Software Implementation for Wulfsohn-Tsiatis

Pete Philipson

University of Newcastle Upon Tyne

Outline

- Longitudinal data

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- Survival data

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- Joint model

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- Computational implementation

Longitudinal data

- Random effects model

$$Y_{ij} = \boldsymbol{x}_{1i}(t_{ij})^T \boldsymbol{\beta}_1 + U_{0i} + U_{1i} t_{ij} + \epsilon_{ij}$$

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$$\begin{pmatrix} U_{0i} \\ U_{1i} \end{pmatrix} \sim \left(N \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{00} & \sigma_{01} \\ \sigma_{10} & \sigma_{11} \end{pmatrix} \right),$$

  or, alternatively,

$$\boldsymbol{U_i} \sim N(\boldsymbol{0}, \boldsymbol{V}).$$

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    or, alternatively,

$$\boldsymbol{U_i} \sim N(\boldsymbol{0}, \boldsymbol{V}).$$

  - Also,

$$\epsilon_{ij} \sim N(0, \sigma_\epsilon^2).$$

Survival data

- Survival data with hazard

$$\alpha(t; \boldsymbol{x}_{2i}, \boldsymbol{U_i}) = \alpha_0(t)\text{exp}\{\boldsymbol{x}_{2i}(t)^T\boldsymbol{\beta}_2 + \gamma(U_{0i} + U_{1i}t) + U_{2i}\}$$

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- Observe time s_i with associated failure indicator

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- Joint model - idea is to combine longitudinal and survival elements in a larger meta-model, see Wulfsohn and Tsiatis (1997)

Joint model I

- Observed data likelihood

$$\prod_{i=1}^{m}\left[\int_{-\infty}^{\infty}\left\{\prod_{j=1}^{n_i} f(y_{ij} \mid \cdot)\right\} f(s_i, \Delta_i \mid \cdot) f(\boldsymbol{U}_i \mid \boldsymbol{V}) \; d\boldsymbol{U}_i\right]$$

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- Complete data likelihood

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- Complete data likelihood
- EM algorithm
- Maximisation (M) step

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- Complete data likelihood
- EM algorithm
- Maximisation (M) step
  - score equations

Joint model I

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- Complete data likelihood
- EM algorithm
- Maximisation (M) step
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- Complete data likelihood
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  - Newton-Raphson iterative algorithm

Joint model II

- Expectation (E) step

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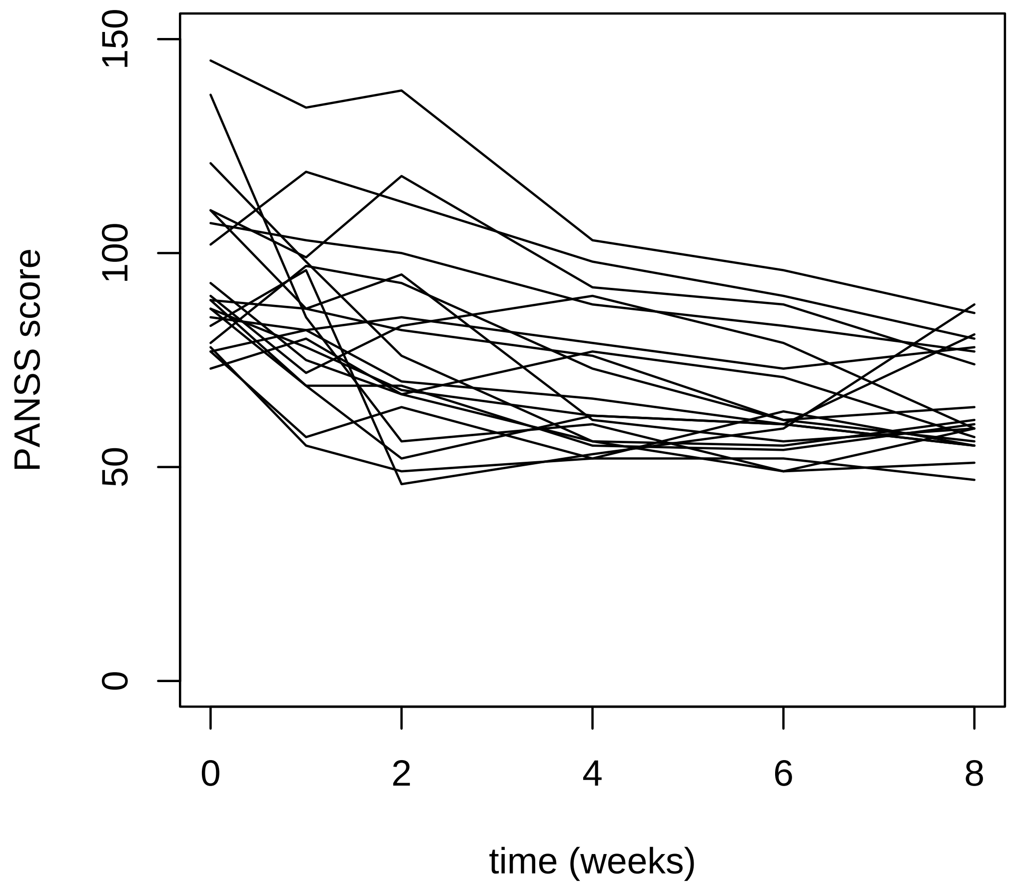
- Comprehensive R Archive Network (CRAN)

PANSS data



Bibliography

Wulfsohn, M. S. and Tsiatis, A. A. (1997). A Joint Model for Survival and Longitudinal Data Measured with Error. *Biometrics*, **53**, 330-339.