Chapter 6

Decision Making Using Probability

Outline

- Conditional Probability
- Tree Diagrams
- Optimal Decisions

Conditional Probability

If we have two events A and B then

$$P(A|B)$$

is the probability of A given that B has occurred.

Utility companies – forecast periods of high demand:

$$P(\text{High demand}|\text{air temperature is below normal}) = 0.6$$
$$P(\text{High demand}|\text{air temperature is normal}) = 0.2$$
$$P(\text{High demand}|\text{air temperature is above normal}) = 0.05.$$

Conditional Probability

General formula:

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

Example

Sales of CD singles at a local outlet:

	< 30	$30 - 50$	$50+$
Male	0.275	0.125	0.025
Female	0.325	0.175	0.075

From this table, we can calculate

$$\begin{aligned} P(\text{Male}) &= P(\text{Male and } < 30) + P(\text{Male and } 30-50) \\ &\qquad + P(\text{Male and } 50+) \\ &= 0.275 + 0.125 + 0.025 = 0.425 \end{aligned}$$

and

$$\begin{aligned} P(\text{Female}) &= P(\text{Female and } < 30) + P(\text{Female and } 30-50) \\ &\qquad + P(\text{Female and } 50+) \\ &= 0.325 + 0.175 + 0.075 = 0.575. \end{aligned}$$

Also, the age distribution of the customers is

$$\begin{aligned} P(<30) &= Pr(\text{Male and } < 30) + Pr(\text{Female and } < 30) \\ &= 0.275 + 0.325 = 0.6 \end{aligned}$$

$$\begin{aligned} P(30-50) &= Pr(\text{Male and } 30-50) + Pr(\text{Female and } 30-50) \\ &= 0.125 + 0.175 = 0.3 \end{aligned}$$

$$\begin{aligned} P(50+) &= Pr(\text{Male and } 50+) + Pr(\text{Female and } 50+) \\ &= 0.025 + 0.075 = 0.1. \end{aligned}$$

Also

$$P(\text{Male}|30-50) = \frac{P(\text{Male and } 30-50)}{P(30-50)} = \frac{0.125}{0.3} = 0.4167$$

$$P(\text{Female}|30-50) = 1 - P(\text{Male}|30-50) = 1 - 0.4167 = 0.5833$$

and

$$P(<30|\text{Male}) = \frac{P(\text{Male and } < 30)}{P(Male)} = \frac{0.275}{0.425} = 0.6471$$

$$P(30-50|\text{Male}) = \frac{P(\text{Male and } 30-50)}{P(Male)} = \frac{0.125}{0.425} = 0.2941$$

$$\begin{aligned} P(50+|\text{Male}) &= 1 - P(<30|\text{Male}) - P(30-50|\text{Male}) \\ &= 1 - 0.6471 - 0.2941 = 0.588. \end{aligned}$$

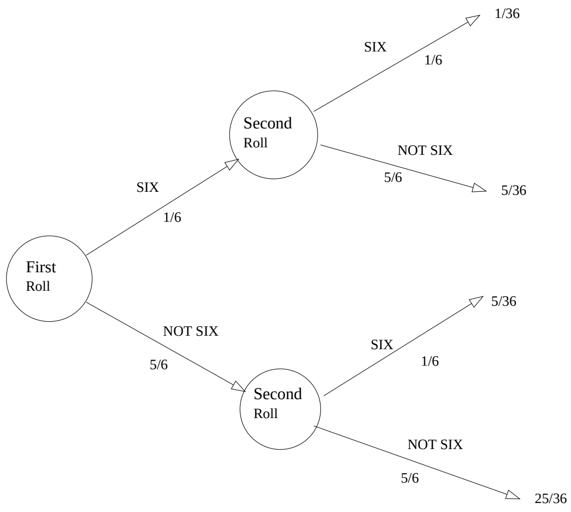
Tree Diagrams

- Experiment with multiple outcomes
- Represent each experiment by a circle
- Branches from it represent outcomes
- Each outcome has a probability associated with it

Tree Diagrams

Consider the probability of throwing two consecutive 6's on a die.

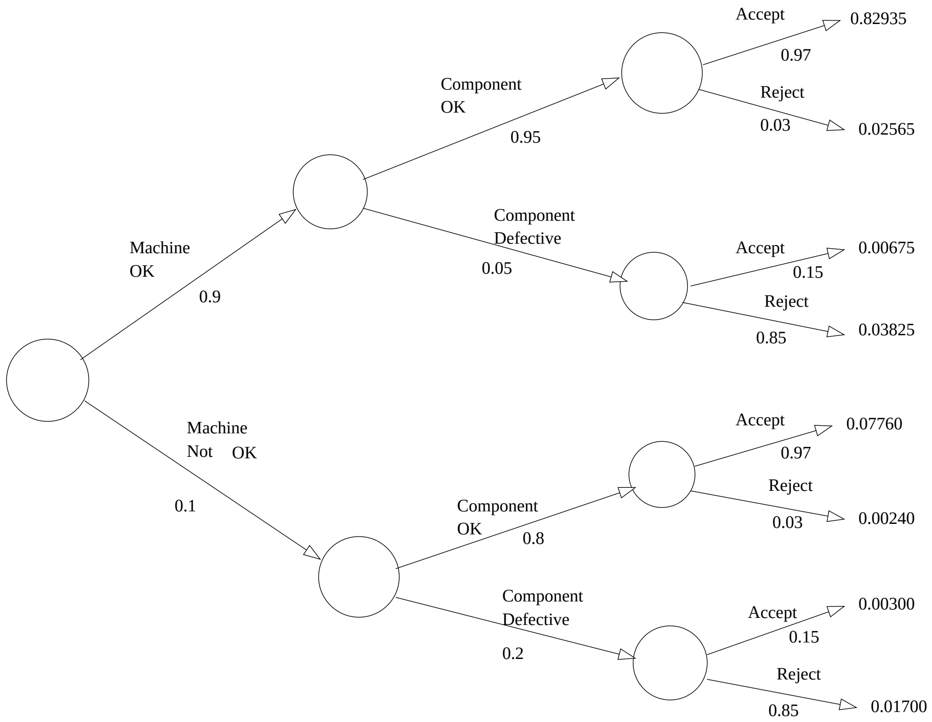
$$P(\text{Six and Six}) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$



Example

- A machine produces components. The machine may be OK or not.
- The components may be defective or not.
- The components are tested and may be accepted or rejected.

$$\begin{aligned}
P(\text{Machine OK}) &= 0.9 \\
P(\text{Component OK} \mid \text{Machine OK}) &= 0.95 \\
P(\text{Component OK} \mid \text{Machine faulty}) &= 0.8 \\
P(\text{Accept component} \mid \text{Component OK}) &= 0.97 \\
P(\text{Accept component} \mid \text{Component defective}) &= 0.15
\end{aligned}$$



Example ctd

$$
\begin{aligned}
P(\text{accepted}) &= 0.82935 + 0.00675 + 0.07760 + 0.00300 \\
&= 0.9167 \\
P(\text{defective}) &= (0.9 \times 0.05) + (0.1 \times 0.2) \\
&= 0.045 + 0.02 = 0.065 \\
P(\text{defective and accepted}) &= 0.00675 + 0.00300 = 0.00975 \\
P(\text{accepted} \mid \text{defective}) &= \frac{0.00975}{0.065} = 0.15 \\
P(\text{defective} \mid \text{accepted}) &= \frac{0.00975}{0.9167} = 0.010636 \\
P(\text{machine OK and accepted}) &= 0.82935 + 0.00675 = 0.8361 \\
P(\text{machine OK} \mid \text{accepted}) &= \frac{0.8361}{0.9167} = 0.9121 \\
P(\text{machine OK and rejected}) &= 0.02565 + 0.03825 = 0.0639 \\
P(\text{rejected}) &= 1 - P(\text{accepted}) = 0.0833 \\
P(\text{machine OK} \mid \text{rejected}) &= \frac{0.0639}{0.0833} = 0.7671
\end{aligned}
$$

Expected Monetary Value (EMV)

For a single event

$$EMV = P(Event) \times \text{ Monetary value of Event}$$

The expected monetary value of a project with several possible outcomes is

$$EMV = \sum P(\text{Event}) \times \text{ Monetary value of Event}$$

where the sum is over all possible events.

Optimal Decisions

Best decision $\longleftrightarrow$ Largest EVM

Example: launch of a new product

- Three options
  - Direct
  - Internet
  - Licence

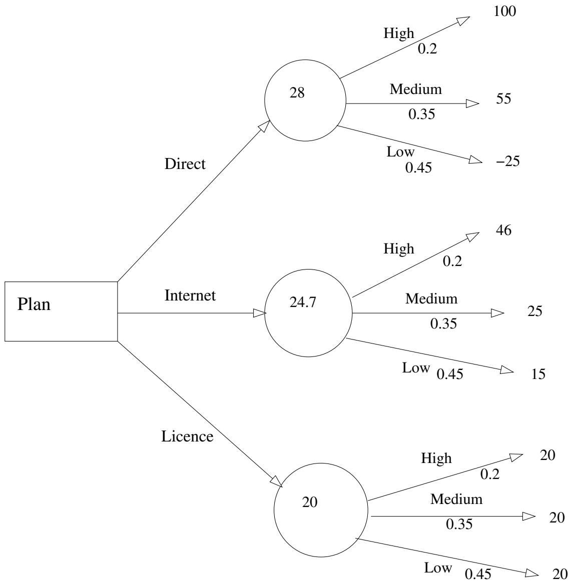
Decision Trees

High, medium or low chance of success with probabilities 0.2, 0.35 and 0.45 respectively

Likely profits:

	High	Medium	Low
Direct	100	55	-25
Internet	46	25	15
Licence	20	20	20

What should the company do?



Plan
Direct
Internet
Licence
28
24.7
20
High
0.2
Medium
0.35
Low
0.45
100
55
−25
46
25
15
20
20
20