PARTNERS Summer School
Mathematics & Statistics
Solutions to Tests

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Test 1: Algebra

1. $$5(3 + 3 \times 2^2) = 5(3 + 12) = 5 \times 15 = \underline{75}$$

2. $$(x + 3)(x - 5) = x^2 + 3x - 5x - 15 = x^2 - 2x - 15$$

3. $$\frac{2p}{q} + \frac{7pr}{3} = \frac{6p + 7pqr}{3q} = \frac{p(6 + 7qr)}{3q}$$

Test 1: Algebra

Question 4:

(a)

$$\frac{E_2}{E_1} = \frac{\frac{1}{2}mv_2^2}{\frac{1}{2}mv_1^2} = \left(\frac{v_2}{v_1}\right)^2$$

(b)

$$E_2 - E_1 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \frac{1}{2}m(v_2^2 - v_1^2)$$

(c)

$$\frac{E_2 - E_1}{E_1} = \frac{\frac{1}{2}m(v_2^2 - v_1^2)}{\frac{1}{2}mv_1^2} = \frac{v_2^2 - v_1^2}{v_1^2} = \left(\frac{v_2}{v_1}\right)^2 - 1$$

Test 1: Algebra

Question 5:

$$\left(\frac{a\sqrt{a}}{a^2}\right)^2 = \frac{a^2 \times a}{a^4} = \frac{1}{a} = a^{-1}$$

Question 6:

$$T \propto \sqrt{\frac{l}{g}}$$

Test 1: Algebra

Question 7:

$$\begin{aligned} x - 3 < 2x + 1 \quad &\Rightarrow \quad x > -4 \\ 2x + 1 \leq 5 \quad &\Rightarrow \quad x \leq 2 \end{aligned}$$

Hence

$$\underline{-4 < x \leq 2}$$

Test 1: Algebra

Question 8:

(a) $h < 1$:

$$x(1-h) \geq a \Rightarrow x \geq \frac{a}{1-h}$$

(b) $h > 1$:

$$x(1-h) \geq a \Rightarrow x \leq \frac{a}{1-h}$$

What if $h = 1$:

$$x - x \geq 0 \Rightarrow a \leq 0$$

but we are told that $a > 0$ so this is a contradiction. We can not have $h = 1$.

Test 2: Functions and Graphs

Question 1:

$$y = \frac{1}{1 + x^2 + x^4}$$

Domain: $-\infty < x < \infty$

Range: $0 < y \leq 1$

Test 2: Functions and Graphs

Question 2:

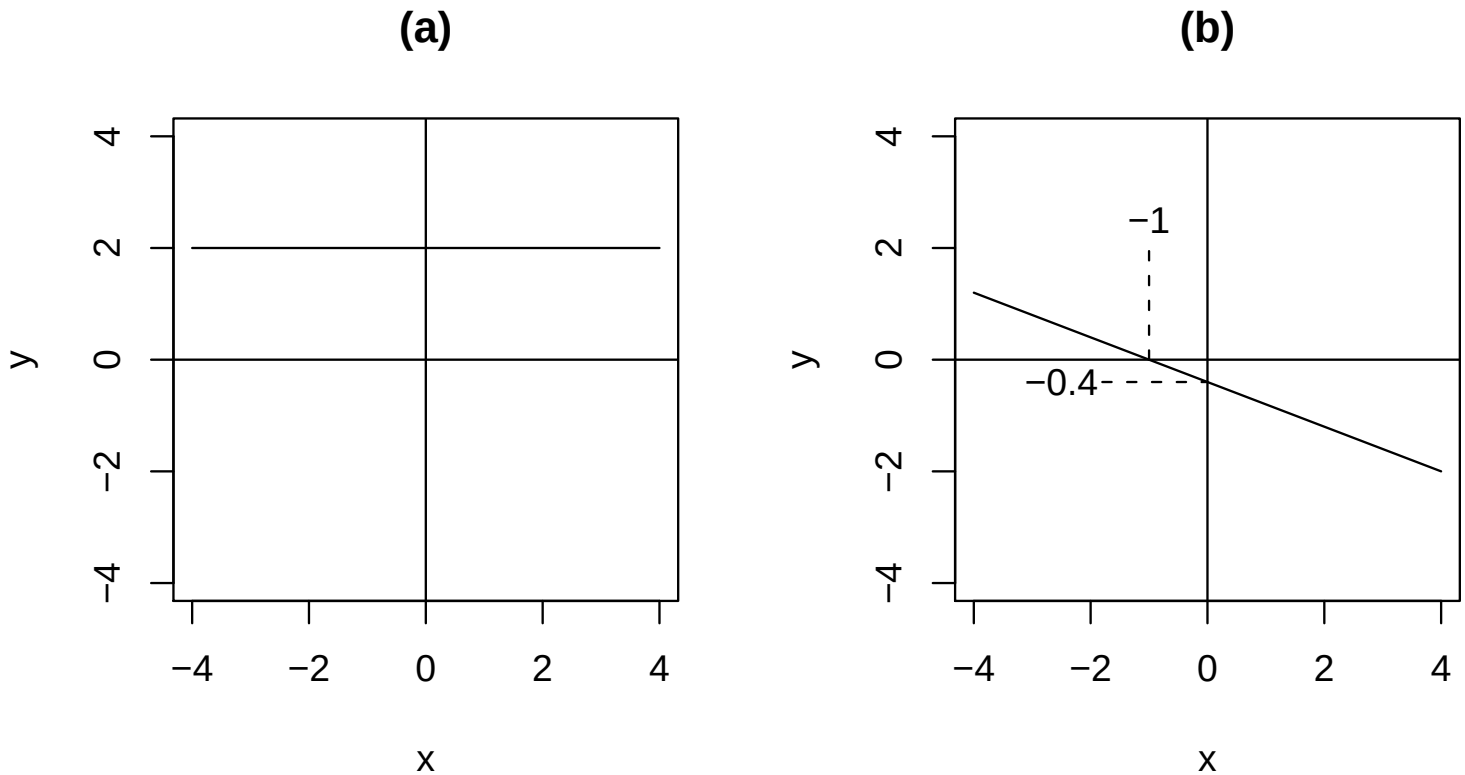
(a) $y = 2$

(b)

$$
\begin{aligned}
0 &= -m + c \quad (1) \\
-2 &= 4m + c \quad (2) \\
(2) - (1) \quad -2 &= 5m \\
m &= -\frac{2}{5} \\
c &= -\frac{2}{5} \\
y &= -\frac{2}{5}(x+1)
\end{aligned}
$$

Test 2: Functions and Graphs

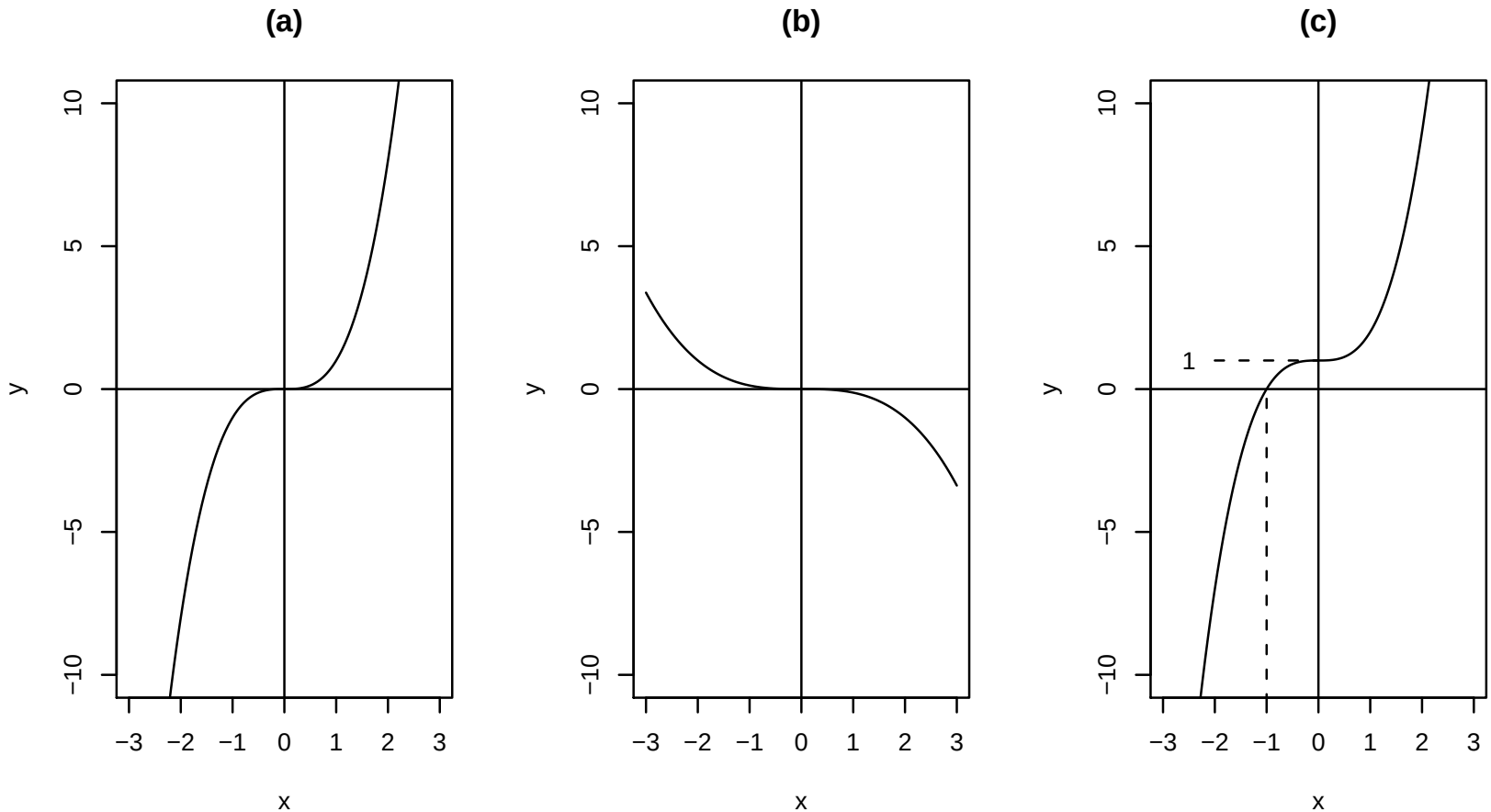
Question 2:



Test 2: Functions and Graphs

Question 3:

Plots:



Test 2: Functions and Graphs

Question 4:

(a)

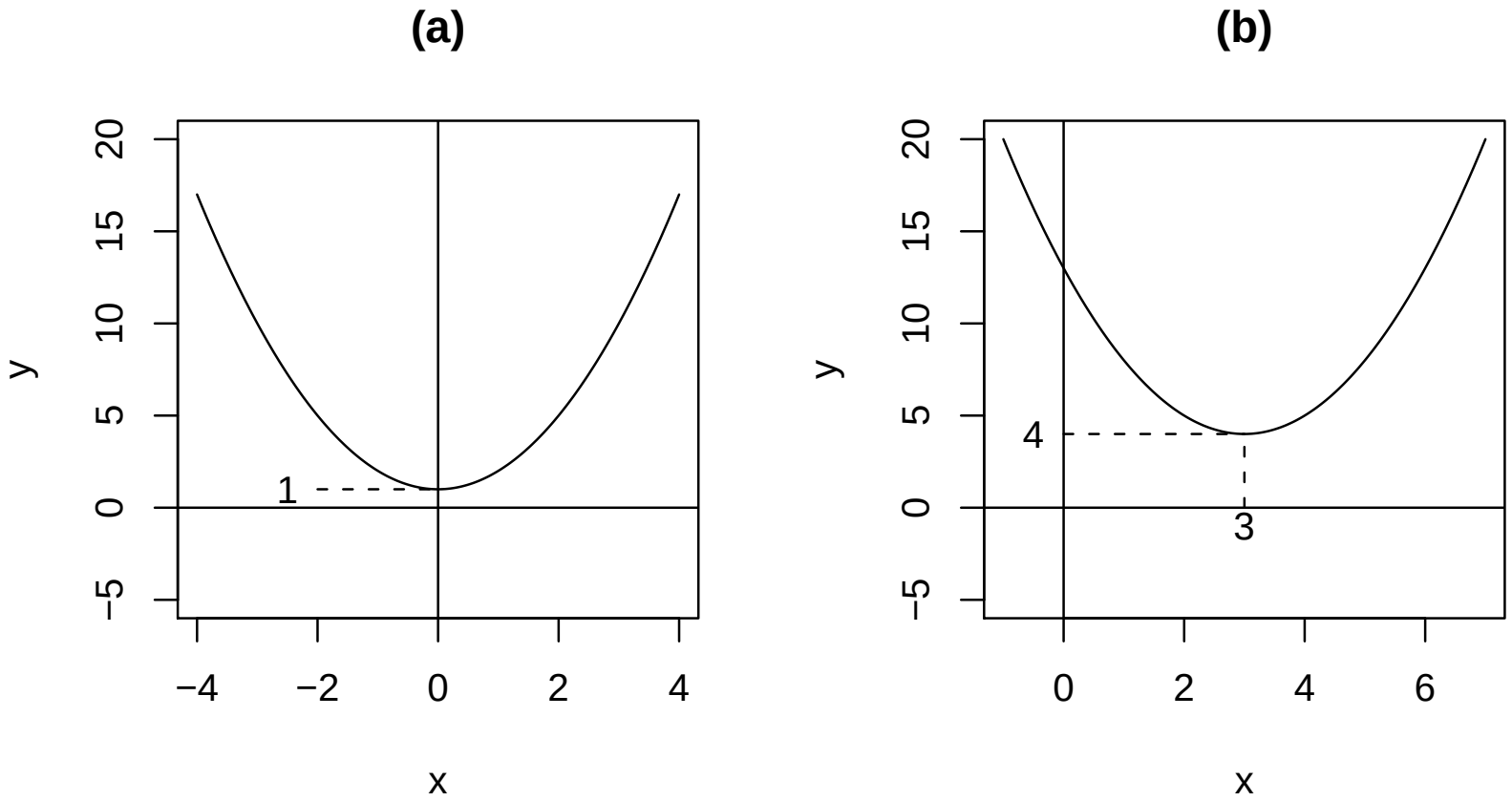
$$f[g(x)] = x^2 + 4 - 3 = x^2 + 1$$

(b)

$$g[f(x)] = (x-3)^2+4 = x^2-6x+9+4 = x^2-6x+13$$

Test 2: Functions and Graphs

Question 4:



Test 3: Solving Equations

Question 1:

$$c = \sqrt{\frac{E}{\rho}} \Rightarrow c^2 = \frac{E}{\rho} \Rightarrow \rho = \frac{E}{c^2}$$

Question 2:

$$\begin{aligned} 2(x-1) + 3(x-6) &= 0 \\ \text{so} \quad 2x - 2 + 3x - 18 &= 0 \\ \text{so} \quad 5x &= 20 \\ \text{so} \quad x &= 4 \end{aligned}$$

Test 3: Solving Equations

Question 3:

$$\begin{aligned} \frac{1}{f} &= \frac{1}{u} + \frac{1}{v} = \frac{u+v}{uv} \\ f &= \frac{uv}{u+v} \end{aligned}$$

Test 3: Solving Equations

Question 4:

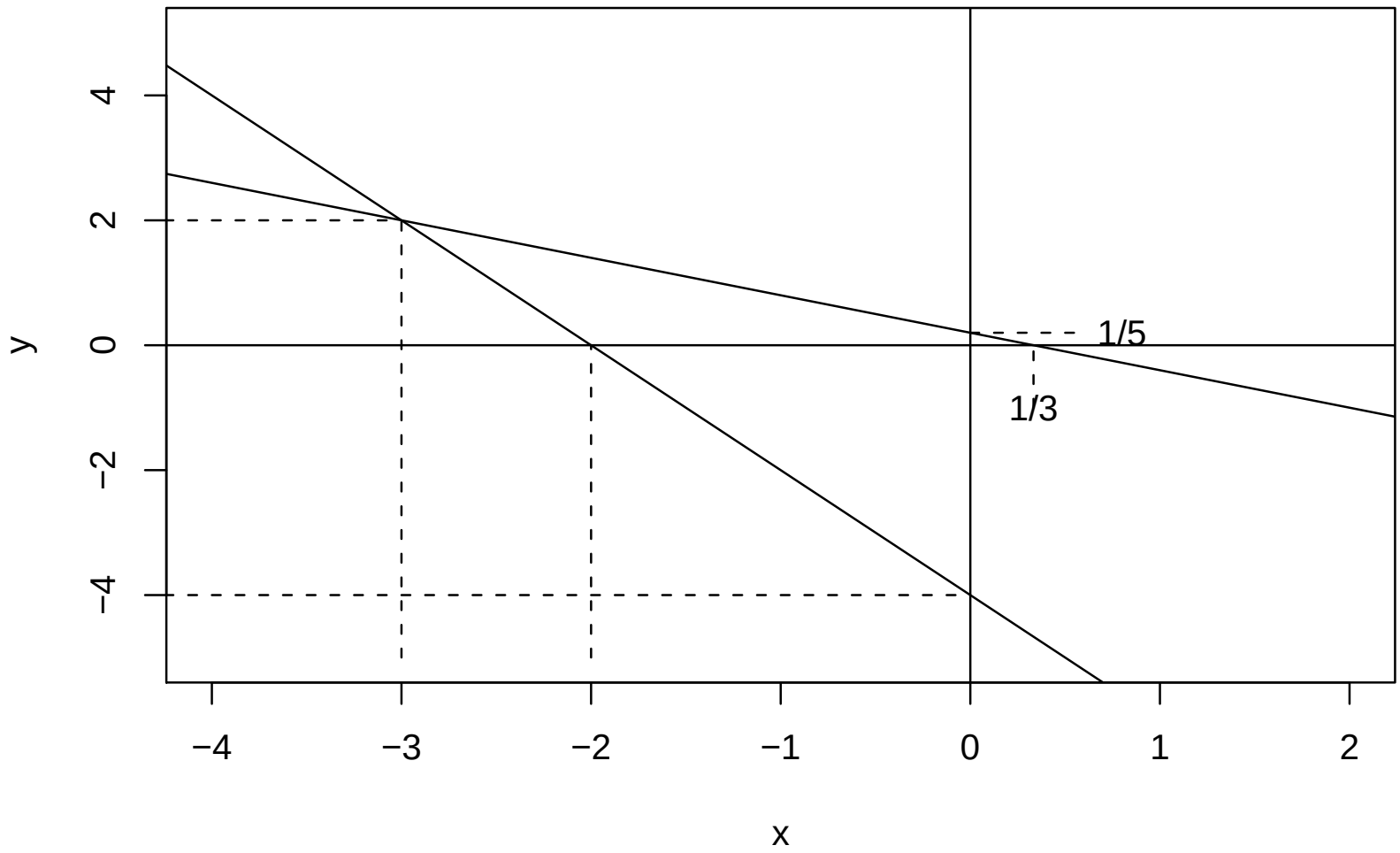
$$\begin{aligned} y + 2x &= -4 \quad (1) \\ 5y + 3x &= 1 \quad (2) \\ 5(1) - (2) \quad 7x &= -21 \\ x &= -3 \end{aligned}$$

Substitute in (1): $y - 6 = -4$ so $y = 2$.

$$\underline{x = -3, \quad y = 2}$$

Test 3: Solving Equations

Question 4:



Test 3: Solving Equations

Question 5(a):

Either $(x+7)(x-2)=0$ so

$$x=-7 \quad \text{or} \quad x=2.$$

Or

$$\begin{aligned} x &= \frac{-5\pm\sqrt{25+56}}{2} \\ &= \frac{-5\pm\sqrt{81}}{2} \\ &= \frac{-5\pm 9}{2} \\ &= \begin{cases} \frac{4}{2} &= 2 \\ -\frac{14}{2} &= -7 \end{cases} \end{aligned}$$

Test 3: Solving Equations

Question 5(b):

$$\begin{aligned} x &= \frac{2 \pm \sqrt{4+4}}{2} \\ &= 1 \pm \frac{\sqrt{8}}{2} \\ &= 1 \pm \sqrt{2} \end{aligned}$$

Test 3: Solving Equations

Question 6:

- $x = 1$?

$$1 + 4 + 2 - 4 = 3 \neq 0$$

Test 3: Solving Equations

Question 6:

- $x = 1$?

$$1 + 4 + 2 - 4 = 3 \neq 0$$

- $x = -1$?

$$-1 + 4 - 2 - 4 = -3 \neq 0$$

Test 3: Solving Equations

Question 6:

- $x = 1$?

$$1 + 4 + 2 - 4 = 3 \neq 0$$

- $x = -1$?

$$-1 + 4 - 2 - 4 = -3 \neq 0$$

- $x = 2$?

$$8 + 16 + 4 - 4 = 24 \neq 0$$

Test 3: Solving Equations

Question 6:

- $x = 1$?

$$1 + 4 + 2 - 4 = 3 \neq 0$$

- $x = -1$?

$$-1 + 4 - 2 - 4 = -3 \neq 0$$

- $x = 2$?

$$8 + 16 + 4 - 4 = 24 \neq 0$$

- $x = -2$?

$$-8 + 16 - 4 - 4 = 0$$

So $x = -2$ is a solution and $(x + 2)$ is a factor.

Test 3: Solving Equations

Question 6:

$$x^3 + 4x^2 + 2x - 4 \equiv (x+2)(x^2 + 2x - 2)$$

Solve $x^2 + 2x - 2 = 0$.

$$\begin{aligned} x &= \frac{-2 \pm \sqrt{4+8}}{2} = -1 \pm \frac{\sqrt{12}}{2} \\ &= -1 \pm \sqrt{3} \end{aligned}$$

Overall:

$$x = \begin{cases} -2 \\ -1 - \sqrt{3} \\ -1 + \sqrt{3} \end{cases}$$