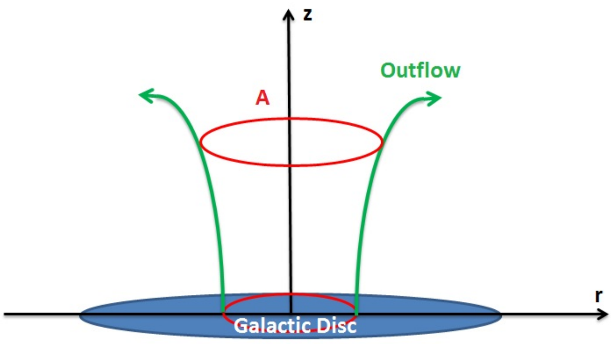
(c) Combined Model.

Figure 1.1: Diagrams of the different galactic models.

Galactic fountains are the other possiblility for the outflow of which unlike the galactic winds the speed of the outflow is less than the escape speed and therefore the gas falls back into the galactic disc. An example of this is shown in Fig. 1.1b. The final possibility is combining both the galactic wind model with the galactic fountain model so we have a model with both outflow and inflow. This can be seen in Fig. 1.1c.

1.3 Magnetohydrodynamic Galactic Winds

When regarding MHD with galactic winds we will mainly be more interested in the magnetic field than the electric field as observations from the magnetic field can provide a better insight into flow properties if the velocity is difficult or impossible to measure. Also we will be focused on creating a model for that uses the combined model. We will be deriving the wind velocity, $\vec{v}$, and the magnetic field, $\vec{B}$, which will both be vectors in cylindrical coordinates (r, ρ, z). Also we will find the pressure, p, density, ρ, and the cross-sectional area of the, A, which will be functions dependent on z and/or r. Since we are using cylindrical coordinates our model shall also be axisymmetric as seen in Fig. 1.2a.



(a) Diagram of our model showing symmetry around the axis, the cross sectional area, the outflow and the galactic disc.

(b) Galaxy NGC 253 - a spiral starburst galaxy with high star formation and has outflow driven by starbursts.

One main galaxy we will be using is NGC 253 (shown in Fig. 1.2b) also known as the Sculptor Galaxy. Currently the galaxy is in a age where there is a lot of star formation going on in its plane.

Chapter 2

Modelling Galactic Outflow

2.1 Basic Equations

The basic equations used to model galactic outflow involves the equation of mass conservation,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0, \tag{2.1}$$

which we can then modify by letting ρ not depend on t so the first term disappears and introducing the cross-sectional area of the outflow A as a function of z. This leads to a simplier equation of mass conservation as used by Hodgers (2013) from Everett *et al.* (2008),

$$\frac{d}{dz}(\rho v_z A) = 0. \tag{2.2}$$

To find the pressure P in terms of the density ρ we use the equation that describes variations in gas and cosmic-ray pressures as described and used in the paper by Everett *et al.* (2008) which when neglecting the cosmic-ray pressure gives us a simple equation for the gas pressure,

$$\frac{dP_g}{dz} = c_g^2 \frac{d\rho}{dz}, \tag{2.3}$$

where z is the height above the galactic plane, c_g is the sound speed and P_g is the gas pressure. Integrating both sides with respect to a dummy variable z' between $z' = z$ and $z' = 0$ the equation can be written as,

$$\int_0^z \frac{dP_g}{dz'}\, dz' = c_g^2 \int_0^z \frac{d\rho}{dz'}\, dz'. \tag{2.4}$$

Hence, when canceling z on both sides, the equation integrates to,

$$P_g = c_g^2 \rho. \tag{2.5}$$

The Navier-Stokes equation,

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = -\frac{\nabla P}{\rho} - \vec{g}, \tag{2.6}$$

can be reduced by assuming that the outflow velocity v and the gravity acceleration g are all functions of z. Hence, the equation becomes,

$$\frac{\partial v_z}{\partial t} + (v_z \cdot \nabla)v_z = -\frac{\nabla P}{\rho} - g, \tag{2.7}$$

and using,

$$\frac{\partial v_z}{\partial t} = 0, \tag{2.8}$$

and that the pressure only depends on z, substituting $P = P_g = c_g^2\rho$ and using the above result the N–S equation becomes,, the equation can be written as,

$$\rho v_z \frac{dv_z}{dz} + c_g^2 \frac{d\rho}{dz} = -\rho g. \tag{2.9}$$

This equation is the one found by Everett *et al.* (2008). Above we have $g \geq 0$ is a gravity acceleration. Also we have c_*^2 being the composite sound speed which involves P_g. The equation for the composite sound speed,

$$c_*^2 = \frac{dP_g}{d\rho}. \tag{2.10}$$

Also when comparing with Equation (2.3) we can see that $c_g^2 = c_*^2$. Now we can substitute a polytropic equation of state for the gas pressure,

$$P_g = P_0 \left(\frac{\rho}{\rho_0}\right)^\gamma. \tag{2.11}$$

Next, substituting this into Equation (2.10), we can see that

$$c_*^2 = \frac{dP_g}{d\rho} = \frac{d}{d\rho}\left[P_0 \left(\frac{\rho}{\rho_0}\right)^\gamma\right] = \gamma\frac{P_g}{\rho} = c_g^2. \tag{2.12}$$

From this we have that γ is the polytropic index of the thermal gas as in the paper by Everett *et al.* (2008).

2.2 Gas Density in the Outflow

To derive the wind density $\rho(z)$ we must assume P_g is distributed exponentially, e.g. $P_g = P_0 e^{-z/H}$, in which H is the gas scale height as in the paper by Hodgers (2013). We do this because Everett *et al.* (2008), found that approximately both the density and pressure of the gas decrease almost expontentially with height. Differentiating this with respect to z gives,

$$\frac{dP_g}{dz} = \frac{d}{dz}(P_0 e^{-z/H}) = -\frac{P_0}{H}e^{-z/H} = -\frac{P_g}{H}. \tag{2.13}$$

Substituting this and Equation (2.12) into Equation (2.3) gives

$$-\frac{P_g}{H} = \frac{\gamma P_g}{\rho}\frac{d\rho}{dz}. \tag{2.14}$$

To solve this equation we need to cancel P_g on both sides, use separation of variables and exponentiating to give an equation for the density,

$$\rho = C \exp\left(-\frac{z}{H\gamma}\right), \tag{2.15}$$

and to find C by letting the density at $z = 0$ be ρ_0 which means that $\rho_0 = C$. Therefore,

$$\rho = \rho_0 \exp\left(-\frac{z}{H\gamma}\right). \tag{2.16}$$

2.3 Outflow Velocity

To derive the outflow velocity we need to rewrite Equation (2.9) by replacing $\rho v_z \dfrac{dv_z}{dz}$ with $\dfrac{1}{2}\rho\dfrac{dv_z^2}{dz}$ as used by Hodgers (2013) and dividing by ρ which gives,

$$\frac{1}{2}\frac{dv_z^2}{dz} = -\left(g + \gamma\frac{P_g}{\rho^2}\frac{d\rho}{dz}\right). \tag{2.17}$$

Since we already know what ρ is, we can find the derivative with respect to z,

$$\frac{d\rho}{dz} = \frac{d}{dz}\left(\rho_0 e^{-z/(H\gamma)}\right) = -\frac{\rho_0}{H\gamma}e^{-z/(H\gamma)}. \tag{2.18}$$

Now, substituting Equations (2.18), (2.16) and (2.11) into Equation (2.17) we get,

$$\frac{1}{2}\frac{dv_z^2}{dz} = -g - \frac{\gamma}{\rho_0^2 e^{-2z/H\gamma}}P_0\left(\frac{\rho_0 e^{-z/H\gamma}}{\rho_0}\right)^\gamma\left(-\frac{\rho_0}{H\gamma}e^{-z/(H\gamma)}\right), \tag{2.19}$$

or

$$\frac{1}{2}\frac{dv_z^2}{dz} = -g + \frac{P_0}{\rho_0 H}\exp\left(-\frac{z(1-1/\gamma)}{H}\right). \tag{2.20}$$

Now we will let g be the gravity acceleration in the solar vicinity, as used in the paper by Fletcher and Shukurov (2001) which was originally derived by Ferrière (1998) from Kuijken and Gilmore (1989),

$$g = A_1\frac{z}{\sqrt{z^2+Z_1^2}} + A_2\frac{z}{Z_2}, \tag{2.21}$$

where $A_1 = 4.4\times10^{-9}$ cm s^{-2}, $A_2 = 1.7\times10^{-9}$ cm s^{-2}, $Z_1 = 0.2$ kpc and $Z_2 = 1$ kpc. Equation (2.21) is similar to the equation found by Hodgers (2013). By substituting this into Equation (2.20) and multiplying by two gives

$$\frac{dv_z^2}{dz} = -2A_1\frac{z}{\sqrt{z^2+Z_1^2}} - 2A_2\frac{z}{Z_2} + 2\frac{P_0}{\rho_0 H}\exp\left(-\frac{z(1-1/\gamma)}{H}\right) \tag{2.22}$$

To find v_z^2 we will have to integrate the equation with respect to a dummy variable between $z'=0$ and $z'=z$,

$$v_z^2 = -2A_1\int_0^z \frac{z'}{\sqrt{z'^2+Z_1^2}}dz' - 2A_2\int_0^z\frac{z'}{Z_2} + \frac{P_0}{\rho_0 H}\int_0^z e^{-\frac{z'(1-1/\gamma)}{H}}\,dz'. \tag{2.23}$$

To find v_z we take these integrals separately and solve them. Starting with the first integral,

$$-2A_1\int_0^z\frac{z'}{\sqrt{z'^2+Z_1^2}}\,dz', \tag{2.24}$$

which integrates to,

$$-2A_1\left[\sqrt{z^2+Z_1^2} - Z_1\right]. \tag{2.25}$$

Now for the second integral, which is a trivial integration,

$$-\frac{2A_2}{Z_2}\int_0^z z'\,dz' = -A_2\frac{z^2}{Z_2}. \tag{2.26}$$

Finally the third integral,

$$\frac{2P_0}{\rho_0 H}\int_0^z \exp\left(-\frac{z'(1-1/\gamma)}{H}\right)dz', \tag{2.27}$$

which becomes,

$$\frac{2P_0\gamma}{\rho_0(\gamma-1)}\left[1-\exp\left(-\frac{z(1-1\gamma)}{H}\right)\right]. \tag{2.28}$$

Now that we have evaluated the three integrals, we can now find v_z,

$$v_z^2 = -2A_1\left[\sqrt{z^2+Z_1^2}-Z_1\right]-A_2\frac{z^2}{Z_2}+\frac{2P_0\gamma}{\rho_0(\gamma-1)}\left[1-\exp\left(-\frac{z(1-1/\gamma)}{H}\right)\right]. \tag{2.29}$$

To find $v(z)$ we also must add the velocity of the wind at $z=0$ which is $v_z(0)=v_0$ and then square root v_z^2,

$$v_z(z) = \sqrt{-2A_1\left[\sqrt{z^2+Z_1^2}-Z_1\right]-A_2\frac{z^2}{Z_2}+\frac{2P_0\gamma}{\rho_0(\gamma-1)}\left[1-\exp\left(-\frac{z(1-1/\gamma)}{H}\right)\right]+v_0^2}, \tag{2.30}$$

which is similar to the equation found by Hodgers (2013).

2.4 Cross-Sectional Area for the Outflow

To find the cross-sectional area for the outflow we must integrate Equation (2.2) with respect to z as in the paper by Hodgers (2013),

$$\frac{d}{dz}(\rho v_z A) = 0, \tag{2.31}$$

to get

$$A(z) = \frac{\phi}{\rho(z)v_z(z)}, \tag{2.32}$$

where ϕ is a constant. To find $A(z)$ we must find ϕ on the galactic disc, i.e. setting $z=0$, which then gives $A(0)=A_0$, $\rho(0)=\rho_0$ and $v(0)=v_0$ so $\phi=A_0\rho_0 v_0$. Finally substituting Equations (2.16) and (2.30) into $A(z)$ to get,

$$A(z) = \frac{A_0 v_0 e^{z/H\gamma}}{\sqrt{-2A_1\left[\sqrt{z^2+Z_1^2}-Z_1\right]-A_2\frac{z^2}{Z_2}+\frac{2P_0\gamma}{\rho_0(\gamma-1)}\left[1-e^{-\frac{z(1-1/\gamma)}{H}}\right]+v_0^2}}, \tag{2.33}$$

which is close to the equation found by Hodgers (2013).

2.5 The Magnetic Field

In this part we will need to find the z-component of the magnetic field, B_z, by first introducing the magnetic flux conservation law,

$$\frac{d}{dz}(B_z A) = 0, \tag{2.34}$$

and then integrating it with respect to z and re-arranging to get,

$$B_z = \frac{\lambda}{A}, \tag{2.35}$$

where λ is a constant, which for $z = 0$, $B_z|_{z=0} = B_{z0}$ and $A(0) = A_0$ gives $\lambda = B_{z0}A_0$. Then substituting Equation (2.33) into this gives,

$$B_z(z) = \frac{B_{z0}}{v_0} e^{-z/H\gamma} \sqrt{-2A_1 \left(\sqrt{z^2 + Z_1^2} - Z_1\right) - \frac{A_2}{Z_2} z^2 + \frac{2P_0\gamma}{\rho_0(\gamma - 1)} \left[1 - e^{-\frac{z(1+1/\gamma)}{H}}\right] + v_0^2}, \tag{2.36}$$

which is similar to the equation found in Hodgers (2013).

2.6 The Magnetic and Velocity Fields in the Outflow

2.6.1 The Magnetic Field Radial Component

The magnetic field uses cylindrical polar co-ordinates denoted by (B_r, B_ϕ, B_z) as it is a solenoidal of which we already have the z-component from earlier. We can assume $\frac{\partial}{\partial \phi} = 0$ since we have an axi-symmetric model. We shall use this to rewrite Gauss' Law for Magnetism, $\nabla \cdot \vec{B} = 0$, as

$$\frac{1}{r}\frac{\partial}{\partial r}(rB_r) + \frac{\partial}{\partial z}(B_z) = 0. \tag{2.37}$$

To solve this we need to re-arrange and integrate with respect to r which gives,

$$B_r = -\frac{r}{2}\frac{\partial B_z}{\partial z} + \frac{C}{r}. \tag{2.38}$$

Due to B_r being finite for all values of r, for $r = 0$ B_r is finite, therefore $C = 0$. Hence,

$$B_r = -\frac{r}{2}\frac{\partial B_z}{\partial z}, \tag{2.39}$$

or

$$\vec{B} = \left(-\frac{r}{2}\frac{\partial B_z}{\partial z}, 0, B_z\right), \tag{2.40}$$

which is the vector found by Hodgers (2013).

2.6.2 The Velocity Field Radial Component

The radial component for the outflow velocity is calculated using the mass conservation,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 \tag{2.41}$$

with

$$\frac{\partial \rho}{\partial t} = 0 \tag{2.42}$$

Therefore the equation becomes,

$$\nabla \cdot \ (\rho \vec{v}) = 0, \tag{2.43}$$

which can be rewritten as,

$$\nabla\rho \cdot \vec{v} + \rho\nabla\vec{v} = 0, \tag{2.44}$$

and since ρ is only a function of z this implies that

$$\nabla\rho = \left(0, 0, \frac{\partial\rho}{\partial z}\right). \tag{2.45}$$

Therefore, Equation (2.44) can be written as,

$$v_z\frac{\partial\rho}{\partial z} + \rho\left[\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{\partial v_z}{\partial z}\right] = 0 \tag{2.46}$$

and since $\frac{1}{\rho}\frac{\partial\rho}{\partial z} = \frac{\partial \ln\rho}{\partial z}$ we have,

$$\frac{\partial v_z}{\partial z} + \frac{\partial \ln\rho}{\partial z}v_z = -\frac{1}{r}\frac{\partial}{\partial r}(rv_r). \tag{2.47}$$

From here there are two choices we can make to find v_r, one being that ρ and v_z are independent of r or to let ρ and v_z dependent on r. If we make the independent choice, then we can simply integrate Equation (2.47) with respect to r and then rearrange to find that,

$$v_r = -\left(\frac{r}{2} + \frac{C}{r}\right)\left(\frac{\partial v_z}{\partial z} + \frac{\partial \ln\rho}{\partial z}v_z\right). \tag{2.48}$$

To find the constant C we know that v_r is finite as $r \to 0$ therefore $C = 0$. Hence,

$$v_r = -\frac{r}{2}\left(\frac{\partial v_z}{\partial z} + \frac{\partial \ln\rho}{\partial z}v_z\right), \tag{2.49}$$

and then substituting ρ in, we then get,

$$v_r = -\frac{r}{2}\left(\frac{\partial v_z}{\partial z} - \frac{v_z}{H\gamma}\right). \tag{2.50}$$

On the other hand, we could let ρ and v_z depend on r which leads us to integrating Equation (2.47) with respect to a dummy variable r' and then rearranging so the equation becomes,

$$v_r = -\frac{1}{r}\int_0^r r'\left(\frac{\partial v_z}{\partial z} + \frac{\partial \ln\rho}{\partial z}v_z\right)dr'. \tag{2.51}$$

Since we want ρ and v_z dependent on the radius r then we will be solving Equation (2.51) numerically.

2.7 The Azimuthal Magnetic and Velocity Field

2.7.1 The Magnetic Field Azimuthal Component

We now need to find the azimuthal component of the magnetic field or B_ϕ which can be found using the magnetic flux and mass conservation. Using, for C,

$$B_\phi \, dr \, dz = C \tag{2.52}$$

and

$$\phi\, dV = C \tag{2.53}$$

where

$$V = 2\pi r\, dr\, dz. \tag{2.54}$$

Therefore,

$$2\pi r\, dr\, dz = \text{const} \tag{2.55}$$

Now to find B_ϕ,

$$\frac{B_\phi}{r} = \text{const}. \tag{2.56}$$

When $z = 0$, $B_\phi = B_{\phi 0}$ and $r = r_0$, we get that $C = \frac{B_{\phi 0}}{r_0}$. Therefore,

$$B_\phi = B_{\phi 0}\frac{r}{r_0}, \tag{2.57}$$

with

$$B_{\phi 0} = B_\phi(r_0, 0) \tag{2.58}$$

2.7.2 The Velocity Azimuthal Component

To find v_ϕ or the velocity azimuthal component can be found by combining the formulae for angular momentum,

$$L = \rho v_\phi r dV = \text{const}, \tag{2.59}$$

and mass conservation,

$$M = \rho dV = \text{const}, \tag{2.60}$$

which gives,

$$v_\phi r = \text{const}. \tag{2.61}$$

Then at the point when $z = 0$, we get $v_\phi = v_{\phi 0}$ and $r = r_0$. Hence, we have const$= v_{\phi 0} r_0$ and then the equation becomes,

$$v_\phi = \frac{v_{\phi 0} r_0}{r}. \tag{2.62}$$

However as this does not work for all values of r, because at $r = 0$, $v_\phi \to \infty$ or the equation is singular at $r = 0$. Therefore we must try with Navier–Stokes again,

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla \vec{v}) = -\frac{\nabla P}{\rho}, \tag{2.63}$$

with

$$\frac{\partial \vec{v}}{\partial t} = 0. \tag{2.64}$$

Now, to find v_ϕ we must find the ϕ component of $(\vec{v} \cdot \nabla \vec{v})_\phi$,

$$(\vec{v} \cdot \nabla \vec{v})_\phi = v_r \frac{\partial v_\phi}{\partial r} + v_z \frac{\partial v_\phi}{\partial z} + \frac{v_\phi v_r}{r}. \tag{2.65}$$

Therefore, the Navier-Stokes equation can be written as,

$$v_r \frac{\partial v_\phi}{\partial r} + v_z \frac{\partial v_\phi}{\partial z} = -\frac{v_r}{r} v_\phi - \frac{1}{\rho}\frac{\partial P}{\partial z}, \tag{2.66}$$

and when $z = 0$ this gives $v_\phi(r,0) = v_0(r)$. To solve for v_ϕ we must rewrite the equation using $v_\phi = u$, $\partial r = \partial x$, $\partial z = \partial y$ and constants $v_r = a$, $v_z = b$, $v_r/r = c$, $v_0(r) = u(x,0)$ and $-\frac{1}{\rho}\frac{\partial P}{\partial z} = d$, then find the characteristics, introduce two new variables, ξ and η, and solve for u. To begin, Equation (2.66) is written as,

$$a\frac{\partial u}{\partial x} + b\frac{\partial u}{\partial y} = -cu + d. \tag{2.67}$$

Now to introduce $\xi = x$ and $\eta = \eta(x,y) = $ const on the characteristics and hence integrating,

$$\frac{dy}{dx} = \frac{b}{a}, \tag{2.68}$$

and solving on the characteristics gives,

$$\eta = y - \frac{b}{a}x. \tag{2.69}$$

Also we have to solve the equation,

$$a\frac{\partial u}{\partial \xi} = -cu + d. \tag{2.70}$$

Dividing by a, rearranging and then integrating both sides gives,

$$\int \frac{a}{cu - d}\, du = \int -\, d\xi. \tag{2.71}$$

We will need to use $f(\eta)$ as the constant of integration, then divide both sides by a/c and next exponentiating both sides of the solution to get,

$$cu - d = \exp\left[-\frac{c}{a}\xi + \frac{c}{a}f(\eta)\right]. \tag{2.72}$$

Now rearranging to get u, substituting Equation (2.69) and $\xi = x$ the equation now is,

$$u = \frac{d}{c} + \frac{1}{c}\exp\left[\frac{c}{a}\left\{-x + f\left(y - \frac{b}{a}x\right)\right\}\right]. \tag{2.73}$$

Using $u(x,0) = v_0(x)$, the above equation becomes,

$$v_0(x) = \frac{d}{c} + \frac{1}{c}\exp\left\{\frac{c}{a}\left[-x + f\left(-\frac{b}{a}x\right)\right]\right\}, \tag{2.74}$$

which can be rewritten as,

$$(cv_0(x) - d)\exp\left(\frac{c}{a}x\right) = \exp\left[\frac{c}{a}f\left(-\frac{b}{a}x\right)\right]. \tag{2.75}$$

Now, by letting $A = -bx/a$ and substituting into Equation (2.75), we obtain,

$$(cv_0(x) - d)\exp\left\{-\frac{c}{b}A\right\} = \exp\{f(A)\}. \tag{2.76}$$

After letting $A = y - \dfrac{b}{a}x$ in Equation (2.76) and substituting back into u from Equation (2.73), the equation becomes,

$$u = \frac{d}{c} + \frac{(cv_0(x) - d)}{c}\exp\left\{-\frac{c}{b}\left(y - \frac{b}{a}x\right) - \frac{c}{a}x\right\}. \tag{2.77}$$

Now cancelling the cx/a's in the exponential and rewriting this with the original variables and constants we have,

$$v_\phi = \frac{1}{\rho}\frac{\partial P}{\partial z}\frac{r}{v_r} + \left[v_0(r) - \frac{1}{\rho}\frac{\partial P}{\partial z}\frac{r}{v_r}\right]\exp\left\{-\frac{v_r}{r}\frac{z}{v_z}\right\}. \tag{2.78}$$

To prove that this is correct, we can let $z = 0$ therefore the solution to our equation should be $v_\phi(r, 0) = v_0(r)$. So solving our equation we get that,

$$\begin{aligned} v_\phi(r,0) &= \frac{1}{\rho}\frac{\partial P}{\partial z}\frac{r}{v_r} + \left[v_0(r) - \frac{1}{\rho}\frac{\partial P}{\partial z}\frac{r}{v_r}\right]\exp\left\{-\frac{v_r}{r}\frac{0}{v_z}\right\} \\ &= \frac{1}{\rho}\frac{\partial P}{\partial z}\frac{r}{v_r} + v_0(r) - \frac{1}{\rho}\frac{\partial P}{\partial z}\frac{r}{v_r} \\ &= v_0(r). \quad \square \end{aligned} \tag{2.79}$$

Which is correct but now we must also check if our equation works for all values of r, as that was the problem with using angular momentum and mass conservation. The equation now also works for all values of r, specifically when $r = 0$, $\exp(-v_r z/0) \to 0$, therefore we can see for $r = 0$, v_ϕ has no singularities.

2.8 Galactic Parameters

Now that we have found the wind velocity, magnetic field, cross-sectional area and the density we can now find out what they look graphically but first we need to non-dimensionalize certain variables. We shall start by defining what our dimensionless quantities will be for length x, time, t and velocity v,

$$\begin{aligned} [x] &= 1 \text{ kpc} = 3 \times 10^{21} \text{ cm}, \\ [t] &= 10 \text{ Myr} = 3 \times 10^{14} \text{ s}, \\ [v] &= 100 \text{ km/s} = 10^7 \text{ cm/s}. \end{aligned} \tag{2.80}$$

To check these, we shall check that the velocity is length/time,

$$[v] = \frac{[x]}{[t]} = \frac{3 \times 10^{21} \text{ cm}}{3 \times 10^{14} \text{ s}} = 10^7 \text{ cm/s} = 100 \text{ km/s}. \checkmark \tag{2.81}$$

From Equation (2.80) we have that,

$$\begin{aligned} 1 \text{ cm} &= \frac{[x]}{3 \times 10^{21}} \\ 1 \text{ s} &= \frac{[t]}{3 \times 10^{14}} \end{aligned} \tag{2.82}$$

Now we can find the dimensionless quantities for the density, ρ, the magnetic field, $\vec{B}$ and the pressure P. The density is defined by $[\rho] = m_p[n]$ where m_p is the mass of the proton and $[n]$ is the number density. Using $m_p = 1.7 \times 10^{-24}$ g and the number density $[n] = 1 \text{ cm}^{-3}$, we get that,

$$[\rho] = 1.7 \times 10^{-24} \text{ g cm}^{-3}. \tag{2.83}$$

From this we get that,

$$1 \text{ g cm}^{-3} = \frac{[\rho]}{1.7 \times 10^{-24}}. \tag{2.84}$$

The pressure is $[P] = [\rho][v]^2$ with $[\rho] = 1.7 \times 10^{-24}$ g cm^{-3} and $[v] = 10^7$ cm s^{-1}, so we obtain,

$$[P] = 1.7 \times 10^{-10} \text{ g cm}^{-1} \text{ s}^{-2}. \tag{2.85}$$

Which then gives us,

$$1 \text{ g cm}^{-1} \text{ s}^{-2} = \frac{[P]}{1.7 \times 10^{-10}}. \tag{2.86}$$

The magnetic field is calculated using $[B] = \sqrt{4\pi[\rho][v]^2}$ with ρ and v as above we find that,

$$[B] = 4.62 \times 10^{-5} \text{ g}^{1/2} \text{ cm}^{3/2} \text{ s}^{-1/2}, \tag{2.87}$$

which can be converted into Gaussian units for the magnetic field by using $\text{G} \equiv \text{ g}^{1/2} \text{ cm}^{3/2} \text{ s}^{-1/2}$. Hence, we have,

$$[B] = 4.62 \times 10^{-5} \text{ G} = 46.2. \tag{2.88}$$

This then gives us,

$$1 \ \mu\text{G} = \frac{[B]}{46.2\mu\text{G}}. \tag{2.89}$$

Now we can convert the other parameters into a dimensionless form using the values for A_1 and A_2 from the paper by Kuijken and Gilmore (1989),

$$\begin{aligned} A_1 &= 4.4 \times 10^{-9} \frac{\text{cm}}{\text{s}^2} \times \frac{[x] \ 9 \times 10^{28} \text{ s}^2}{3 \times 10^{21} \text{ cm } [t]^2} \\ &= 0.132 \frac{[x]}{[t]^2}. \\ A_2 &= 1.7 \times 10^{-9} \frac{\text{cm}}{\text{s}^2} \times \frac{[x] \times 9 \times 10^{28} \text{ s}^2}{3 \times 10^{21} \text{ cm } [t]^2} \\ &= 0.051 \frac{[x]}{[t]^2}. \end{aligned} \tag{2.90}$$

To check that the conversion to dimensionless form is correct we must do the calculation A_1/A_2 for the original values and the dimensionless values,

$$\begin{aligned} \frac{A_1}{A_2} &= \frac{4.4 \times 10^{-9}}{1.7 \times 10^{-9}} = \frac{44}{17} \\ \frac{A_1}{A_2} &= \frac{0.132}{0.051} = \frac{44}{17}. \end{aligned} \tag{2.91}$$

Therefore our dimensionless forms for A_1 and A_2 are correct. The initial conditions for the density, ρ_0 involving using the mass of the proton, m_p, from before as well as the number density at the base of the wind, i.e. when $z = 0$, which is $n_0 = 10^{-3}$ cm^{-3}. Thus we obtain,

$$\rho_0 = m_p n_0 = 1.7 \times 10^{-27} \text{ g cm}^{-3}. \tag{2.92}$$

Now the initial condition for the pressure, P at $z = 0$, is dependent upon the temperature of the gas as it leaves the galactic disc, T, the number density at the base of the wind, n_0 and $k = 1.38 \times 10{-}16$ erg K^{-1}. We shall use the typical temperature of the hot gas in galactic discs as described by Spitzer (2004), i.e. $T = 10^6$ K. This gives,

$$P = n_0 kT = 1.38 \times 10^{-13} \text{ g cm}^{-1} \text{ s}^{-2}. \tag{2.93}$$

For the components of the magnetic field (B_r, B_ϕ, B_z), we will need to have the conditions on the galactic plane i.e. when $z = 0$. So we shall use $B_r|_{z=0} = B_{r0} \approx 0.5 \ \mu G$, $B_\phi|_{z=0} =$

$B_{\phi 0} \approx 2\ \mu G$ and $B_z|_{z=0} = B_{z0} \approx 0.1\ \mu G$. Hence the dimensionless quantities for these will be found by making each of B_{r0}, $B_{\phi 0}$ and B_{z0} dimensionless,

$$\begin{aligned} B_{r0} &= 0.5\ \mu\text{G}\ \times \frac{[B]}{46.2\ \mu\text{G}} = 0.01082\ [B] \\ B_{\phi 0} &= 2\ \mu\text{G}\ \times \frac{[B]}{46.2\ \mu\text{G}} = 0.04329\ [B] \\ B_{z0} &= 0.1\ \mu\text{G}\ \times \frac{[B]}{46.2\ \mu\text{G}} = 0.00216\ [B]. \end{aligned} \tag{2.94}$$

Other parameters we need are $\gamma = 5/3$, $H = 0.5$ kpc, $Z_1 = 0.2$ kpc and $Z_2 = 1$ kpc which are either already dimensionless or in the dimensional quantities we need from Section 2.3. The other dimensionless quantities will by found by making each of ρ, P and v dimensionless. Also we shall use $v_0 = 300$ km s^{-1} for our initial condition for the outflow velocity. Therefore, we have,

$$\begin{aligned} \rho_0 &= 1.7 \times 10^{-27}\ \text{g cm}^{-3} \times \frac{[\rho]}{1.7 \times 10^{-24}\ \text{g cm}^{-3}} = 10^{-3}[\rho]. \\ P &= 1.38 \times 10^{-13}\ \text{g cm}^{-1}\ \text{s}^{-2} \times \frac{[P]}{1.7 \times 10^{-10}\ \text{g cm}^{-1}\ \text{s}^{-2}} = 8 \times 10^{-4}[P]. \\ v_0 &= 300\ \text{km s}^{-1} \times \frac{[v]}{100\ \text{km s}^{-1}} = 3[v]. \end{aligned} \tag{2.95}$$

Now that we have all the dimensionless parameters and variables that we need, we can now look at the graphical solutions in the next chapter.

Chapter 3

Solutions for Galactic Outflow

We will now be using Matlab to graph our solutions numerically for Equations (2.16), (2.30), (2.33) and (2.36).

3.1 One-Dimensional Solutions

For each of our graphs different properties of the outflow are shown which are the gas density, vertical wind velocity, cross-sectional area and the vertical magnetic field, as shown in Figures (3.1)-(3.4) respectively. For Figure's (3.2)-(3.3) we have used multiple values for v_0, A_0 and B_{z0}.

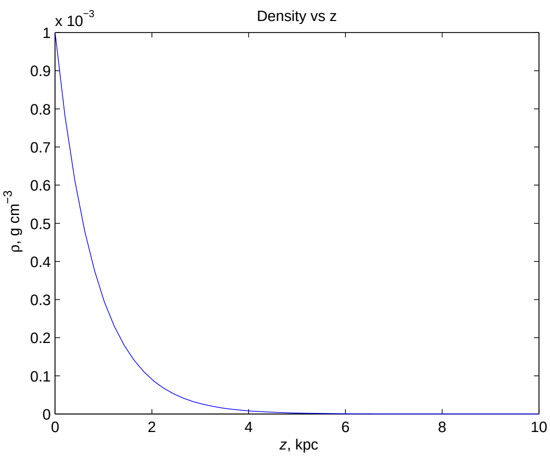


Figure 3.1: The gas density in the outflow ρ against the height of the wind z. We can see that as the wind height increases, the gas density decreases exponentially as we expect.

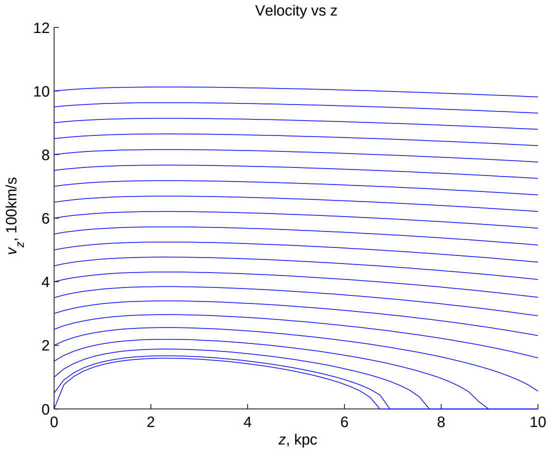


Figure 3.2: The vertical outflow velocity v_z against the height of the wind z. For smaller values of the initial velocity v_0 we can see that the outflow velocity increases but does not achieve the escape speed and then steadily decreases as the wind does not escape from the galaxy. For larger values of v_0, the graph clearly shows that the wind does achieve the escape speed and then steadies out as it escapes the galaxy.

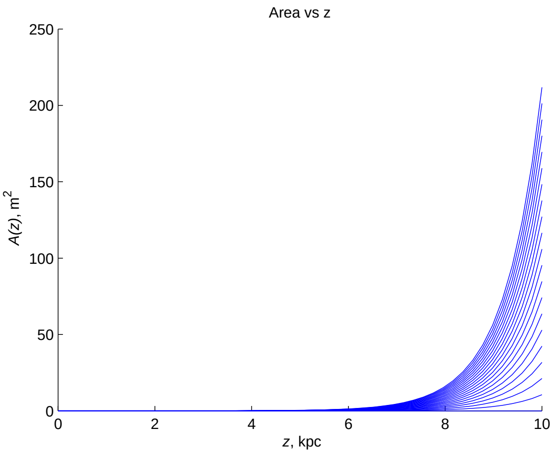


Figure 3.3: The cross-sectional area A against the height of the wind z. We have that as z increases, A increases exponentially and for small A_0 and constant v_0 that the cross-sectional area is also small. The graph suggests that the wind's shape is similar to a cylinder or a funnel.

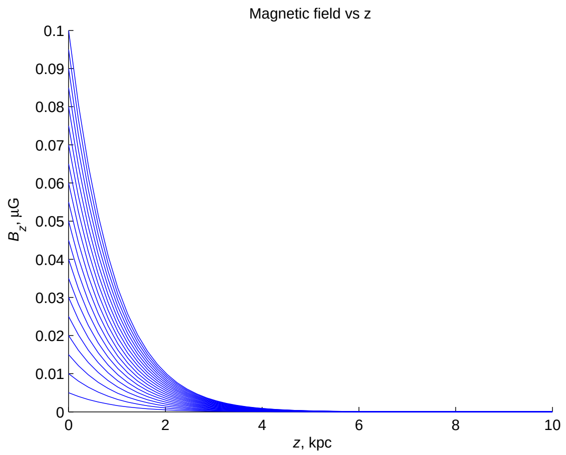


Figure 3.4: The vertical magnetic field B_z against the height of the wind z. B_z decreases with z, very rapidly and steadily tends to zero for large values of z.

Within our range of z, the gas density decreases exponentially with z, as we assumed in Section 2.2. Also we have that the vertical velocities, for small values of v_0, do not reach their escape velocity and thus the wind becomes a fountain. On the other hand, for the larger values of v_0 the vertical outflow velocity is at a steady speed during it's escape from the galaxy and continues outward.

The cross-sectional area increases rapidly only for larger values of z. Also for bigger values of A_0, the cross-sectional area is also bigger. Conversely, the vertical magnetic field decreases rapidly for small values of z .

Before we can look at the 2-dimensional solutions we must first redefine some constants and variables by giving them some r-dependence.

3.2 Radial Dependence

The variables we must give radial dependence are, H, A_2 and v_0 by introducing two new constants, r_a and r_H. The radial dependence for H is exponentially increasing whereas the radial dependence for A_2 and v_0 are exponentially decreasing. The new equation for H is,

$$H = H_0 \exp\left(\frac{r}{r_H}\right), \tag{3.1}$$

where $H_0 = 0.5$ kpc from Section 2.8. The new equation for A_2 is,

$$A_2 = 0.051 \exp\left(\frac{-r}{r_a}\right), \tag{3.2}$$

and finally the equation for v_0,

$$v_0 = 3 \exp\left(\frac{-r}{r_a}\right). \tag{3.3}$$

3.3 Cartesian Form for the Magnetic Field

Since we already have the equations for B_r, B_ϕ and B_z which are given by Equations (2.39), (2.57) and (2.36) respectively. We can find formulae that correspond to the cartesian forms for the magnetic field, by using the transformations from cylindrical polar to cartesian coordinates which are $x = r\cos\phi$, $y = r\sin\phi$ and $z = z$. Using these we can see that,

$$r = \sqrt{x^2 + y^2}, \tag{3.4}$$

and

$$\phi = \arctan\frac{y}{x}. \tag{3.5}$$

Also we can use the transformation rules for unit vectors in cylindrical polar coordinates,

$$\begin{aligned}\vec{\hat{x}} &= \cos\phi\vec{\hat{r}} - \sin\phi\vec{\hat{\phi}},\\ \vec{\hat{y}} &= \sin\phi\vec{\hat{r}} + \cos\phi\vec{\hat{\phi}}.\end{aligned} \tag{3.6}$$

These unit vectors can now be exchanged for the vector components of the magnetic field,

$$\begin{aligned}B_x &= \cos\phi B_r - \sin\phi B_\phi\\ B_y &= \sin\phi B_r + \cos\phi B_\phi.\end{aligned} \tag{3.7}$$

Also since z is the same in cartesian and cylindrical polar, we have B_z unchanged. These equations will be useful for plotting the magnetic field.

3.4 The Magnetic Field

In this part we look at and discuss our results for both the three-dimensional and two-dimensional solutions for the magnetic field which are presented in cartesian coordinates as shown in Section 3.3. Also we have used the parameters for the magnetic field, wind velocity, cross-sectional area, pressure and density from Section 2.8 alongside the updated equations from Section 3.2.

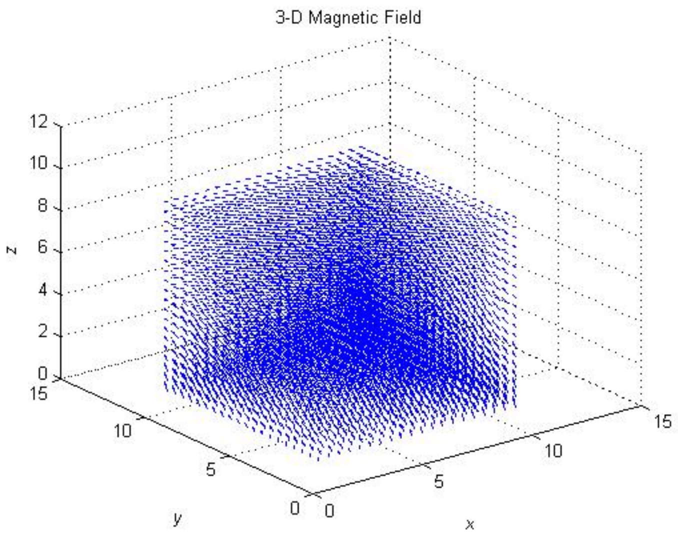


Figure 3.5: 3-d magnetic field shown in cartesian coordinates which shows that there is definite symmetry.

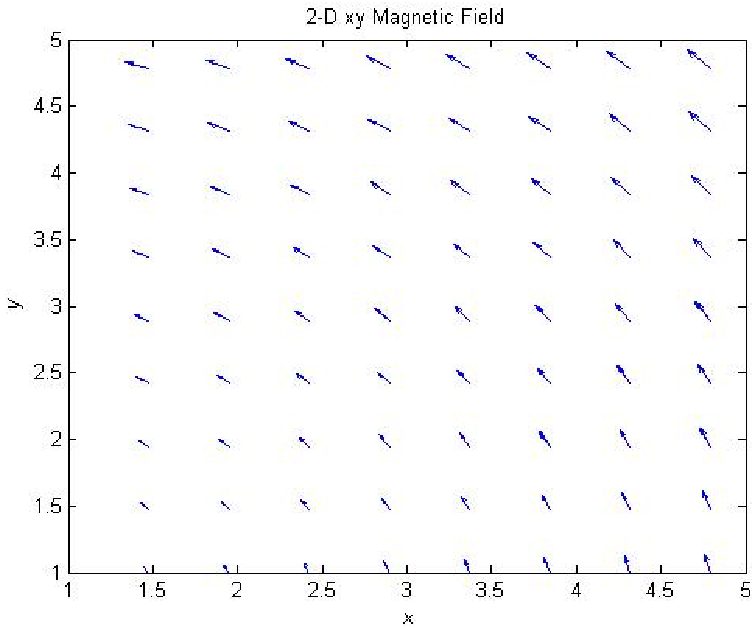
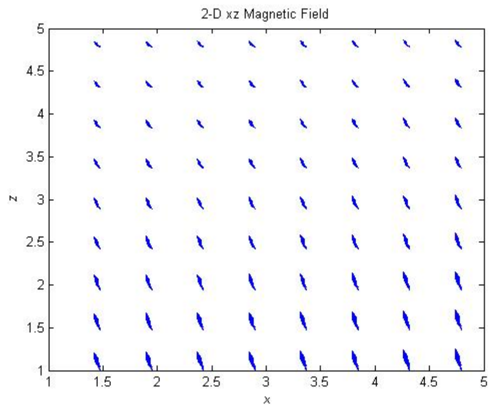
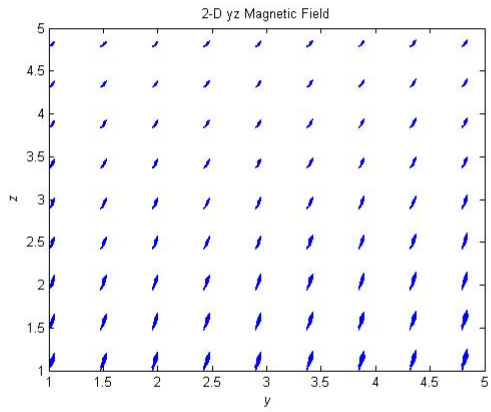


Figure 3.6: 2-d magnetic field for the cartesian $x-y$ axis which shows that in this quadrant the magnetic field lines fan out in a radial direction from the z-plane.



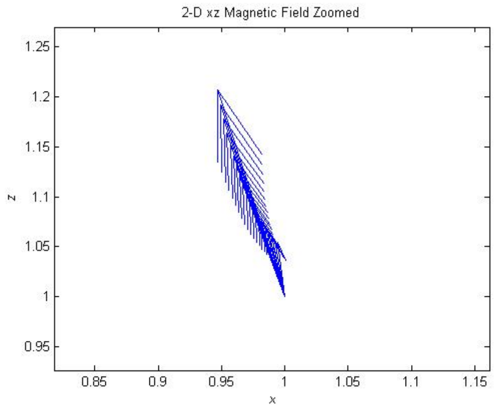
(a) 2-d magnetic field for the $x-z$ axis



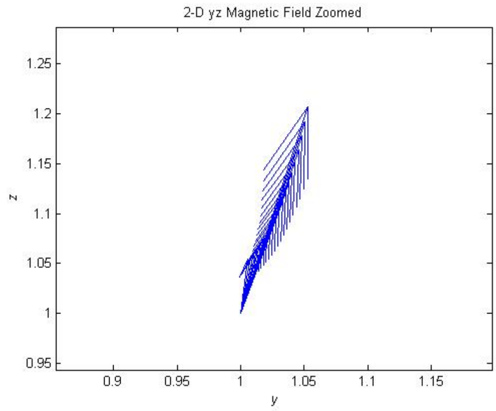
(b) 2-d magnetic field for the $y-z$ axis

Figure 3.7: The magnetic field lines for the $x-z$ and $y-z$ axis that shows that when comparing the two together we can see that appear to be symmetrical to one another.

Now we will look at the final two plots for the magnetic field zoomed in.



(a) Zoomed $x - z$ magnetic field lines.



(b) Zoomed $y - z$ magnetic field lines.

Figure 3.8: As before we can see that on a smaller scale the two plots are indeed symmetric to one another.

From Figures 3.6 and 3.7a we can see that the magnetic field lines for the $x - y$ and $x - z$ axes flow in an anti-clockwise direction whereas the the field lines for the $y - z$ flow in a clockwise direction. Also from Figure 3.5 we can see that the majority of the field lines are focused in the centre and along each of the axes.

From Figures 3.8a and 3.8b we can identify that there is symmetry between the two sets of axis. Also from the plots above we can see that on each the magnetic field lines move away from their respective axis at certain angles and then continue on in which they should theoretically spiral out as true to a magnetic field as our plots only show the first quadrants. Also we can see that from the plots the magnetic field points outward for our model.

Chapter 4

Comparison with NGC 253

In this chapter, we compare our model with observations of NGC 253 as shown in the paper by Hessen *et al.* (2009). The reason for using NGC 253 is since it is a spiral, starburst galaxy with high star formation in its galactic plane i.e. there is a huge amount of stars being created within the galactic plane of the galaxy. NGC 253's outflow is starburst-driven i.e. the outflow is dependent on the formation of stars and supernovae in the galaxy. From this the gas in the ouflow is hot and mainly emits thermal X-ray emissions instead of radiation, therefore the velocity would be near impossible to take measurements of. Instead observations of the magnetic field provides the necessary information for the outflow properties.

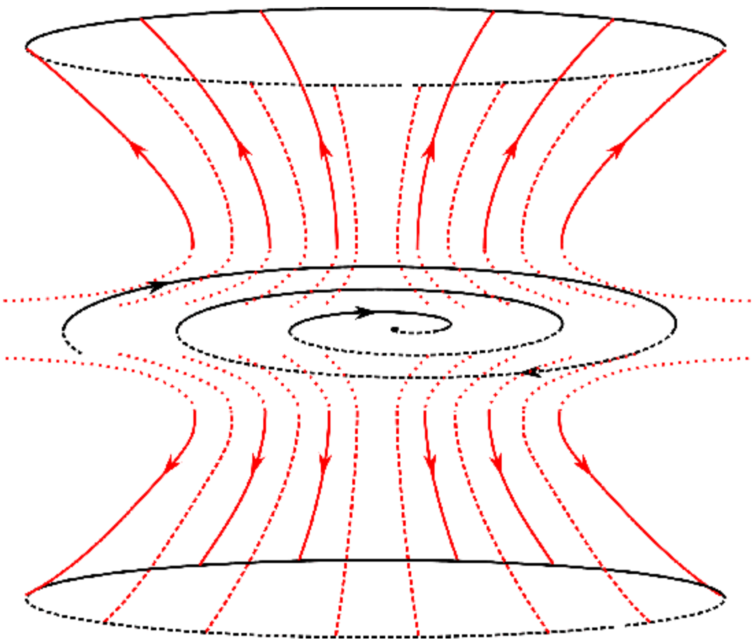


Figure 4.1: Figure 16 from Hessen *et al.* (2009) which is a sketch of the observable magnetic field structure for NGC 253 on a large scale.

The sketch from Hessen *et al.* (2009) in Figure 4.1 shows that in the galactic plane i.e. the black horizontal spiral field, is similar in shape to our plot of the $x - y$ axis Figure 3.6 specifically the top left hand side which corresponds directly. Similarly the sketch resembles our plots directions for the $x - z$ and $y - z$ axes as shown by the red field lines. The only difference from our model is that the magnetic field points inwards in the sketch and outwards in our model. Also from this sketch we have that the magnetic field is even inside and outside the galactic disc which supports our symmetry of the magnetic field.

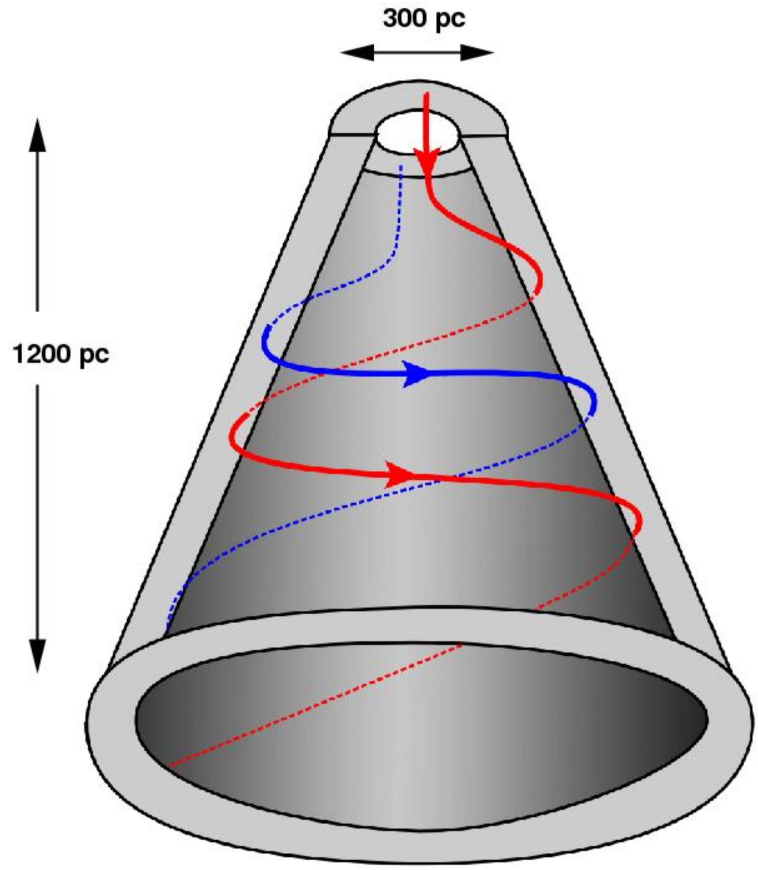


Figure 4.2: Figure 15 from Hessen *et al.* (2011) showing a sketch of the magnetic field in the walls around the outflow cone. Solid lines show field lines on front of cone whilst dotted lines show them on rear of cone. The galactic plane is at the top and the outflow fans out to the bottom.

The sketch from Hessen *et al.* (2011) in Figure 4.2 shows that the magnetic field lines, when close to the galactic disc, are vertical and then open into a winding helix with increasing height. This is similar to our findings from the 2-dimensional plots Figures 3.6, 3.7a and 3.7b.

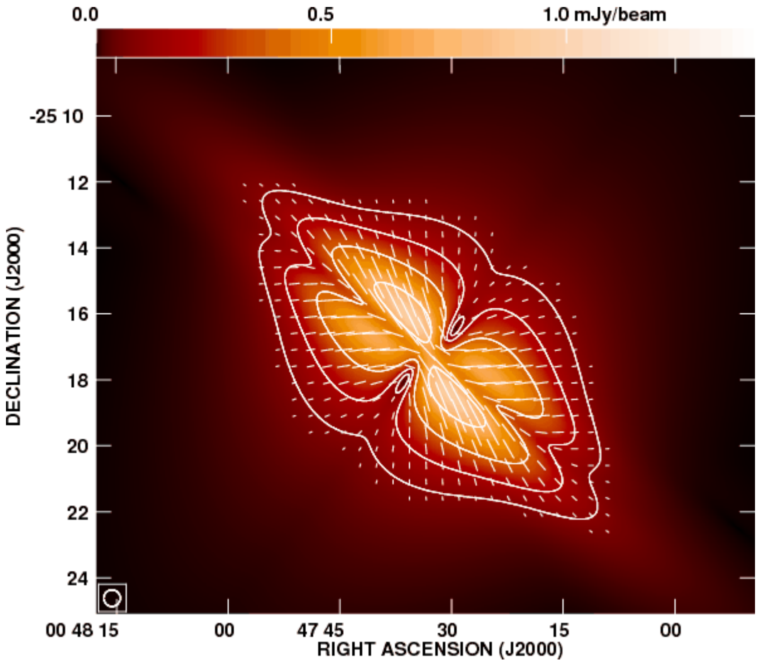


Figure 4.3: Figure 14 from Hessen *et al.* (2009) showing modeled polarized intensity of the combined even disc and even halo magnetic field. Vectors indicate the orientation of the magnetic field.

Figure 4.3 shows the magnetic field lines travelling in opposite directions as in our plots in Figures 3.7a and 3.7b. Overall, according to Figures 4.1, 4.2 and 4.3 we can say that our model fits accordingly with the observations done in the papers by Hessen *et al.* (2009) and Hessen *et al.* (2011).

Chapter 5

Summary and Conclusions

First we introduced magnetohydrodynamics; describing the equations of electromagnetism, Maxwell's equations and the induction equation, and the main equations of fluid dynamics, the Navier–Stokes and mass conservation equations. We also introduced what each of the individual variables of the equations were, specifically the velocity, area, magnetic field, pressure and density. Then we introduced and discussed the different types of galactic winds, wherein the outflow can achieve a high enough speed for the wind to escape into galactic space, and fountains, wherein the outflow does not escape into space, then we generalised into a combined model with both scenarios happening. Next we brought the two ideas together and neglected the electric field in favour of the magnetic field. Then we introduced the cylindrical coordinates we used and the main galaxy we focus on NGC 253.

Desiring a model we looked into previous researched models by Everett *et al.* (2008) and Hodgers (2013) and decided to follow the model used in Hodgers (2013) as the model by Everett *et al.* (2008) was found to be complicated. The model we used follows the model outlined by Hodgers (2013) wherein the density $\rho(z)$ is assumed to be exponential as in Hodgers (2013). Whereas the equation for the gravity acceleration for the solar vicinity is not negative, which then affects the vertical wind velocity $v_z(z)$, cross-sectional area A and the vertical magnetic field $B_z(z)$; all found using $\rho(z)$, solving Navier-Stokes and solving mass conservation. Also, we found the radial magnetic field B_r as found by Hodgers (2013) but for the radial and azimuthal velocity v_r and v_ϕ respectively we decided on a more complicated model that was dependent on the radius. Next we solved the partial differentiation equation to find the azimuthal magnetic field B_ϕ. Then we found dimensionless parameters so we could have a solution using realistic data.

Investigating our model for one-dimension, we found that: the gas density decreases exponentially; vertical outflow velocity is dependent on the height of the wind if it can or cannot achieve escape velocity; the cross-sectional area increases exponentially with respect to the height of the wind; and the vertical magnetic height decreases exponentially with respect to the wind height. For two-dimensions we found that there is symmetry within our field lines; the field lines fan out in opposite directions from their respective planes and that the magnetic field points outward.

Finally, we compared our results with observations of NGC 253 as documented by Hessen *et al.* (2009) and Hessen *et al.* (2011). We find that our results were corroborated with the diagrams and sketches from the two papers.

Overall, we found out that our model is not perfect but is accurate to a certain degree and is similar to observational data. We can say that our model is a sufficient description of galactic outflows for the assumptions we have made. Further work could entail updating or adjusting our model to get a better precision and accuracy for the observations as well as looking into how the velocity could be incorporated.

5.1 Acknowledgements

The paper by Everett *et al.* (2008) and the project by Hodgers (2013) were extremely helpful in constructing and refining our model specifically in the earlier stages of the project. Many thanks to Prof. Anvar Shukurov and Dr. Chris Graham for their continuous help and support throughout the project.

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