Please solve the following problems:

Qu.4 and Qu.6;

hand in your solutions on Tuesday the 5th of May by 16.00. Tutorial is on Thursday the 30th of April at 16.00 in TR2, Level 4, Herschel Building. **Z.A.Lykova**

MAS8220 Topology and Functional Analysis (2015)

Examples Sheet 8

Qu. 1. Let I be an ideal in an algebra A. Show that there is a bijective correspondence between the set of ideals in A/I and the set of ideals of A that contain I.

Qu. 2. Construct an example to show that maximal ideals in a commutative Banach algebra need not be closed.

Hint: Consider a zero multiplication algebra. You may assume that on any infinite-dimensional Banach space there is a linear functional with non-closed kernel.

Qu. 3. Let I be the ideal $\{f : f(t) = 0 \text{ for } 0 \le t \le \frac{1}{2}\}$ in $C[0,1]$.

Show that

(i) I is a closed ideal,

(ii) $C[0,1]/I$ is isometrically isomorphic to $C[0, \frac{1}{2}]$.

Qu. 4. Let A be a commutative Banach algebra, not necessarily having an identity, and let $\mathcal{M}_A$ be the maximal ideal space of A. If I is an ideal of A we say that an element x is an idempotent mod I if $x^2 - x \in I$. If φ is a character of A and x is an idempotent mod kerφ then what is $\varphi(x)$?

Show that the mapping

$$\hat{A} \to \mathcal{M}_A : \varphi \mapsto \ker \varphi$$

is injective. **50 marks**

Qu. 5. Show that there is a character of ℓ^∞ that is not a co-ordinate evaluation functional (Recall Ex. 6, Qu. 1).

Qu. 6. Show that $\ell^1(\mathbb{Z})$ is semisimple. **50 marks**

Qu. 7. Let A be the subset of $M_n(\mathbb{C})$ consisting of all matrices of the form

$$T = \begin{bmatrix} t_0 & t_1 & \dots & t_{n-1} \\ 0 & t_0 & \dots & t_{n-2} \\ \cdot & \cdot & \dots & \cdot \\ 0 & 0 & \dots & t_0 \end{bmatrix}$$

where $t_0, \dots, t_{n-1}$ are complex numbers (upper triangluar matrices constant on diagonals). Check that A is a commutative Banach algebra with identity.

Show that A has a unique character φ given by $\varphi(T) = t_0$.

What is the Jacobson radical of A?