Chapter 6

Decision Making Using Probability

Outline

- Conditional Probability

- Tree Diagrams

- Optimal Decisions

Conditional Probability

If we have two events A and B then

$$P(A|B)$$

is the probability of A given that B has occurred.

Utility companies – forecast periods of high demand:

$$P(\text{High demand}|\text{air temperature is below normal}) = 0.6$$
$$P(\text{High demand}|\text{air temperature is normal}) = 0.2$$
$$P(\text{High demand}|\text{air temperature is above normal}) = 0.05.$$

Conditional Probability

General formula:

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

Example

Sales of CD singles at a local outlet:

	<30	$30-50$	$50+$
Male	0.275	0.125	0.025
Female	0.325	0.175	0.075

From this table, we can calculate

$$\begin{aligned} P(\text{Male}) &= P(\text{Male and } <30) + P(\text{Male and } 30-50) \\ &\qquad + P(\text{Male and } 50+) \\ &= 0.275 + 0.125 + 0.025 = 0.425 \end{aligned}$$

and

$$\begin{aligned} P(\text{Female}) &= P(\text{Female and } <30) + P(\text{Female and } 30-50) \\ &\qquad + P(\text{Female and } 50+) \\ &= 0.325 + 0.175 + 0.075 = 0.575. \end{aligned}$$

Also, the age distribution of the customers is

$$
\begin{aligned}
P(< 30) &= Pr(\text{Male and } < 30) + Pr(\text{Female and } < 30) \\
&= 0.275 + 0.325 = 0.6
\end{aligned}
$$

$$
\begin{aligned}
P(30-50) &= Pr(\text{Male and } 30-50) + Pr(\text{Female and } 30-50) \\
&= 0.125 + 0.175 = 0.3
\end{aligned}
$$

$$
\begin{aligned}
P(50+) &= Pr(\text{Male and } 50+) + Pr(\text{Female and } 50+) \\
&= 0.025 + 0.075 = 0.1.
\end{aligned}
$$

Also

$$P(\text{Male}|30-50) = \frac{P(\text{Male and } 30-50)}{P(30-50)} = \frac{0.125}{0.3} = 0.4167$$

$$P(\text{Female}|30-50) = 1 - P(\text{Male}|30-50) = 1 - 0.4167 = 0.5833$$

and

$$P(<30|\text{Male}) = \frac{P(\text{Male and } <30)}{P(Male)} = \frac{0.275}{0.425} = 0.6471$$

$$P(30-50|\text{Male}) = \frac{P(\text{Male and } 30-50)}{P(Male)} = \frac{0.125}{0.425} = 0.2941$$

$$\begin{aligned} P(50+|\text{Male}) &= 1 - P(<30|\text{Male}) - P(30-50|\text{Male}) \\ &= 1 - 0.6471 - 0.2941 = 0.588. \end{aligned}$$

Tree Diagrams

- Experiment with multiple outcomes

- Represent each experiment by a circle

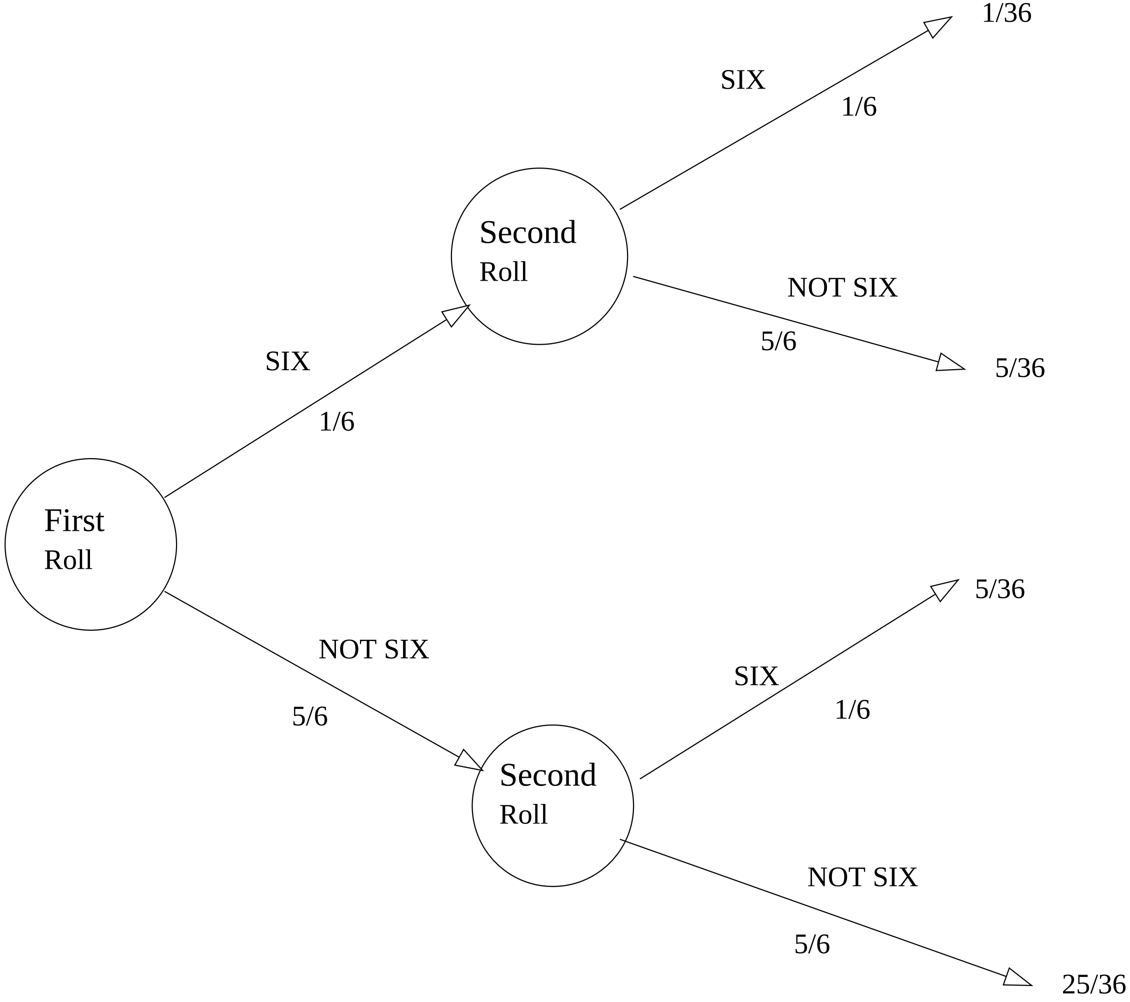
- Branches from it represent outcomes

- Each outcome has a probability associated with it

Tree Diagrams

Consider the probability of throwing two consecutive 6's on a die.

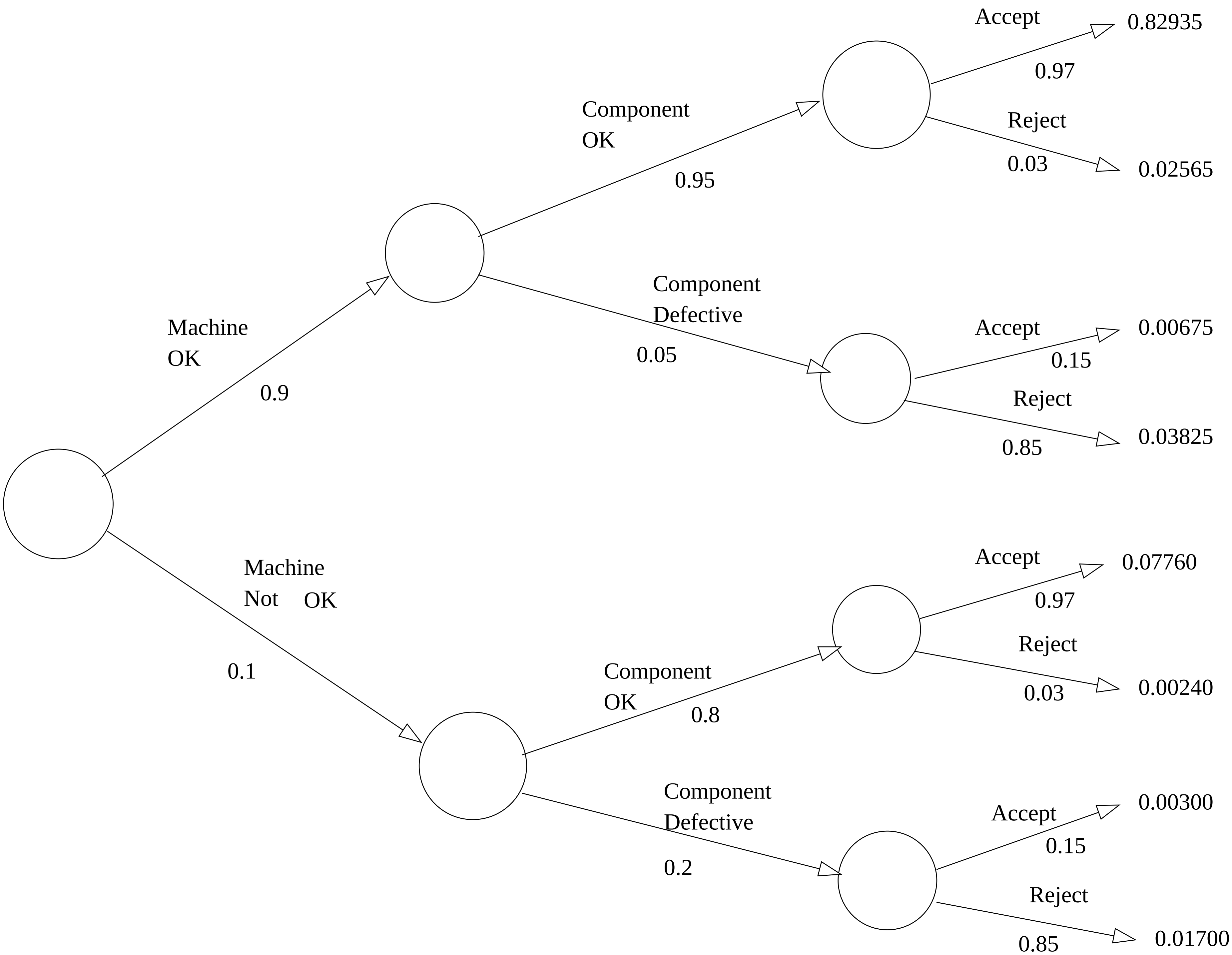
$$P(\text{Six and Six}) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$


First
Roll
SIX
1/6
NOT SIX
5/6
Second
Roll
SIX
1/6
1/36
NOT SIX
5/6
5/36
Second
Roll
SIX
1/6
5/36
NOT SIX
5/6
25/36

Example

- A machine produces components. The machine may be OK or not.
- The components may be defective or not.
- The components are tested and may be accepted or rejected.

$$
\begin{aligned}
P(\text{Machine OK}) &= 0.9 \\
P(\text{Component OK} \mid \text{Machine OK}) &= 0.95 \\
P(\text{Component OK} \mid \text{Machine faulty}) &= 0.8 \\
P(\text{Accept component} \mid \text{Component OK}) &= 0.97 \\
P(\text{Accept component} \mid \text{Component defective}) &= 0.15
\end{aligned}
$$



Accept
0.82935
0.97
Component
OK
Reject
0.03
0.02565
0.95
Component
Defective
Accept
0.00675
Machine
OK
0.05
0.15
Reject
0.9
0.03825
0.85
Accept
0.07760
Machine
Not OK
0.97
Reject
0.1
Component
OK
0.00240
0.03
0.8
Component
Defective
0.00300
Accept
0.15
0.2
Reject
0.85
0.01700

Example ctd

$$
\begin{aligned}
P(\text{accepted}) &= 0.82935 + 0.00675 + 0.07760 + 0.00300 \\
&= 0.9167 \\
P(\text{defective}) &= (0.9 \times 0.05) + (0.1 \times 0.2) \\
&= 0.045 + 0.02 = 0.065 \\
P(\text{defective and accepted}) &= 0.00675 + 0.00300 = 0.00975 \\
P(\text{accepted} \mid \text{defective}) &= \frac{0.00975}{0.065} = 0.15 \\
P(\text{defective} \mid \text{accepted}) &= \frac{0.00975}{0.9167} = 0.010636 \\
P(\text{machine OK and accepted}) &= 0.82935 + 0.00675 = 0.8361 \\
P(\text{machine OK} \mid \text{accepted}) &= \frac{0.8361}{0.9167} = 0.9121 \\
P(\text{machine OK and rejected}) &= 0.02565 + 0.03825 = 0.0639 \\
P(\text{rejected}) &= 1 - P(\text{accepted}) = 0.0833 \\
P(\text{machine OK} \mid \text{rejected}) &= \frac{0.0639}{0.0833} = 0.7671
\end{aligned}
$$

Expected Monetary Value (EMV)

For a single event

$$EMV = P(Event) \times \text{ Monetary value of Event}$$

The expected monetary value of a project with several possible outcomes is

$$EMV = \sum P(\text{Event}) \times \text{ Monetary value of Event}$$

where the sum is over all possible events.

Optimal Decisions

Best decision $\longleftrightarrow$ Largest EVM

Example: launch of a new product

- Three options
  - Direct
  - Internet
  - Licence

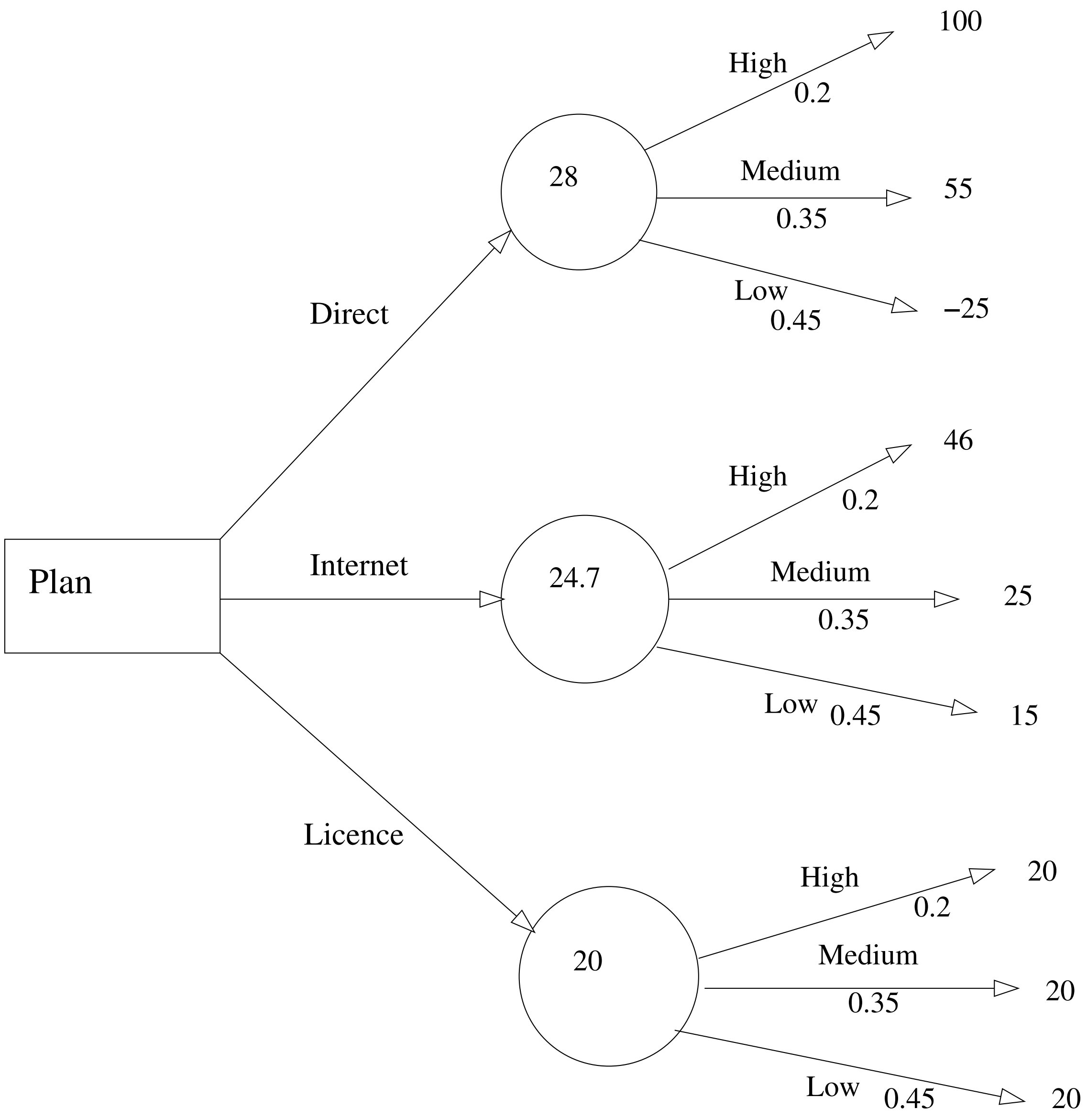
Decision Trees

High, medium or low chance of success with probabilities 0.2, 0.35 and 0.45 respectively

Likely profits:

	High	Medium	Low
Direct	100	55	-25
Internet	46	25	15
Licence	20	20	20

What should the company do?


Plan
Direct
Internet
Licence
28
High
0.2
100
Medium
0.35
55
Low
0.45
−25
24.7
High
0.2
46
Medium
0.35
25
Low
0.45
15
20
High
0.2
20
Medium
0.35
20
Low
0.45
20