PARTNERS Assessed Summer School 2010
Mathematics & Statistics

Dr Malcolm Farrow

July 2010

Week 1: 5th - 9th July

All Mathematics and Statistics sessions this week will be in Lecture Theatre 4 in the King George VI Building.

Date	Time	Topic
Tuesday 6th July	14.00-15.00	Maths & Stats 1: Algebra
Tuesday 6th July	15.15-16.15	Maths & Stats 2: Functions and Graphs
Wednesday 7th July	10.30-11.30	Maths & Stats 3: Solving Equations
Wednesday 7th July	11.45-12.45	Maths & Stats 4: Complex Numbers
Wednesday 7th July	14.00-15.00	Maths & Stats 5: Geometry and Vectors
Wednesday 7th July	15.15-16.15	Maths & Stats 6: Differential Calculus
Thursday 8th July	14.00-15.00	Maths & Stats 7: Solutions to Problems
Thursday 8th July	15.15-16.15	Maths & Stats Test

Week 2: 12th-16th July

All Mathematics and Statistics sessions this week will be in Room 302 in the Agriculture Building, except for Session 14 which will be in the Naiad PC Cluster (Room 2.33/2.34) in the King George VI Building).

Date	Time	Topic
Monday 12th July	14.00-15.00	Maths & Stats 8: Probability and Statistics
Monday 12th July	15.15-16.15	Maths & Stats 9: Probability and Statistics
Tuesday 13th July	10.30-11.30	Maths & Stats 10: Mathematical Modelling
Tuesday 13th July	11.45-12.45	Maths & Stats 11: Mathematical Modelling
Wednesday 14th July	14.00-15.00	Maths & Stats 12: Waves
Wednesday 14th July	15.15-16.15	Maths & Stats 13: Waves
Thursday 15th July	10.30-11.30	Maths & Stats 14: Maple **(Naiad Cluster)**
Thursday 15th July	11.45-12.45	Maths & Stats 15: Project Briefing

Lecturers

Sessions	Lecturer
1-9	Dr M. Farrow
10-11	Dr A. Fletcher
12-13	Prof R.S. Johnson
14	Dr A.J. Youd
15	Dr M. Farrow

Attendance

Attendance at all sessions is compulsory for satisfactory completion of the Summer School. Attendance lists will be kept.

Assessment

For **Mathematics and Statistics** students there are two elements of assessment, as follows.

- A 50-minute **TEST** on Thursday 8th July at 15.15 in Lecture Theatre 4, King George VI Building.
- A project. Further details of the project are given below. Project reports are to be submitted by **12.00 noon** on **Tuesday 27th July**.

In addition, students will be given sheets marked "Achievement & Assessment Test." There are six of these. You should do each one after the corresponding lecture. These are for practice and will not count towards your assessment. However I will provide solutions in due course.

Information on the Web

There is a Mathematics & Statistics Partners Web page at the following address.

`http://www.mas.ncl.ac.uk/~nmf16/teaching/partners/`

This handout and various other bits of information will be made available there.

Project (Maths & Stats only)

You should do EITHER A OR B as given below.

A: Topic in Mathematics or Statistics

Investigate and write a report, of about 10-15 pages, including references and any figures and tables, on a topic of your choice in the general area of Mathematics or Statistics. You can choose your own topic but you might like to discuss your idea with Dr Farrow or Dr Toms before starting work on it. It is a good idea to find your own topic as this is likely to be something which interests you but here are some possible titles for illustration.

- History of Algebra
- Mathematics of the Ancient Greeks
- The Invention of Calculus
- Probability and Statistics before the Twentieth Century
- Industrial Applications of Statistics
- Applications of Calculus
- Fibonacci Numbers in Nature
- Newton's Laws of Motion
- Statistics in Medicine
- Florence Nightingale and Statistics
- Secure Cryptography

You can, of course, obtain information using Internet search engines or Wikipedia. There is a lot to be found in libraries as well. However be careful to use information properly.

- Give credit to your sources of information, giving proper references. (If you obtain material from the Web, you can give the Web address as a reference).

- Do not copy material word for word but interpret it.
- Do not attempt to give material copied word for word from somewhere as though you wrote it yourself.
- Try not to be too dependent on one source of information but combine information from several sources.
- Try to structure your report, organising bits of information in a clear and sensible way.
- If you can, try to contribute some ideas of your own.

We are not expecting the "last word" on a topic but you should convince us that you have made a serious effort to produce something worthwhile and which, for example, might be interesting for another student to read.

B: Hospital Ward Bed Occupancy

You are to investigate a system as follows. You should present your findings in a report with clear structure and explanations of what you have done and what your findings are.

The investigation will involve the use of some data. Each student will be provided with a different set of data. Each data set has a reference number. You must write the reference number of your data set on the front of your report.

A specialised hospital ward deals only with patients suffering from certain particular conditions. Patients arrive at random times and stay for different lengths of time. This may be regarded as a kind of queueing system where the number of servers is the number of beds in the ward. We would like the number of beds to be great enough so that it is only rarely necessary to send a patient to another hospital because all beds are occupied. However, for reasons of cost, we would not wish to maintain an unnecessarily large number of beds.

For the purpose of this investigation, to get an idea of how many beds are likely to be needed, suppose that the number of beds in unlimited so that no patient has to be transferred to another hospital.

We assume that arrivals follow a Poisson process. That is, the number of arrivals in a time interval of length t days has a Poisson distribution with mean λt where $\lambda > 0$ is a parameter. So, if X is the number of arrivals, then the probability that $X = j$ is

$$\Pr(X = j) = \frac{e^{-\lambda t}(\lambda t)^j}{j!}$$

for $j = 0, 1, 2, \ldots$. The numbers of arrivals in non-overlapping time intervals are independent.

1. Write down the probability that $X = 0$ and hence show that the probability density function of the time to the next arrival is

$$f_T(t) = \begin{cases} 0 & (t < 0) \\ \lambda e^{-\lambda t} & (t \geq 0) \end{cases} .$$

   Note that this is also the probability density function for inter-arrival times and that inter-arrival times are independent. The distribution of inter-arrival times is called an exponential distribution with parameter λ.

2. Show that the mean of the distribution of inter-arrival times is $1/\lambda$.

3. Your data include 100 observed inter-arrival times, in seconds.
   (a) Display the data in a histogram.
   (b) Calculate the sample mean of your data and, by equating this to the expression for the mean of the distribution of inter-arrival times, find an estimate of λ.

(c) Plot a graph of the probability density function of the inter-arrival times, using your estimate of λ, and compare this with your histogram.

4. A normal distribution with mean μ and variance σ^2, that is a $N(\mu,\ \sigma^2)$ distribution, has probability density function

$$f_X(x) = (2\pi\sigma^2)^{-1/2} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}$$

$(-\infty < x < \infty)$.

Show that

$$e^x f_X(x) = (2\pi\sigma^2)^{-1/2} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu^*}{\sigma}\right)^2\right\} \exp(\mu + \sigma^2/2) \qquad (1)$$

where $\mu^* = \mu + \sigma^2$.

5. Suppose that X has a normal distribution with mean μ and variance σ^2 and that $T = e^X$. We say that T has a lognormal($\mu,\ \sigma^2$) distribution. Notice that we must have $T \geq 0$. Find the mean of T.

   (Hint: Use equation (1) and find the expectation of e^X).

6. We assume that the bed-occupancy times, in days, are independent and have a lognormal($\mu,\ \sigma^2$) distribution. A bed-occupancy time is the time elapsed from the arrival of a patient until that patient has left and the bed is ready for a new patient. So there is no need to allow additional time for bed preparation.

   Your data include 100 observed bed occupancy times in days.

   (a) Display the data in a histogram.

   (b) Calculate the (natural) logarithms of your data.

   (c) Display your log-data in a histogram.

   (d) Calculate the sample mean $\bar{x}$ and sample standard deviation s of your log-data and use these as estimates of μ and σ.

   (e) Plot a graph of the probability density function of the log-bed-occupancy times, using your estimates of μ and σ, and compare this with your histogram.

7. Simulate the arrivals and bed occupancy, starting from a state where there are no patients in the ward and continuing until the arrival of the thirtieth patient.

   (a) Your data include 30 random observations from the exponential distribution with parameter 1. Generate your inter-arrival times by dividing these by your estimate of λ.

   (b) Your data include 30 random observations from the $N(0, 1)$ distribution. If these are $z_1, \ldots, z_{30}$, generate your bed occupancy times using $t_i = \exp(\bar{x} + sz_i)$ where $\bar{x}$ and s are from Part 6.

   (c) Illustrate your results using suitable graphs and summaries and comment on what you have found.

8. Present your results in the form of a report.

Data

The data can be downloaded from the Mathematics & Statistics Partners Web page at the following address.

`http://www.mas.ncl.ac.uk/~nmf16/teaching/partners/`

Reference numbers

Nadia Alali	1	Abigail Forster	9	Liam Jordan	17	Rebecca Salkeld	25
Emily-Rose Alexanders	2	Benjamin Gibbin	10	Scott March	18	Charlotte Todhunter	26
Thomas Anderson	3	Natasha Gidda	11	Ryan Nicholson	19	Sean Toland	27
Richard Anderson	4	Michael Gough	12	Faye Oughton	20	Jonathan Turnbull	28
Philippa Bartle	5	Fraser Gray	13	Aaron Pankhurst	21	Hok Wan	29
Arron Crossman	6	Jonathan Hayden	14	Joseph Perry	22	Aimee Wilde	30
Nathan Embleton	7	Claire Hornsby	15	Stuart Raw	23	Shi Wu	31
Matthew English	8	Matthew Hyde	16	Rebecca Riley	24		

Project Contacts:

For questions about the project you may contact either Dr Farrow or Dr Toms.

Dr Malcolm Farrow	Herschel Building, Room 3.12	`malcolm.farrow@ncl.ac.uk`
Dr David Toms	Herschel Building, Room 2.14	`d.j.toms@ncl.ac.uk`

School of Mathematics and Statistics,
Newcastle University,
Newcastle upon Tyne.
NE1 7RU