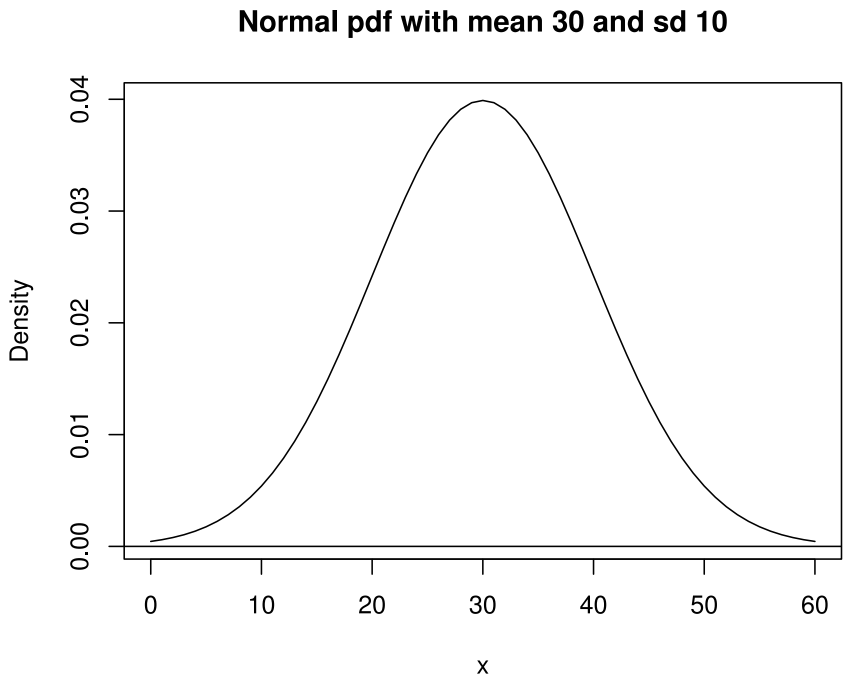


The Normal Distribution

There are four important characteristics of the normal distribution.

- It is symmetrical about it's mean, μ.
- It's mean, median and mode all coincide.
- The area under the curve is equal to 1.
- The curve extends in both direction to infinity (∞).

Outline

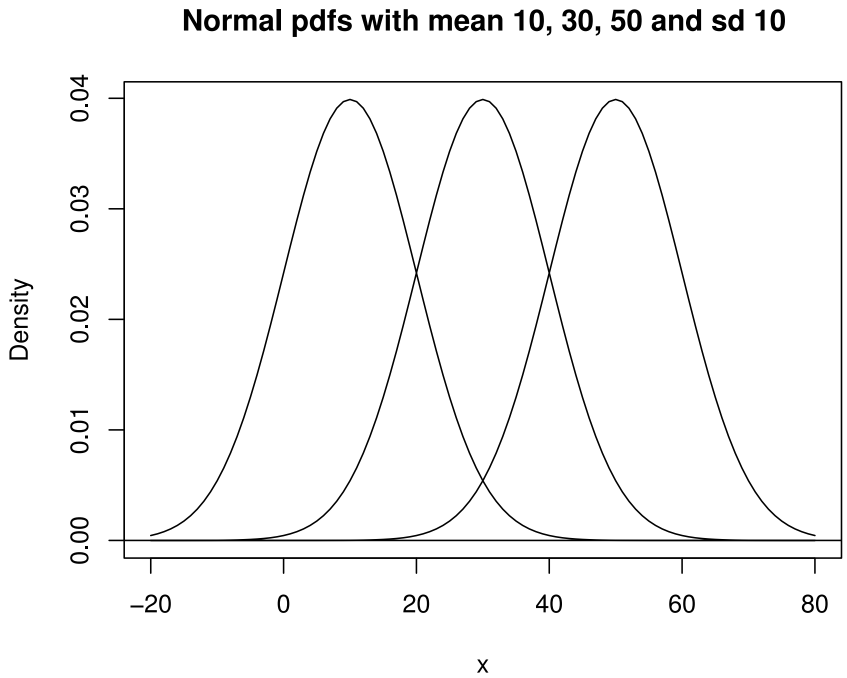
- The Normal Distribution.
- The Standard Normal Distribution.
- Calculating Probabilities

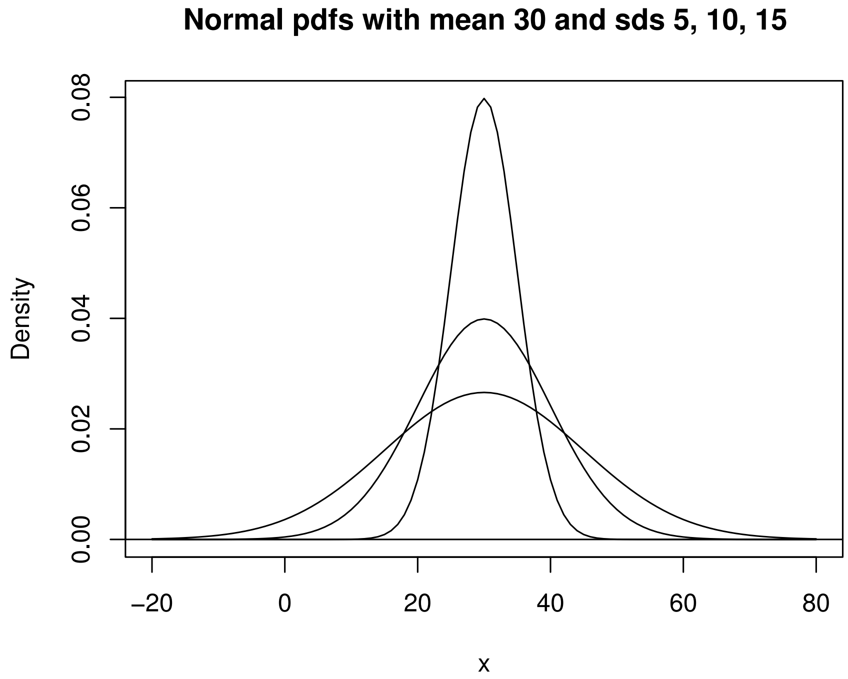
Chapter 10

The Normal Distribution

The Standard Normal Distribution

$$Z \sim N(0, 1).$$





Notation for the Normal Distribution

$$X \sim N\left(\mu, \sigma^2\right).$$

Standardising Random Variables

Any random variable X with mean, μ and variance, σ^2 can be standardised as follows

1. Take the variable X.
2. Subtract the mean, μ.
3. Divide by the standard deviation, σ.

Hence a random variable X becomes a standard normal random variable Z.

$$Z = \frac{X - \mu}{\sigma}$$

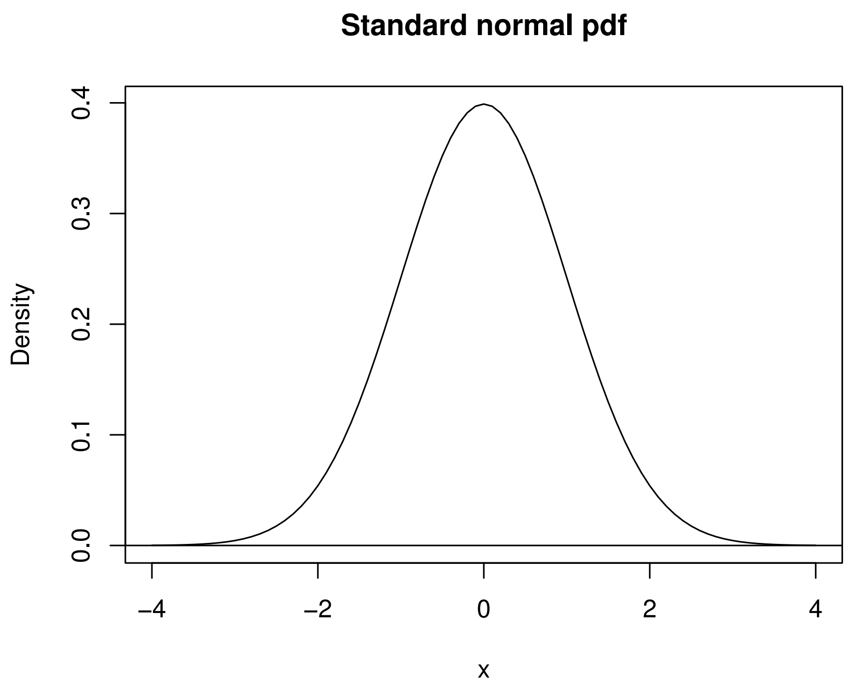
Standard Normal Distribution Tables

$P(Z < z)$, where $Z \sim N(0, 1)$.

z	−0.09	−0.08	−0.07	−0.06	−0.05	−0.04	−0.03	−0.02	−0.01	0.00
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
−1.4	0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808
−1.3	0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Calculating Probabilities under the Standard Normal Distribution

1. Find the row of interest.
2. Find the column of interest.
3. Read off the probability.



Example

The weights of the packages are independent. Find the probability that the total weight of 10 packages is less than 42kg.

Solution

Let the total weight be T.

$$T \sim N(10 \times 4,\ 10 \times 0.6^2)$$

That is

$$T \sim N(40,\ 3.6)$$

$$\Pr(T < 42) = \Pr\left(\frac{T-40}{\sqrt{3.6}} < \frac{42-40}{\sqrt{3.6}}\right) = \Pr(Z < 1.054)$$

From tables $\Pr(Z < 1.05) = 0.8531$ and $\Pr(Z < 1.06) = 0.8554$ so $\Pr(Z < 1.054) \approx 0.8531 + 0.4 \times (0.8554 - 0.8531) = 0.8540$.

So the probability that the total weight is less than 42 kg is 0.8540.

Example

The weights of packages have a normal distribution with mean 4kg and standard deviation 0.6kg. Find the probability that a package weighs more than 4.75kg.

Solution

$$X \sim N(4, 0.6^2)$$

$$\Pr(X > 4.75) = \Pr\left(\frac{X-4}{0.6} > \frac{4.75-4}{0.6}\right) = \Pr(Z > 1.25)$$

From tables $\Pr(Z < 1.25) = 0.8944$ so $\Pr(Z > 1.25) = 1 - 0.8944 = 0.1056$.

So the probability that a package weighs more than 4.75 kg is 0.1056.

Example

Find an approximate value for the probability that, out of a batch of 80 packages, fewer than 10 weigh more than 4.75kg.

<u>Solution</u>

The probability that a package weighs more than 4.75 kg is 0.1056. Let Y be the number in the batch which weigh more than 4.75kg. Then

$$Y \sim \text{Bin}(80, 0.1056)$$

Now $np = 80 \times 0.1056 = 8.448$ and $n(1-p) = 80 \times (1 - 0.1056) = 80 \times 0.8944 = 71.552$. Therefore we can use a normal approximation with $\mu = np = 8.448$ and $\sigma^2 = np(1-p) = 7.556$. The standard deviation is $\sqrt{7.556} = 2.749$.

The probability that fewer than 10 packages weigh more than 4.75kg is

$$\Pr\left(Z < \frac{9.5 - 8.448}{2.749}\right) = \Pr(Z < 0.3827)$$

From tables $\Pr(Z < 0.38) = 0.6480$ and $\Pr(Z < 0.39) = 0.6517$ so $\Pr(Z < 0.3827) = 0.6480 + 0.27 \times (0.6517 - 0.6480) = 0.6470$.

The required probability is approximately 0.647.