Please solve the following problems:

Qu.1, Qu.2 (iii), Qu.3, Qu.5 and Qu.6;

hand in your solutions on Tuesday the 21st of April by 16.00. Tutorial is on Thursday the 5th of March at 16.00 in TR2, Level 4, Herschel Building. **Z.A.Lykova**

MAS8220 Topology and Functional Analysis (2015)

Examples Sheet 6

Qu. 1. A co-ordinate evaluation functional on the commutative Banach algebra ℓ^∞ is a functional of the form

$$v_j : \ell^\infty \to \mathbb{C} : (x_n)_{n=1}^\infty \mapsto x_j$$

for some $j \in \mathbb{N}$. Use Theorem 7.17 to show that ℓ^∞ contains a maximal ideal that is not of the form $\ker v_j$ for any $j \in \mathbb{N}$. **10 marks**

Qu. 2. Does the sequence of functions (f_n) converge (a) pointwise (b) uniformly on $[0, 1)$, where

(i) $f_n(x) = x^n$;

(ii) $f_n(x) = xe^{-nx}$;

(iii) $f_n(x) = nxe^{-nx}$; **10 marks**

(iv) f_n such that $f_n(t) = nt$ for $t \in [0, 1/n]$, $f_n(t) = 2-nt$ for all $t \in [1/n, 2/n]$ and $f_n(t) = 0$ for all $t \in [2/n, 1]$.

Qu. 3. Construct a sequence of functions (f_n) in $C[0, 1]$ such that $f_n \to 0$ pointwise on $[0, 1]$ but $\|f_n\| \to \infty$. **10 marks**

Qu. 4. Let X be a set and Y a topological space. For any functions f_n, $n \in \mathbb{N}$, and $g : X \to Y$, show that $f_n \to g$ pointwise on X if and only if $f_n \to g$ in the product topology Y^X.

Qu. 5. Show that if K_1 and K_2 are homeomorphic compact Hausdorff spaces then $C(K_1)$ and $C(K_2)$ are isometrically isomorphic Banach algebras. [A linear mapping between Banach spaces is isometric if it preserves norms.] **10 marks**

Qu. 6. Let $K = \{0\} \cup \{\frac{1}{n} : n \in \mathbb{N}\}$ with the relative topology induced by $\mathbb{R}$. Show that c_0, the Banach algebra of complex sequences tending to 0, is isometrically isomorphic to a maximal ideal of $C(K)$. **10 marks**

Qu. 7. Does the geometric series $1 + z + z^2 + ...$ converge uniformly on $\mathbb{D}$? On compact subsets of $\mathbb{D}$?