

STATEMENT OF RESEARCH INTERESTS

by Z.A.Lykova

PURE MATHEMATICS

My pure mathematical research has been on the structure and properties of topological algebras from the viewpoint of homological algebra. Substantial progress in these questions in the context of topological algebras has been made since the seventies by mathematicians from various countries. Now the theory is widely developed and has applications in many branches of mathematics, including noncommutative geometry, K -theory, the theory of de Rham homology from differential geometry, the structure theory of Banach and topological algebras, the representation theory of Banach algebras, automatic continuity theory, the theories of derivations and extensions of algebras.

The higher-dimensional amenability of tensor products of Banach algebras

In [35] I investigate the higher-dimensional amenability of tensor products $\mathcal{A} \hat{\otimes} \mathcal{B}$ of Banach algebras \mathcal{A} and \mathcal{B} . I prove that the weak bidimension db_w of the tensor product $\mathcal{A} \hat{\otimes} \mathcal{B}$ of Banach algebras \mathcal{A} and \mathcal{B} with bounded approximate identities satisfies $db_w \mathcal{A} \hat{\otimes} \mathcal{B} = db_w \mathcal{A} + db_w \mathcal{B}$. I show that it cannot be extended to arbitrary Banach algebras. For example, for a biflat Banach algebra \mathcal{A} which has a left or right, but not two-sided, bounded approximate identity, we have $db_w \mathcal{A} \hat{\otimes} \mathcal{A} \leq 1$ and $db_w \mathcal{A} + db_w \mathcal{A} = 2$. We describe explicitly the continuous Hochschild cohomology $\mathcal{H}^n(\mathcal{A} \hat{\otimes} \mathcal{B}, (\mathcal{X} \hat{\otimes} \mathcal{Y})^*)$ and the cyclic cohomology $\mathcal{HC}^n(\mathcal{A} \hat{\otimes} \mathcal{B})$ of certain tensor products $\mathcal{A} \hat{\otimes} \mathcal{B}$ of Banach algebras \mathcal{A} and \mathcal{B} with bounded approximate identities; here $(X \hat{\otimes} Y)^*$ is the dual bimodule of the tensor product of essential Banach bimodules X and Y over \mathcal{A} and \mathcal{B} respectively.

Hochschild cohomology of certain nuclear Fréchet and DF algebras

There is much current interest in the computation of cyclic (co)homology groups. Most of the literature has been devoted to the purely algebraic context, but there have also been papers addressing the calculation of the continuous version of these groups for topological algebras. However, it remains the case that these groups can only be calculated for a restricted range of algebras.

In [33] I describe explicitly continuous Hochschild and cyclic cohomology groups of simplicially trivial nuclear Fréchet and DF algebras. In [34] I show the existence of a topological isomorphism in the Künneth formula for the cohomology groups of complete nuclear DF -complexes such that all their boundary maps have closed range. This allows me to prove the existence of a topological isomorphism in the Künneth formula for continuous Hochschild cohomology groups of nuclear Fréchet algebras. In [34] I describe explicitly the continuous Hochschild and cyclic-type homology and cohomology of certain tensor products of topological algebras which are Fréchet or nuclear Fréchet or nuclear DF -spaces.

Cyclic-type cohomology of locally convex topological algebras

My work [31] describes techniques for the calculation of the cyclic type cohomology groups of projective limits of locally convex algebras and applies them to some natural classes of algebras. Let $(A_\alpha, T_{\alpha, \beta})_{(\Lambda, \leq)}$ be a reduced projective system of complete Hausdorff locally convex algebras with jointly continuous multiplications, and let A be the projective limit

algebra $A = \varprojlim_{\alpha} A_{\alpha}$. I prove that, for the continuous cyclic cohomology HC^* and continuous periodic cohomology HP^* of A and $A_{\alpha}, \alpha \in \Lambda$, for all $n \geq 0$, $HC^n(A) = \varinjlim_{\alpha} HC^n(A_{\alpha})$, the inductive limit of $HC^n(A_{\alpha})$, and, for $k = 0, 1$, $HP^k(A) = \varinjlim_{\alpha} HP^k(A_{\alpha})$. For a projective limit algebra $A = \varprojlim_m A_m$ of a countable reduced projective system $(A_m, T_{m,\ell})_{\mathbf{N}}$, I also establish relations between the cyclic-type continuous homology of A and $A_m, m \in \mathbf{N}$.

In [32] I show methods for the computation of the Hochschild and cyclic-type continuous homology and cohomology of some locally convex strict inductive limits $A = \varinjlim_m A_m$ of Fréchet algebras A_m .

A Künneth formula for the simplicial cohomology groups Fréchet algebras

A Künneth formula for bounded chain complexes \mathcal{X} and \mathcal{Y} of Fréchet spaces and continuous boundary maps with closed ranges and, in particular, for the simplicial cohomology groups of Fréchet algebras is proved in [29, 30]. These papers are joint with Dr. F. Gourdeau (Univ. of Laval, Canada) and Dr. M.C. White (Univ. of Newcastle). For the semigroup algebras $\ell^1(\mathbf{Z}_+^k)$ and $L^1(\mathbf{R}_+^k)$, we apply the Künneth formula to describe the simplicial cohomology and homology groups explicitly.

Excision in the Hochschild and cyclic cohomology of Fréchet and Banach algebras

The excision property for continuous Hochschild, cyclic and periodic cyclic (co)homology in the category of Fréchet algebras is proved in [27] joint with Dr. J. Brodzki (Univ. of Exeter). For continuous Hochschild and cyclic cohomology groups, it was an open problem. This is particularly aimed at cohomological problems arising in differential geometry and K -theory.

My work [25] (jointly with Dr. M.C. White) describes techniques for the calculation of continuous Hochschild (co)homology groups of Banach algebras. We prove the existence of long exact sequences associated with extensions of algebras for continuous cohomology of Banach algebras with coefficients in dual bimodules (under certain conditions on the kernel of the extension and on the bimodule). Thus we can break the calculation down by making use of extensions of Banach algebras. We also show that the existence of an associated long exact sequence of continuous homology groups with coefficients in bimodules is equivalent to the existence of one for cohomology groups with coefficients in dual bimodules. We apply these results to study of the n -amenability and the simplicial triviality of Banach algebras.

Cyclic and simplicial (co)homology of Banach algebras

My works [24, 26] describe techniques for the calculation of the simplicial and cyclic cohomology groups of Banach algebras and apply them to some natural classes of algebras. The key result of [26] is that, for a Banach algebra A with a bounded approximate identity the cyclic cohomology groups of A are isomorphic to those of A relative to an amenable closed subalgebra of A . In the other paper [24] I prove that, for every extension of Banach algebras $0 \rightarrow B \rightarrow A \rightarrow D \rightarrow 0$ such that B has a left or right bounded approximate identity, the existence of an associated long exact sequence of Banach simplicial or cyclic cohomology groups is equivalent to the existence of one for homology groups. It follows from the continuous version

of a result of Wodzicki that associated long exact sequences exist. In particular, they exist for every extension of C^* -algebras.

Extensions of Banach algebras and automatic continuity

A research monograph [1] joint with Prof. W.G. Bade (UC at Berkeley) and Prof. H.G. Dales (Leeds Univ.) on algebraic and strong splittings of extensions of Banach algebras is published in Memoirs of the American Mathematical Society. The book includes many examples of Banach algebras A for which it is proved that there is an extension of A which does not split algebraically or strongly. Thus we consider the generalization of the Wedderburn decomposition problem. Another direction is automatic continuity theory. For example, we show that, for many classes of Banach algebras, including C^* -algebras, every extension of these algebras which splits algebraically also splits strongly. There are still many open questions in this area. For example, for many classes of algebra A (in particular the class of all C^* -algebras) it is not known if there is an extension of A which does not split strongly.

Homological dimension of operator algebras

A typical concern in homological algebra is the calculation of global homological dimension. Besides being of independent interest, this question is closely connected with the continuous Hochschild cohomology, which is very useful in the study of derivations, extensions and perturbations of Banach algebras. Most progress had been made in the case of commutative Banach algebras, and in the non-commutative case Banach algebras of zero global homological dimension were fairly well understood, but there were few results otherwise. I was able to show that if a C^* -algebra is such that the images of its irreducible representations consist of compact operators then its global homological dimension is at least 2 [8, 10, 15]. The same conclusion holds for some other classes of operator algebras on Banach spaces. These results give information about the existence of non-split extensions of an algebra.

Relative homology over commutative Banach algebras

The major part of my research between 1985 and 1999 was been on the development of the theory of relative homology of Banach algebras [12, 13, 14, 16, 17, 18, 19, 21]. In the algebraic context these notions were introduced by Hochschild and were further developed by Cartan and Eilenberg. The study of some central homological characteristics of Banach algebras was started by Liddell, Phillips and Raeburn. I was able to find analogues for many of the notions of relative homological theory in a Banach algebra context, and to use these analogues to prove numerous results about homological structure. For example, for a C^* -algebra A , all central cohomology groups of A for $n \geq 1$ vanish if and only if A has an identity and a continuous trace [16]. Another series of results relates the cohomology of a Banach algebra A relative to its centre to the usual cohomology of A for dual modules [17, 19]. Further results show the existence of Banach algebras with prescribed homological dimension relative to a commutative subalgebra [13, 14, 23].

Projectivity of operator algebras and their ideals

Another typical concern in homological algebra is the identification of projective algebras and their ideals. It is again true that much had been known for commutative Banach algebras. The focus of much of my research has been to investigate this homological property for a natural class of non-commutative algebras arising in functional analysis: operator algebras on

Banach spaces. For example, I showed that all closed left ideals in a separable C^* -algebra are projective, but that every infinite-dimensional von Neumann algebra has a closed left ideal which is not projective [9, 10]. Results of this type were previously known for C^* -algebras only in the commutative case.

A familiar technique in operator algebras is to reduce properties of an algebra to those of its commutative subalgebras. I succeeded in doing this for certain homological properties. For example, if a C^* -algebra has the property that every closed left ideal is projective then the same is true of all its commutative subalgebras [28].

APPLIED MATHEMATICS

After getting an MSc in Mathematics I was involved in research in two areas of Applied Mathematics. The first (1976-1980) one was the numerical solution of the differential equations for the motion of a particle in a gravitational field [2, 3, 4].

The second (1982-1988) was on methods of assessing the capacity of building contractors and on planning their productivity [5, 6].

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May 2009