Chapter 5

Introduction to Probability

Definitions

- Event
- $P(Event)$
- Expressed as a fraction, decimal number or percentage

$$P(Rain) = \frac{1}{20} = 0.05 = 5\%$$

Probability

- Business world is full of uncertainty
- Helps to quantify uncertainty
- Aids in decision making

Probability scale

- $0 \leq prob \leq 1$
- $prob = 0 \longleftrightarrow$ event can't happen
- $prob = 1 \longleftrightarrow$ event must happen

Definitions

- Mutually Exclusive Events
- Independent Events

Measuring Probability

- Classical
- Frequentist
- Subjective/Bayesian

Classical

- All outcomes are equally likely
- 10 possible outcomes $\longleftrightarrow P(\text{each outcome}) = \frac{1}{10}$
- General case

$$P(\text{Event}) = \frac{\text{Number of outcomes in which event occurs}}{\text{Number of possible outcomes}}$$

Frequentist

- All events are not equally likely
- Conduct experiment a **large** number of times
- $prob \longleftrightarrow proportion$ of times $Event$ occurs:

$$P(\text{Event}) = \frac{\text{Number of times event occurs}}{\text{Total number of times experiment done}}$$

Subjective/Bayesian

- Personal belief
- Can be calibrated using betting arguments
- Consider $prob = P(\text{Newcastle win next game})$. What is your value?

  How much would you pay for a bet which gives you £1 if Newcastle wins next game?

How much would you pay for a bet which gives you £1 if Newcastle wins next game?

If $prob = 80\%$ then you would pay up to 80p

How much would you need to take a bet in which you give £1 if Newcastle wins next game?

If $prob = 80\%$ then you would need to take at least 80p

$\longrightarrow$ 80p is a fair price

$\longrightarrow$ $prob = 80\%$ is your probability

Multiplication Law

- For *independent* events E_1 and E_2

$$P(E_1 \text{ and } E_2) = P(E_1) \times P(E_2)$$

Addition Law

- For two events E_1 and E_2

$$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) - P(E_1 \text{ and } E_2)$$

- If E_1 and E_2 are *mutually exclusive* then

$$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2)$$

as $P(E_1 \text{ and } E_2) = 0$

Example

- A building has three rooms.
- Each room has two separate electric lights.
- Probability of 0.1 that a given light will have failed.
- All lights are independent.

Find the probability that there is at least one room in which both lights have failed.

For a given room, the probability that it is not true that both lights have failed, that is the probability that at least one of the two lights is working, is

$$1 - 0.01 = 0.99.$$

Solution

For a given light, the probability that it has failed is 0.1.

For a given room, the probability that *both* lights have failed is

$$0.1 \times 0.1 = 0.01.$$

The probability that at least one light is working in every one of the three rooms (that is, in Room A *and* in Room B *and* in Room C) is

$$0.99 \times 0.99 \times 0.99 = 0.99^3 = 0.970299.$$

The probability that there is at least one room in which both lights have failed (that is the probability that it is not true that there is at least one light working in every room) is

$$1 - 0.970299 = 0.029701$$

or just under 3%.

N.B. We also can obtain this answer by extending the addition law to cover three events. Let A, B, C be the events "both lights have failed in Room A," " both lights have failed in Room B," "both lights have failed in Room C." We can show that

$$\begin{aligned} P(A \text{ or } B \text{ or } C) &= P(A) + P(B) + P(C) \\ &\quad -P(A \text{ and } B) - P(A \text{ and } B) \\ &\quad -P(B \text{ and } C) \\ &\quad +P(A \text{ and } B \text{ and } C) \end{aligned}$$

So, the required probability is

$$\begin{aligned} P(A \text{ or } B\text{or } C) &= 0.01 + 0.01 + 0.01 \\ &\quad -(0.01 \times 0.01) - (0.01 \times 0.01) \\ &\quad -(0.01 \times 0.01) \\ &\quad +(0.01 \times 0.01 \times 0.01) \\ &= 3 \times 0.01 - 3 \times 0.0001 + 0.000001 \\ &= 0.03 - 0.0003 + 0.000001 = 0.029701. \end{aligned}$$