**Please solve the following problems:**

**Qu.2, Qu.3, Qu.6 and Qu.10;**

**hand in your solutions on Tuesday the 24th of February by 16.00. Tutorial is on Thursday the 12th of February at 16.00 in TR2, Level 4, Herschel Building.** **Z.A.Lykova**

# MAS8220 Topology and Functional Analysis (2015)

## Examples Sheet 3

In Q1-5 $R$ is a ring with identity.

Qu. 1. Show that if $x \in R$ has both a left inverse and a right inverse then $x$ is a regular element of $R$.

Qu. 2. Show that if $x, y \in R$ and both $xy$ and $yx$ are regular elements then $(xy)^{-1}x = x(yx)^{-1}$.

**10 marks**

Qu. 3. Let $x, y \in R$. Show that $x$ and $y$ are regular elements if and only if $xy$ and $yx$ are regular elements. **15 marks**

Qu. 4. Let $x, y$ be commuting elements of $R$ (i.e. $xy = yx$). Show that if $x$ is singular then so is $xy$.

Qu. 5. If $x, y \in R$ and $x$ is singular, does it follow that $xy$ is singular? [Recall Ex.2, Qu.7].

Qu. 6. Let $\mathbb{D}$ denote the open unit disc in $\mathbb{C}$ :

$$\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$$

Let $x(z) = z$ for all $z \in \mathbb{D}$. Find the spectrum $\sigma(x)$ of $x$ when $x$ is regarded as an element of the algebra $H^\infty(\mathbb{D})$. [Recall Ex. 2, Qu.5] **15 marks**

Qu. 7. Same as 6, but for $x$ as an element of $\ell^\infty(\mathbb{D})$. [Ex. 2, Qu.4].

Qu. 8. Let $x(z) = (3 - z)^{-1}$ for $z \in \mathbb{D}$. Show that $x \in H^\infty(\mathbb{D})$. What is the spectrum of $x$ in $H^\infty(\mathbb{D})$? In $\ell^\infty(\mathbb{D})$?

Qu. 9. Show that, for $x \in H^\infty(\mathbb{D})$, the spectrum of $x$ in $H^\infty(\mathbb{D})$ is the closure in $\mathbb{C}$ of $x(\mathbb{D})$.

Qu. 10. Let $X$ be a Banach space and let $T \in \mathcal{B}(X)$ [Ex. 1, Qu.5]. Show that if $\lambda \in \mathbb{C}$ is an eigenvalue of $T$ then $\lambda \in \sigma(T)$. Give an example to show that elements of $\sigma(T)$ need not be eigenvalues of $T$ [The forward shift $S$ on $\ell^2$ has no eigenvalues]. **10 marks**