Experiment II:
Magnetic Fields due to Currents

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I. INTRODUCTION

Electric currents generate magnetic fields. The magnitude and direction of the magnetic field generated depend on the specific geometry of the wire in which the current is flowing. In this experiment, we study three different types of geometry: (1) a circular coil, (2) two circular coils with current flowing in either the same or opposite directions and (3) a torus. Using the Biot-Savart Law and Ampère's Law, the magnetic field $\vec{B}$ of these configurations can be calculated:

1. a circular coil of radius a with current I,

$$B = \frac{\mu_0 N I}{2} \frac{a^2}{(a^2 + x^2)^{3/2}} \tag{1}$$

   where μ_0 is the magnetic permeability of vacuum, N is the number of loops in the coil and x is the distance from the center of the coil along the axis of the coil.

2. two circular coils separated by a distance of $2b$,

$$B = \frac{\mu_0 N I}{2} \left\{ \frac{a^2}{[a^2 + (x-b)^2]^{3/2}} \pm \frac{a^2}{[a^2 + (x+b)^2]^{3/2}} \right\} , \tag{2}$$

   the origin is taken to be at the midpoint between the two coils.

3. a torus,

$$B = \frac{\mu_0 N I}{2\pi} \frac{1}{R} \tag{3}$$

   where R is the distance from the center of the torus.

We experimentally verify Eq. (1) – Eq. (3) and also use them to determine some system parameters.

II. APPARATUS AND PROCEDURE

Apparatus

1. Battery eliminator power supply
2. Circular coils
3. Toroidal coil
4. Gaussmeter
5. Digital multimeters

Procedure

The circuit shown in FIG. 1 was setup.

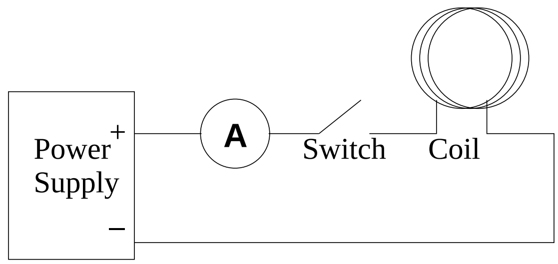

FIG. 1: The circuit used to generate magnetic field in the experiment

A. A single circular coil

The Gaussmeter was first calibrated for the longitudinal Hall probe. With the Hall probe placed at the center of the coil, the strength of the magnetic field was measured as a function of current. The direction of the current was reversed and the direction of the magnetic field was noted.

The current was then fixed at 7.5 A. The center of the coil was located as the position where the magnetic field strength was maximum. Then, the magnetic field strength was measured as a function of position along the axis of the coil. The number of turns in the coil was counted.

B. Two circular coils

The circuit was setup so that a current of 5.05 A was flowing through the two coils in the same sense. The magnetic field strength as a function of position along the axis of the coils was measured. The circuit was then modified so that a current of 5.05 A was flowing through the two coils in the opposite sense. The magnetic field strength as a function of position along the axis of the coils was again measured.

C. A toroidal coil

A current of fixed magnitude 10.15 A was setup to flow through the torus. The Gaussmeter was then calibrated for the transverse Hall probe. Using the transverse Hall probe, the magnetic field strength was measured as a function of R. The number of turns in the torus was also counted.

III. DATA AND NUMERICAL ANALYSIS

A. A single circular coil

The measured values of the magnetic field strength B and current I, along with their uncertainties σ_B and σ_I are shown in TABLE I. FIG. 2 shows the plot of B versus I. We fit the data with a straight line. The y-intercept is forced to be zero in the fitting process.

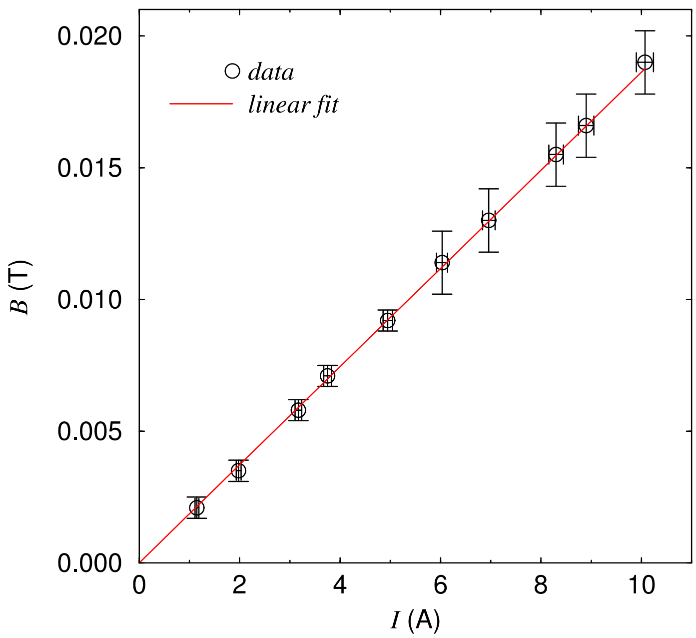

FIG. 2: Plot of B versus I. The red line is a linear fit to the data

From Eq. (1), the number of turns N is related to the slope m of the linear fit as follow,

$$N = \frac{2am}{\mu_0} \ . \tag{4}$$

The measured values of m and a and the corresponding uncertainties σ_m and σ_a are,

$$m = 0.0019 \text{ T/A} \ , \qquad \sigma_m = 0.00004 \text{ T/A}$$
$$a = 0.034 \text{ m} \ , \qquad \sigma_a = 0.0005 \text{ m} \ .$$

By Eq. (4), we obtain

$$N = 101 \text{ (to the nearest integer)}$$
$$\sigma_N = \frac{2}{\mu_0}\sqrt{a^2\sigma_m^2 + m^2\sigma_a^2} = 3 \ .$$

TABLE I: magnetic field strength B at different current I for a single circular coil

I (A)	σ_I (A)	B ($\times 10^{-4}$ T)	σ_B ($\times 10^{-4}$ T)
1.15	0.04	21	4
1.98	0.05	35	4
3.17	0.07	58	4
3.75	0.08	71	4
4.95	0.09	92	4
6.03	0.11	114	12
6.96	0.12	130	12
8.30	0.14	155	12
8.90	0.15	166	12
10.07	0.17	190	12

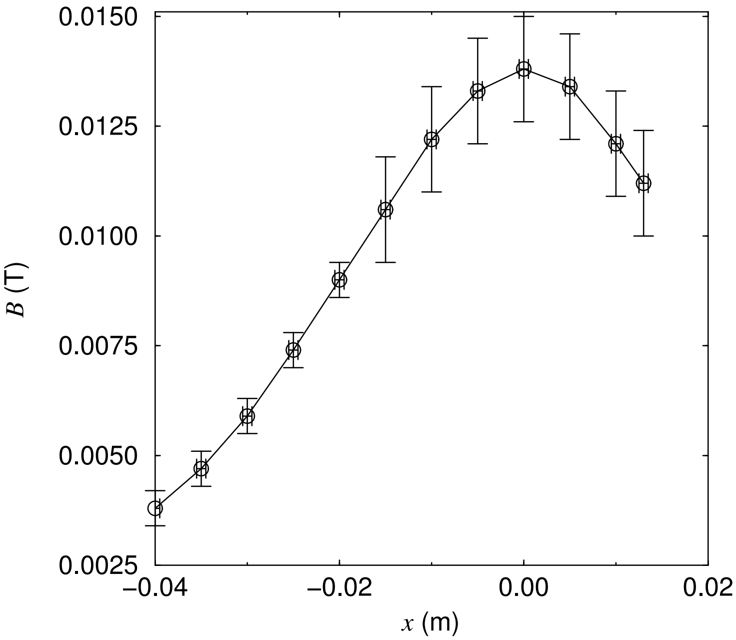

FIG. 3: B as a function of x.

Therefore, the number of turns in the coil is estimated to be 101 ± 3. We also find that when the direction of the current flow is reversed, the direction of the magnetic field is reversed.

Next, we fix the current at $I = 7.5 \pm 0.13$ A and measure B as a function of x. The results are tabulated in TABLE II and plotted in FIG. 3. By taking the natural log of both sides of Eq. (1), we obtain

$$\ln B = -\frac{3}{2}\ln(a^2 + x^2) + \ln\left(\frac{\mu_0 N I a^2}{2}\right) \ . \tag{5}$$

Therefore, in order to estimate N, we plot $\ln B$ versus $\ln(a^2 + x^2)$ in FIG. 4. We then fit the

TABLE II: magnetic field strength B at different positions along the axis of a single circular coil

x (m)	σ_x (m)	B ($\times 10^{-4}$ T)	σ_B ($\times 10^{-4}$ T)
-0.040	0.0005	38	4
-0.035	0.0005	47	4
-0.030	0.0005	59	4
-0.025	0.0005	74	4
-0.020	0.0005	90	4
-0.015	0.0005	106	12
-0.010	0.0005	122	12
-0.005	0.0005	133	12
0.000	0.0005	138	12
0.005	0.0005	134	12
0.010	0.0005	121	12
0.013	0.0005	112	12

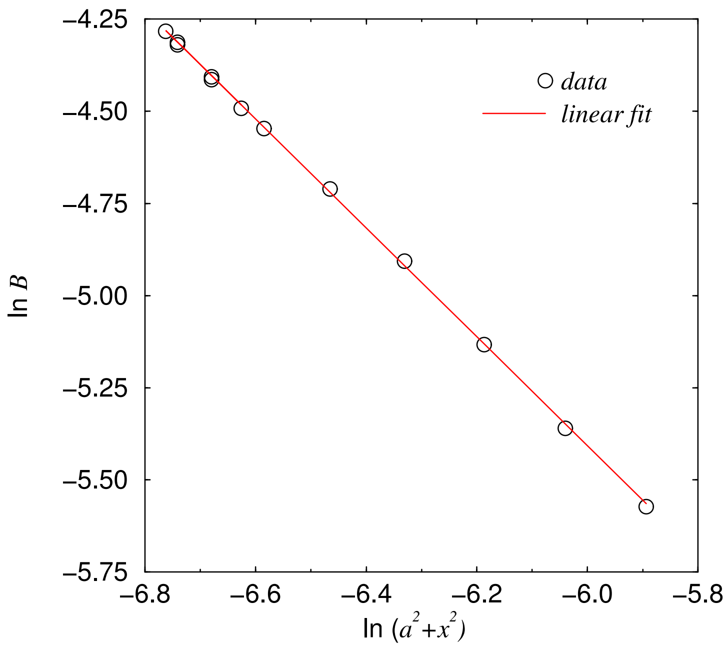

FIG. 4: $\ln B$ versus $\ln(a^2 + x^2)$ for a single circular coil.

data with a straight line and obtain the following estimates for the slope m and y-intercept c with uncertainties σ_m and σ_c respectively,

$$\begin{aligned} m &= -1.48 \ , & \sigma_m &= 0.099 \\ c &= -14.27 \ , & \sigma_c &= 0.64 \ . \end{aligned}$$

Hence,

$$\begin{aligned} N &= \frac{e^c}{\mu_0 I a^2} \\ &= 117 \text{ (to the nearest integer)} \\ \sigma_N &= N\sqrt{\sigma_c^2 + \left(\frac{\sigma_I}{I}\right)^2 + 2\left(\frac{\sigma_a}{a}\right)^2} = 75 \ . \end{aligned} \tag{6}$$

By counting the number of turns in the coil, we obtain the approximate value of $N \approx 100$.

B. Two circular coils

We first consider the case when current is flowing through the two coils in the same sense. The magnetic field strength B measured at different positions along the axis of the coils are listed in TABLE III. The theoretical values of B given by Eq. (2) are also given. The origin is taken to be at where B is minimum and b in Eq. (2) is estimated to be 0.035 ± 0.0005 m. From Eq. (4), we take $N = 101$. Both the experimental and theoretical values of B are plotted in FIG. 5.

The results for the case when current flows through the coils in opposite sense are similarly presented in TABLE IV and FIG. 6.

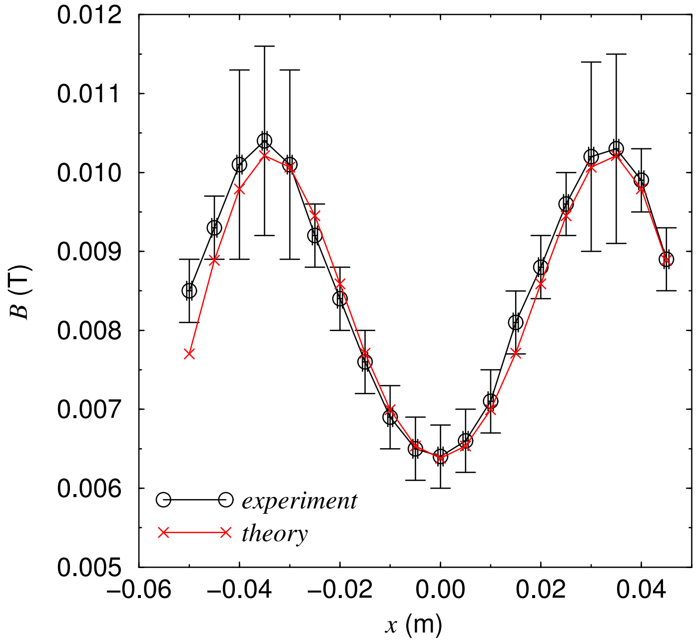

FIG. 5: B versus x for two circular coils with current flowing in the same sense

TABLE III: magnetic field strength B at different positions due to two circular coil with current flowing in the same sense

x (m)	σ_x (m)	Measured B ($\times 10^{-4}$ T)	σ_B ($\times 10^{-4}$ T)	Theoretical (Eq. (2)) B ($\times 10^{-4}$ T)
-0.050	0.0005	85	4	77
-0.045	0.0005	93	4	89
-0.040	0.0005	101	12	98
-0.035	0.0005	104	12	102
-0.030	0.0005	101	12	101
-0.025	0.0005	92	4	95
-0.020	0.0005	84	4	86
-0.015	0.0005	76	4	77
-0.010	0.0005	69	4	70
-0.005	0.0005	65	4	65
0.000	0.0005	64	4	64
0.005	0.0005	66	4	65
0.010	0.0005	71	4	70
0.015	0.0005	81	4	77
0.020	0.0005	88	4	86
0.025	0.0005	96	4	95
0.030	0.0005	102	12	101
0.035	0.0005	103	12	102
0.040	0.0005	99	4	98
0.045	0.0005	89	4	89

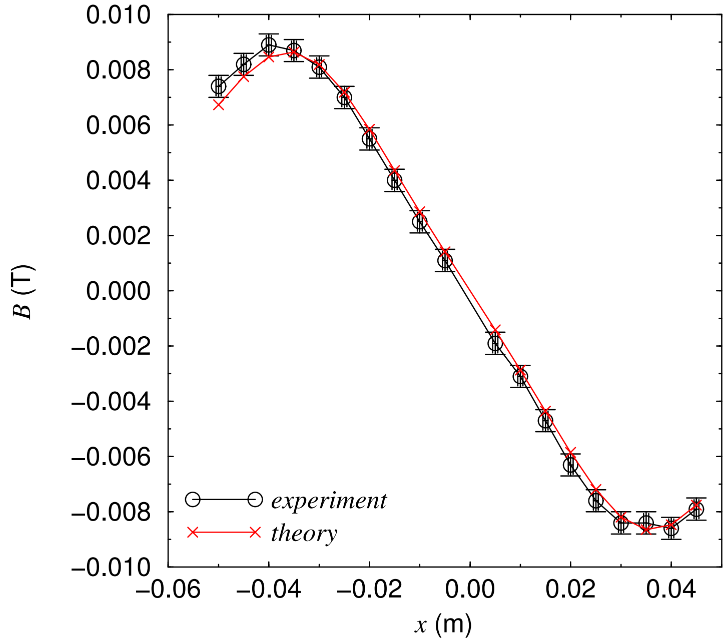

FIG. 6: B versus x for two circular coils with current flowing in the opposite sense

TABLE IV: magnetic field strength B at different positions due to two circular coil with current flowing in the opposite sense

x (m)	σ_x (m)	Measured B ($\times 10^{-4}$ T)	σ_B ($\times 10^{-4}$ T)	Theoretical (Eq. (2)) B ($\times 10^{-4}$ T)
-0.050	0.0005	74	4	67
-0.045	0.0005	82	4	78
-0.040	0.0005	89	4	85
-0.035	0.0005	87	4	86
-0.030	0.0005	81	4	82
-0.025	0.0005	70	4	72
-0.020	0.0005	55	4	58
-0.015	0.0005	40	4	44
-0.010	0.0005	25	4	29
-0.005	0.0005	11	4	14
0.005	0.0005	-19	4	-14
0.010	0.0005	-31	4	-29
0.015	0.0005	-47	4	-44
0.020	0.0005	-63	4	-59
0.025	0.0005	-76	4	-72
0.030	0.0005	-84	4	-82
0.035	0.0005	-88	4	-86
0.040	0.0005	-86	4	-85
0.045	0.0005	-79	4	-78

TABLE V: magnetic field strength B at different R for a toroidal coil

R (m)	σ_R (m)	B ($\times 10^{-4}$ T)	σ_B ($\times 10^{-4}$ T)
0.005	0.0005	3	0.0004
0.010	0.0005	6	0.0004
0.015	0.0005	24	0.0004
0.020	0.0005	62	0.0004
0.025	0.0005	70	0.0004
0.030	0.0005	62	0.0004
0.035	0.0005	54	0.0004
0.040	0.0005	49	0.0004
0.045	0.0005	44	0.0004
0.050	0.0005	39	0.0004
0.055	0.0005	36	0.0004
0.060	0.0005	33	0.0004
0.065	0.0005	28	0.0004
0.070	0.0005	16	0.0004
0.075	0.0005	3	0.0004

C. A toroidal coil

A current of 10.15 ± 0.17 A is flowing through the toroidal coil. The magnetic field strength B is measured as a function of R. The results are given in TABLE V. When the direction of the current is reversed, the direction of the magnetic field is reversed.

Taking the natural log of both side of Eq. (3), we get

$$\ln B = -\ln R + \ln\left(\frac{\mu_0 N I}{2\pi}\right) \ . \tag{7}$$

In FIG. 7, we plot $\ln B$ versus $\ln R$ together with the linear fit to the linear portion of the data. The values of the fitted slope m and y-intercept c are,

$$m = -0.90 \ , \qquad \sigma_m = 0.14$$
$$c = -8.23 \ , \qquad \sigma_c = 0.47 \ ,$$

and from which we obtain,

$$N = \frac{2\pi e^c}{\mu_0 I} = 132 \text{ (to the nearest integer)}$$
$$\sigma_N = N\sqrt{\sigma_c^2 + \left(\frac{\sigma_I}{I}\right)^2} = 62 \ .$$

We count the number of turns in the toroidal coil and find that $N = 110$.

IV. DISCUSSION

In Part A of the experiment, we found the experimental results agree extremely well with the theoretical prediction that the magnetic field strength B at the center of a circular coil

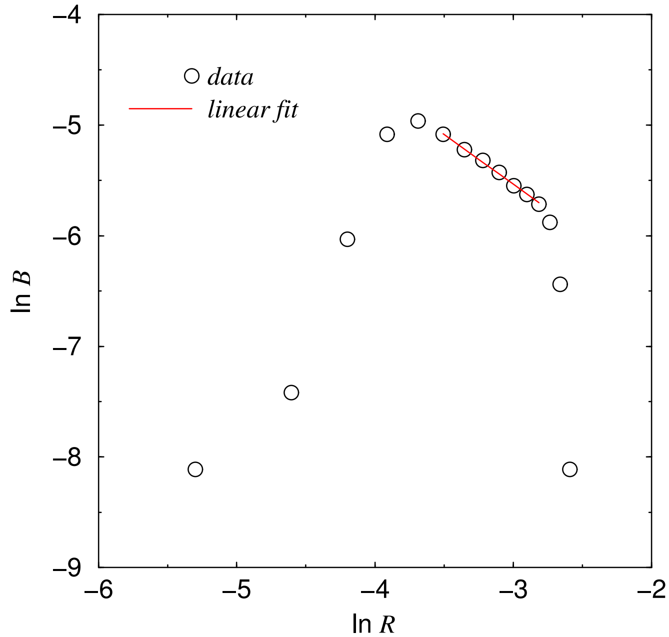

FIG. 7: Magnetic field strength due to a toroidal coil

is proportional to the current I in the coil. The deduced number of turns N in the coil also agrees with our estimate by counting.

When B is measured as a function of x at fixed I, the deduced N is much bigger. We expect the error in this estimate of N to be bigger because it relies on the accurate estimation of the position of the center of the coil. The fact that the estimated $m = -1.48$ is close to the theoretical value of -1.5 indicates that we have located the the center of the coil fairly accurately. Also note that, the factor a^2 in Eq. (6), as compared to a in Eq. (4), makes the result more sensitive to the error in the measurement of a. The uncertainty σ_N is very large in this case, due to fact that N depends on e^c.

In Part B, we study the two coils configuration. The experimental results show correct qualitative behavior both when current is flowing in the same and opposite sense in the coils. The data also agrees quantitatively with the theory within the uncertainties of the experiment, except for a few data points at large negative x.

In Part C, the magnetic field inside the toroidal coil is found to be decreasing as $R^{-0.9}$, instead of the theoretical prediction of $1/R$ for an ideal toroidal coil. The estimated value of N agrees fairly well with our count. The derivation of Eq. (3) is based on the symmetry argument that inside an ideal torus, only the azimuthal component of $\vec{B}$ is non-zero and is dependent on R only. The argument is only approximately true for a real toroidal coil, as indicated by the non-zero value of B outside the torus, see FIG. 7.

V. CONCLUSION

We have studied the magnetic field generated by three different current configurations involving coils of wire. The experimental results agree quantitatively with the theoretical predictions within the uncertainties of the experiment. Reasonable values for the number of coils in each configuration are deduced from our experimental data.