1. A factory contains six machines. The probability that one of these machines develops a fault in a week's operation is 0.2. Each machine behaves independently of the others. Find the probability that at least four of the machines develop faults in a week.

   Solution

   Let $\Pr(i)$ be the probability that i machines develop faults. Then the probability that at least four develop faults is

$$P = \Pr(4) + \Pr(5) + \Pr(6).$$

   Now

$$\Pr(i) =^6 C_i \times 0.2^i \times 0.8^{6-i}.$$

   It is easy to see that

$${}^nC_i =^n C_{n-i}$$

   so

$$\begin{aligned} {}^6C_4 &= {}^6C_2 = \frac{6\times 5}{2\times 1} = 15 \\ {}^6C_5 &= {}^6C_1 = 6 \\ {}^6C_6 &= {}^6C_0 = 1 \end{aligned}$$

   and

$$\begin{aligned} P &= 15 \times 0.2^4 \times 0.8^2 + 6 \times 0.2^5 \times 0.8 + 1 \times 0.2^6 \\ &= 0.01536 + 0.001536 + 0.000064 \\ &= 0.017 \end{aligned}$$

2. The weights of packets produced by a machine follow a normal distribution. The mean and standard deviation of the distribution can be altered by adjusting the machine. It is required to adjust the machine so that the proportion of packets with weights less than 490g is 5% and the proportion of packets with weights greater than 505g is 5%. Find the mean and standard deviation which must be used.

   Solution

   The upper 95% point of the standard normal distribution is 1.6449 so 490 and 505 must be respectively 1.6449 standard deviations below and above the mean. So the mean must be $(490 + 505)/2 = 497.5$ and the standard deviation must be $(505 - 497.5)/1.6449 = 4.56$.

3. A company called "Prune," which currently markets personal computers, will need to choose between at most three options in three years' time:

   - a_1 – continue to market its current machine (called Prunejuice);
   - a_2 – market an improved version of Prunejuice instead (called Pruneplus);
   - a_3 – market a much more powerful machine instead (called Superprune).

   Prune can choose between a_1, a_2, a_3 but can not implement more than one of them.

   A machine can be marketed only if it has been researched and developed (R & D) successfully. The event that a R & D programme could be successful for Pruneplus and the event that it would be successful for Superprune are considerd independent with current respective probabilities 0.9 and 0.6. If neither of the new machines has been successfully researched and developed in the next three years then Prune would be forced to choose option a_1 – to market its current machine. Prune now needs to choose whether to:

(a) R & D neither machine (decision d_1) at a cost of \$ 0.

(b) R & D Pruneplus only (decision d_2) at a cost of \$ 3,000,000.

(c) R & D Superprune only (decision d_3) at a cost of \$ 5,000,000.

(d) R & D both Pruneplus and Superprune (decision d_4) at a cost of \$ 8,000,000.

Given successful R & D, Prune expects to make \$ 2 000 000 net profit from Prune juice, \$ 10 000 000 net profit from Pruneplus and \$ 18 000 000 net profit from Superprune. Draw a decison tree representing Prune's *current* decision problem and advise Prune on its best plan of action given that the aim is to maximise the expected net profit less costs.

<u>Solution</u>

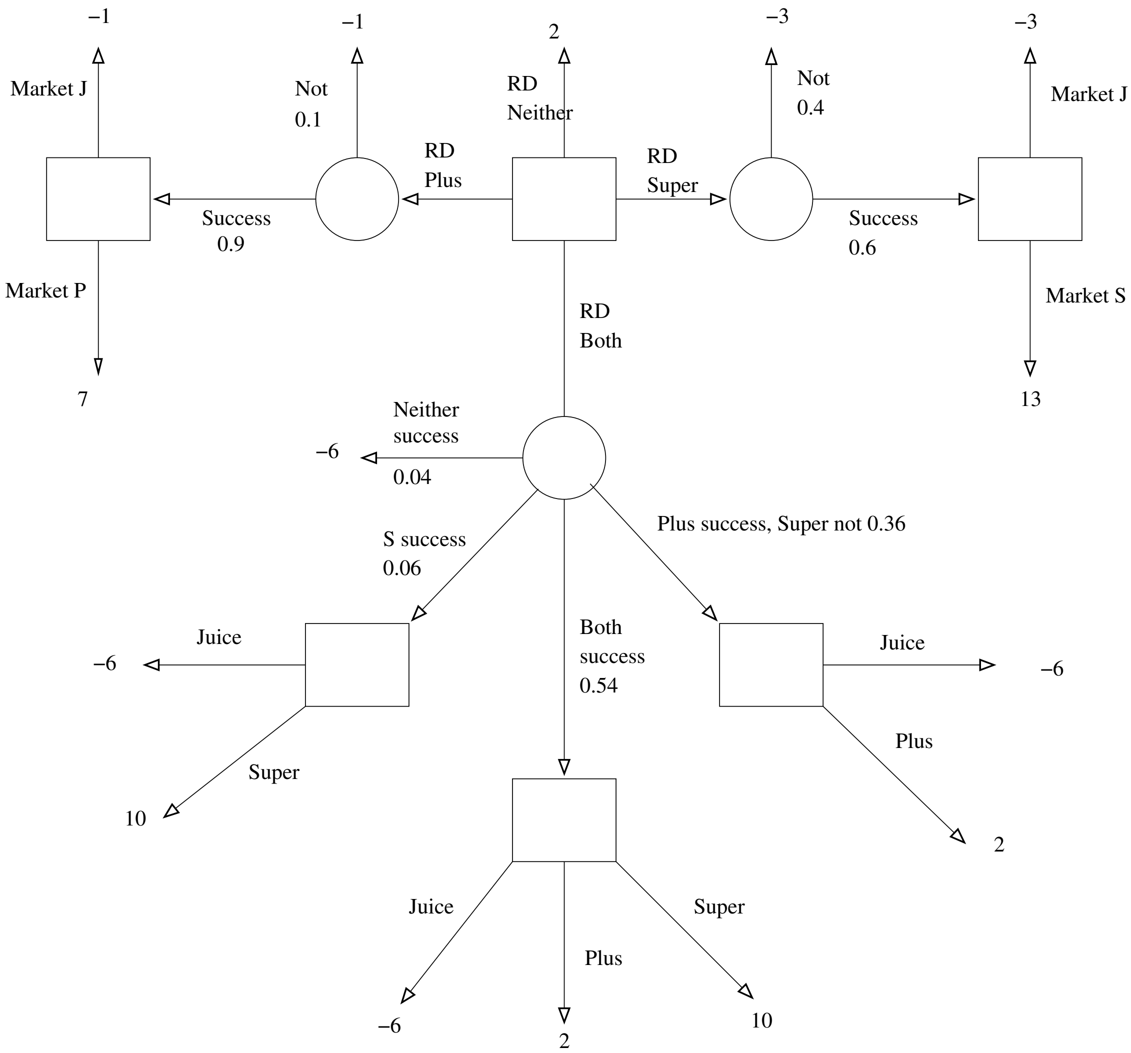
R & D:

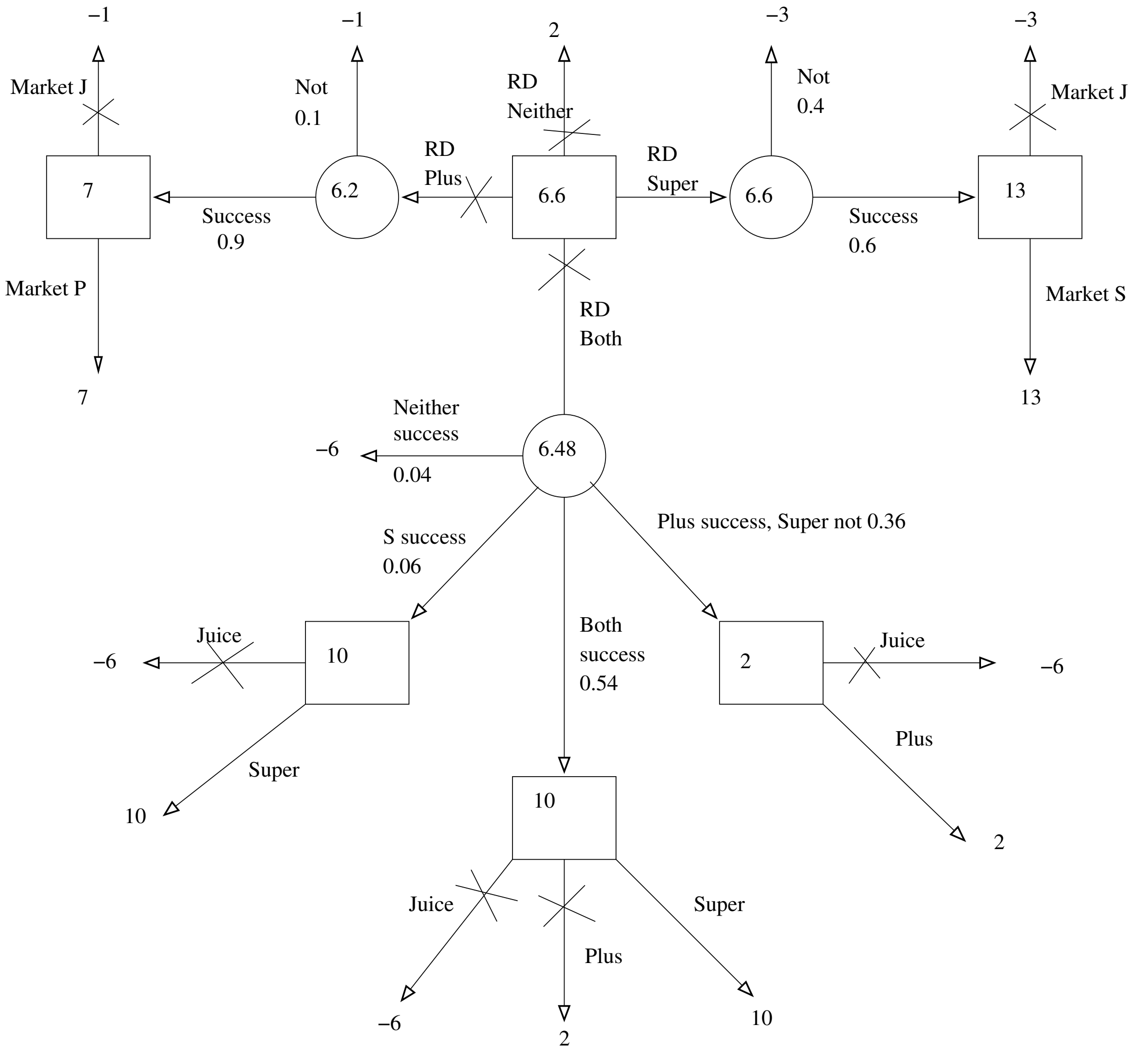
The probability that both Pruneplus and Superprune are successful is $0.9 \times 0.6 = 0.54$.

The probability that Pruneplus is a success and Superprune is not is $0.9 \times 0.4 = 0.36$.

The probability that Superprune is a success but Pruneplus is not is $0.1 \times 0.6 = 0.06$.

The probability that neither Superprune nor Pruneplus is a success is $0.1 \times 0.4 = 0.04$.





So, we R & D Superprune.

If it is a success, we market Superprune.

Otherwise we market Prunejuice.