Determining the depth of Jupiter's dynamo region

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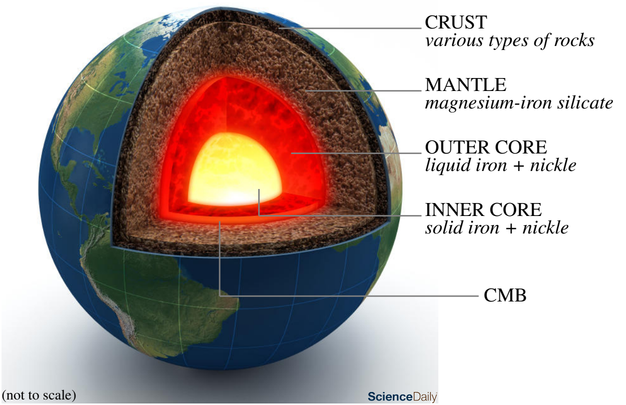
Let's start on Earth . . .



- **core-mantle boundary** (CMB): *sharp boundary* between the non-conducting mantle and the conducting outer core
- location of CMB r_{dyn}: the depth at which dynamo action starts
- one way to deduce r_{dyn} from observation on the surface: spectrum of magnetic energy

Gauss coefficients g_{lm} and h_{lm}

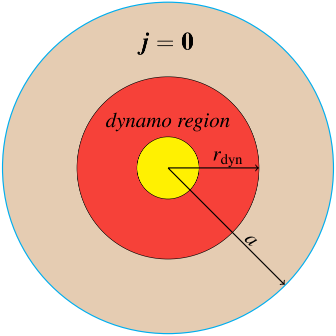
- Outside the dynamo region, $r > r_{\text{dyn}}$:

$$\boxed{\boldsymbol{j} = \boldsymbol{0}}$$

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{j} = \boldsymbol{0} \implies \boldsymbol{B} = -\nabla\Psi$$

$$\nabla \cdot \boldsymbol{B} = 0 \implies \nabla^2\Psi = 0$$

$a =$ *radius of Earth*



- Consider only internal sources,

$$\Psi(r,\theta,\phi) = a \sum_{l=1}^{\infty} \sum_{m=0}^{l} \left(\frac{a}{r}\right)^{l+1} \hat{P}_{lm}(\cos\theta)(g_{lm}\cos m\phi + h_{lm}\sin m\phi)$$

$\hat{P}_{lm}$: *Schmidt's semi-normalised associated Legendre polynomials*

- g_{lm} and h_{lm} can be determined from magnetic field measured on the planetary surface ($r \approx a$)

The Lowes spectrum

- *Average magnetic energy* over a spherical surface of radius r

$$E_B(r) = \frac{1}{2\mu_0}\frac{1}{4\pi}\oint |\boldsymbol{B}(r,\theta,\phi)|^2 \sin\theta \,\mathrm{d}\theta\,\mathrm{d}\phi$$

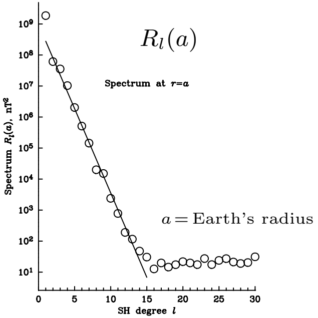
- Inside the source-free region $r_{\text{dyn}} < r < a$,

$$2\mu_0 E_B(r) = \sum_{l=1}^{\infty}\left[\left(\frac{a}{r}\right)^{2l+4}(l+1)\sum_{m=0}^{l}\left(g_{lm}^2 + h_{lm}^2\right)\right]$$

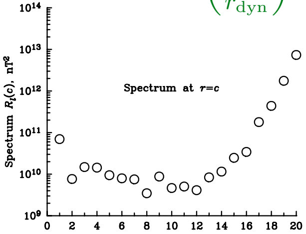
- **Lowes spectrum** (magnetic energy as a function of l):

$$\begin{aligned} R_l(r) &= \left(\frac{a}{r}\right)^{2l+4}(l+1)\sum_{m=0}^{l}\left(g_{lm}^2 + h_{lm}^2\right) \\ &= \left(\frac{a}{r}\right)^{2l+4} R_l(a) \qquad \text{(downward continuation)} \end{aligned}$$

Estimate location of CMB using the Lowes spectrum



$$R_l(r_{\rm dyn}) = \left(\frac{a}{r_{\rm dyn}}\right)^{2l+4} R_l(a)$$



(Robert Parker, UCSD)

- downward continuation through the $\boldsymbol{j} = \mathbf{0}$ region from a to $r_{\rm dyn}$:

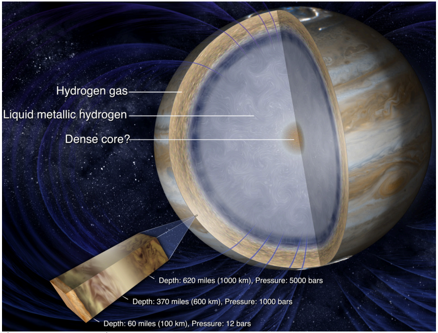
$$\ln R_l(a) = 2\ln\left(\frac{r_{\rm dyn}}{a}\right) l + 4\ln\left(\frac{r_{\rm dyn}}{a}\right) + \ln R_l(r_{\rm dyn})$$

- **white source hypothesis**: turbulence in the core leads to an *even distribution of magnetic energy* across different scales l,

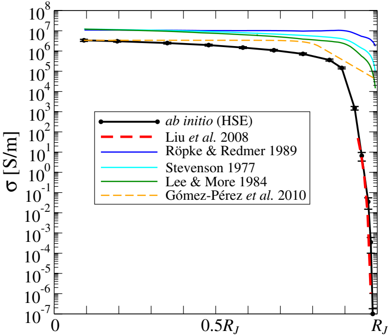
$R_l(r_{\rm dyn})$ is independent of l

- $r_{\rm dyn} \approx 0.55a \approx 3486\,\text{km}$ agrees very well with results from seismic waves observations

Interior structure of Jupiter



(NASA JPL)

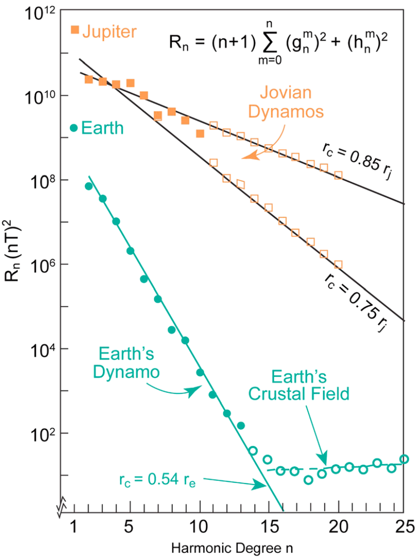


theoretical $\sigma(r)$ (French et al. 2012)

- low temperature and pressure near surface
  $\Rightarrow$ gaseous molecular H/He
- extremely high temperature and pressure inside
  $\Rightarrow$ liquid metallic H
- core?
- **conductivity** $\sigma(r)$ varies smoothly with radius r

At what depth does dynamo action start?

Lowes spectrum from the Juno mission



(Connerney et al. 2018)

- Juno's spacecraft reached Jupiter on 4th July, 2016
- currently in a 53-day orbit, measuring Jupiter's magnetic field (and other data)
- latest results give $R_l(r_J)$ up to $l = 10$ suggesting $r_{\rm dyn} \approx 0.85\, r_{\rm J}$ ($r_{\rm J} =$ Jupiter's radius)

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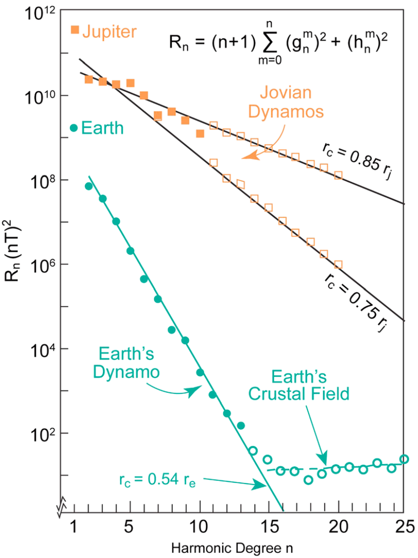

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Questions: given the conductivity profile $\sigma(r)$ is smoothly varying,

- estimation of $r_{\rm dyn}$ using Lowes spectrum the right approach?
- white source hypothesis valid?
- concept of "dynamo radius" $r_{\rm dyn}$ well-defined?

A numerical model of Jupiter

- spherical shell of radius ratio $r_{\text{in}}/r_{\text{out}} = 0.0963$ (small core)
- rotating fluid with electrical conductivity $\sigma(r)$ forced by buoyancy
- convection driven by secular cooling of the planet
- anelastic: linearise about a hydrostatic adiabatic basic state $(\bar\rho, \bar T, \bar p, \ldots)$
- dimensionless numbers: Ra, Pm, Ek, Pr

$$\nabla\cdot(\bar\rho\boldsymbol{u}) = 0$$

$$\frac{Ek}{Pm}\left[\frac{\partial\boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u}\right] + 2\hat{\boldsymbol{z}}\times\boldsymbol{u} = -\nabla\Pi' + \frac{1}{\bar\rho}(\nabla\times\boldsymbol{B})\times\boldsymbol{B} - \left(\frac{EkRaPm}{Pr}\right)S\frac{\mathrm{d}\bar T}{\mathrm{d}r}\hat{\boldsymbol{r}} + Ek\frac{\boldsymbol{F}_\nu}{\bar\rho}$$

$$\frac{\partial\boldsymbol{B}}{\partial t} = \nabla\times(\boldsymbol{u}\times\boldsymbol{B}) - \nabla\times(\eta\nabla\times\boldsymbol{B})$$

$$\bar\rho\bar T\left(\frac{\partial S}{\partial t} + \boldsymbol{u}\cdot\nabla S\right) + \frac{Pm}{Pr}\nabla\cdot\boldsymbol{\mathcal{F}}_Q = \frac{Pr}{RaPm}\left(Q_\nu + \frac{1}{Ek}Q_J\right) + \frac{Pm}{Pr}H_S$$

Boundary conditions: no-slip at r_{in} and stress-free at r_{out}, $S(r_{\text{in}}) = 1$ and $S(r_{\text{out}}) = 0$, electrically insulating outside $r_{\text{in}} < r < r_{\text{out}}$. (Jones 2014)

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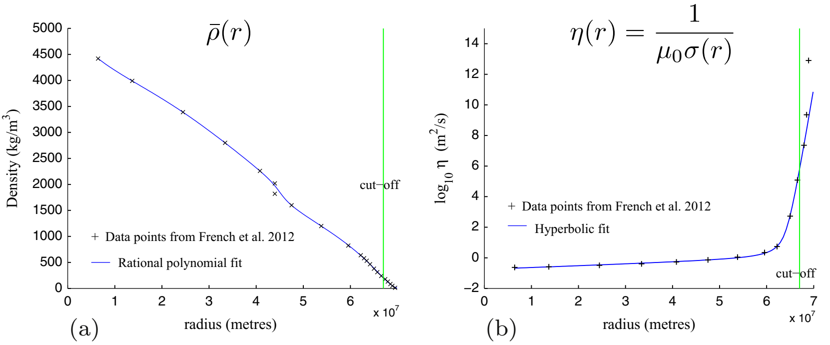
- dimensionless numbers: Ra, Pm, Ek, Pr
- **a Jupiter basic state:**

C.A. Jones / Icarus 241 (2014) 148–159

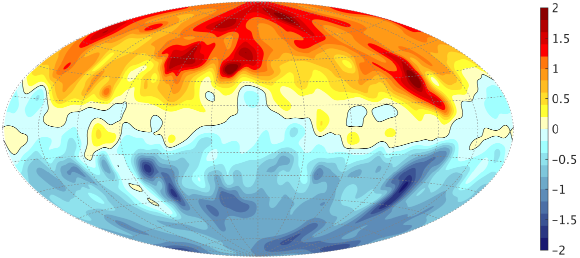


$Ra = 2 \times 10^7,\ Ek = 1.5 \times 10^{-5},\ Pm = 10,\ Pr = 0.1$

radial magnetic field, $B_r(r, \theta, \phi)$

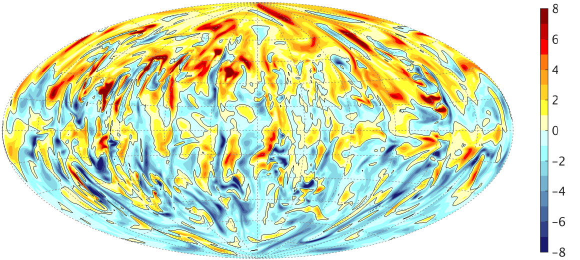
$r = r_{\mathrm{out}}$

dipolar

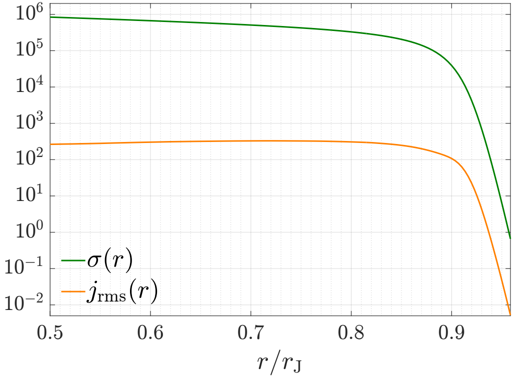


$r = 0.75 r_{\mathrm{out}}$

small scales



Where does the current start flowing?



- average current over a spherical surface of radius r

$$\mu_0 \boldsymbol{j} = \nabla \times \boldsymbol{B}$$

$$j_{\rm rms}^2(r,t) \equiv \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} |\boldsymbol{j}|^2 \sin\theta \mathrm{d}\theta \mathrm{d}\phi$$

- $j_{\rm rms}$ drops quickly but smoothly in the transition region, not clear how to define a characteristic “dynamo radius”

Magnetic power spectrum, $F_l(r)$

- average magnetic energy over a spherical surface:

$$E_B(r) = \frac{1}{2\mu_0}\frac{1}{4\pi}\oint |\boldsymbol{B}(r,\theta,\phi)|^2 \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

- Lowes spectrum: recall that if $\boldsymbol{j} = \boldsymbol{0}$, we solve $\nabla^2\Psi = 0$, then

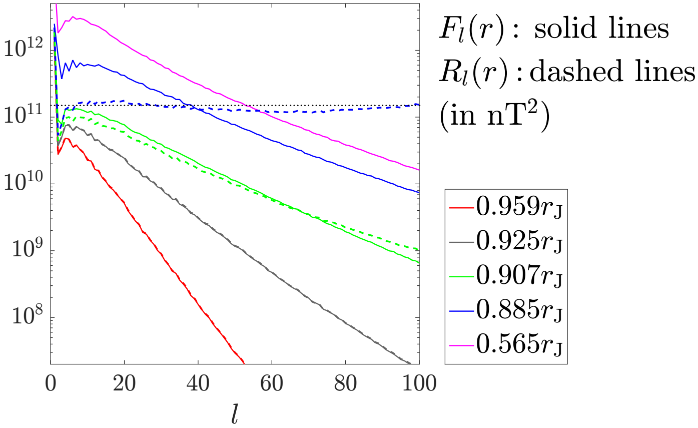
$$2\mu_0 E_B(r) = \sum_{l=1}^{\infty}\left[\left(\frac{a}{r}\right)^{2l+4}(l+1)\sum_{m=0}^{l}\left(g_{lm}^2 + h_{lm}^2\right)\right] = \sum_{l=1}^{\infty} R_l(r)$$

- generally, for the numerical model, $\boldsymbol{B} \sim \sum_{lm} b_{lm}(r) Y_{lm}(\theta,\phi)$,

$$2\mu_0 E_B(r) = \frac{1}{4\pi}\oint |\boldsymbol{B}(r,\theta,\phi)|^2 \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi = \sum_{l=1}^{\infty} F_l(r)$$

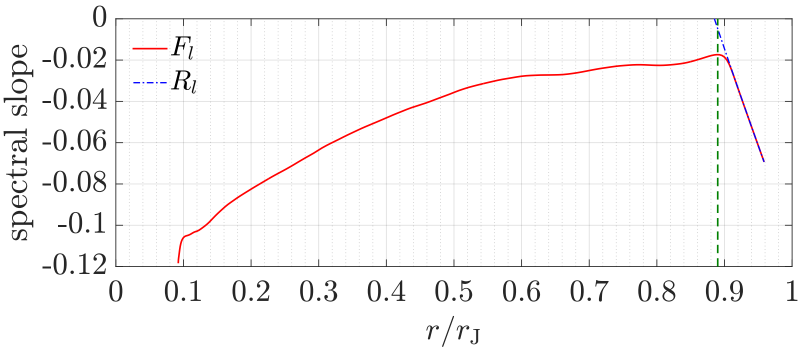
$$\boldsymbol{j}(r,\theta,\phi) = \boldsymbol{0} \ \text{ exactly } \implies R_l(r) = F_l(r)$$

Magnetic power spectrum at different depth r



- near the surface ($r_{out} > r > 0.9r_J$)
  - $F_l(r) \approx R_l(r)$
  - slope of $F_l(r)$ decreases with r
- interior and away from the core ($0.9r_J > r > 0.5r_J$)
  - $F_l(r)$ different from $R_l(r)$
  - $F_l(r)$ is shallow and maintains roughly the same shape

Spectral slope of $F_l(r)$



- $F_l(r)$ indicates a clear transition in dynamics: |slope| minimum at

$$r_{\text{dyn}} = 0.889\, r_J$$

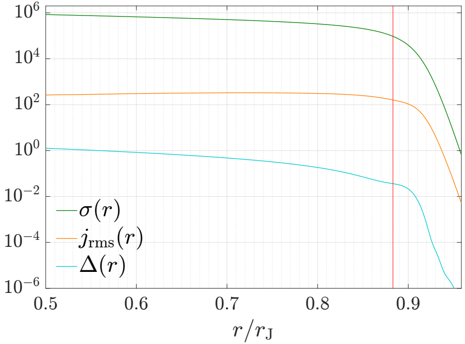
- $F_l(r)$ in the interior is not flat but dependence on l is weak:

$$|\text{slope}| \sim 0.02$$

- downward continuation from spectrum at the surface $F_l(r_{\text{out}})$ predicts:

$$r_{\text{dyn}} = 0.885\, r_J$$

Summary of results from numerical model

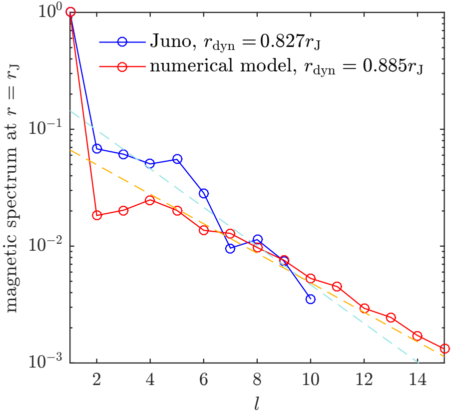


In our numerical model of Jupiter, we find that:

- the magnetic power spectrum provides a characteristic radius of the dynamo action
- in the interior and away from the core, white source hypothesis is approximately valid
- the dynamo radius can be predicted using the magnetic spectrum at the surface

However, ...

Comparison with Juno data



The dynamo radius in the numerical model is too shallow compared to the prediction using the Juno data. The discrepancy suggests:

- the metallic hydrogen layer could be deeper than predicted by theoretical calculation
- the existence of a stably stratified layer above the dynamo region
- our numerical model has more small-scale forcing than Jupiter does