Please solve the following problems:

Qu.1, Qu.3 and Qu.5;

hand in your solutions on Tuesday the 21st of April by 16.00. Tutorial is on Thursday the 12th of March at 16.00 in TR2, Level 4, Herschel Building. **Z.A.Lykova**

MAS8220 Topology and Functional Analysis (2015)

Examples Sheet 7

Qu. 1. $C^1[0,1]$ denotes the Banach algebra of continuously differentiable $\mathbb{C}$-valued functions on [0,1] with pointwise operations and norm

$$\|f\|_{C^1} = \sup_{0\le t\le 1} |f(t)| + \sup_{0\le t\le 1} |f'(t)|.$$

Check that this norm is submultiplicative.

Show that the character space of $C^1[0,1]$ can be naturally identifined with $[0,1]$ (adapt the proof of Theorem 9.2). **20 marks**

Qu. 2. Show that the Gelfand topology on the character space of $C^1[0,1]$ agrees (under the identification obatined in Qu. 1) with the usual topology of $[0,1]$.

Qu. 3. Let K be a compact Hausdorff space.

Show that K is connected if and only if $C(K)$ contains no idempotents other than 0 and the identity. [An *idempotent* in an algebra is an element x such that $x^2 = x$]. **15 marks**

Qu. 4. Let A, B be commutative unital Banach algebras and let $\Phi : A \to B$ be a continuous unital homomorphism.

Show that Φ determines a map $f : \hat{B} \to \hat{A}$.

Show further that f is continuous with respect to the Gelfand topologies on $\hat{A}$ and $\hat{B}$.

Qu. 5. Let e_n denote the "nth standard basis vector" in the convolution algebra $\ell^1(\mathbb{Z})$ (Example 6.2); thus e_n has 1 in the nth place (for $n \in \mathbb{Z}$) and zero elsewhere.

Show that, for any character φ of $\ell^1(\mathbb{Z})$, $\varphi(e_n) = \varphi(e_1)^n$ for $n \in \mathbb{Z}$, and deduce that $|\varphi(e_1)| = 1$.

Show that the character space of $\ell^1(\mathbb{Z})$ is homeomorphic to the unit circle $\mathbb{T}$ of the complex plane.

Describe the Gelfand representation $\Gamma : \ell^1(\mathbb{Z}) \to C(\mathbb{T})$. **15 marks**

Qu. 6. Give an example of an ideal in C[0,1] that is not closed in C[0,1].