MAS187 Exam 2005

Solutions

Question A1

Mean

$$\sum_{i=1}^{12} x_i = 191$$

So the sample mean is

$$\bar{x} = \frac{191}{12} = \underline{15.917}$$

Question A1

Median

The data in increasing order are

8	10	10	12	12	15
16	18	20	21	24	25

So the median is

$$\frac{15+16}{2} = 15.5$$

Question A1

$$\sum_{i=1}^{12} x_i^2 = 3399$$

Hence the sample variance is

$$\begin{aligned} s^2 &= \frac{1}{11}\{3399 - 12\bar{x}^2\} \\ &= \frac{1}{11}\{3399 - 3040.083\} \\ &= 32.629 \end{aligned}$$

So the sample standard deviation is

$$s = \sqrt{s^2} = \underline{5.712}$$

Question A2 (a)

Number of cars sold in a day is X.

$$X \sim \mathrm{Po}(3),$$

a Poisson distribution with mean $\lambda = 3$.

Question A2 (b)

Expected number: $\mathrm{E}(X) = \underline{3}$,

Variance: $\mathrm{var}(X) = 3$,

Standard deviation: $\mathrm{std.dev.}(X) = \sqrt{3} = \underline{1.732}$.

Question A2 (c)

Probability of no cars sold on a particular day:

$$\Pr(X = 0) = \frac{e^{-\lambda}\lambda^0}{0!} = e^{-\lambda} = e^{-3} = \underline{0.0498}.$$

Question A3 (a)

Weekly demand $X \sim N(2000,\ 500^2)$.

The probability of running out of stock is

$$\begin{aligned}
\Pr(X > 2750) &= \Pr\left(\frac{X - 2000}{500} > \frac{2750 - 2000}{500}\right) \\
&= \Pr(Z > 1.5) \\
&= 1 - \Pr(Z < 1.5) \\
&= 1 - 0.9332 = \underline{0.0668}
\end{aligned}$$

Question A3 (b)

We require $\Pr(Z > z) = 0.06$.

That is $\Pr(Z < z) = 0.94$.

From tables
$\Pr(Z < 1.55) = 0.9394$ and $\Pr(Z < 1.56) = 0.9406$.

Now $0.9406 - 0.9394 = 0.0012$
and $0.94 - 0.9394 = 0.0006 = 0.5 \times 0.0012$.

So $\Pr(Z < 1.555) \approx 0.94$.

If

$$\frac{x - 2000}{500} = 1.555$$

then

$$x - 2000 = 500 \times 1.555$$

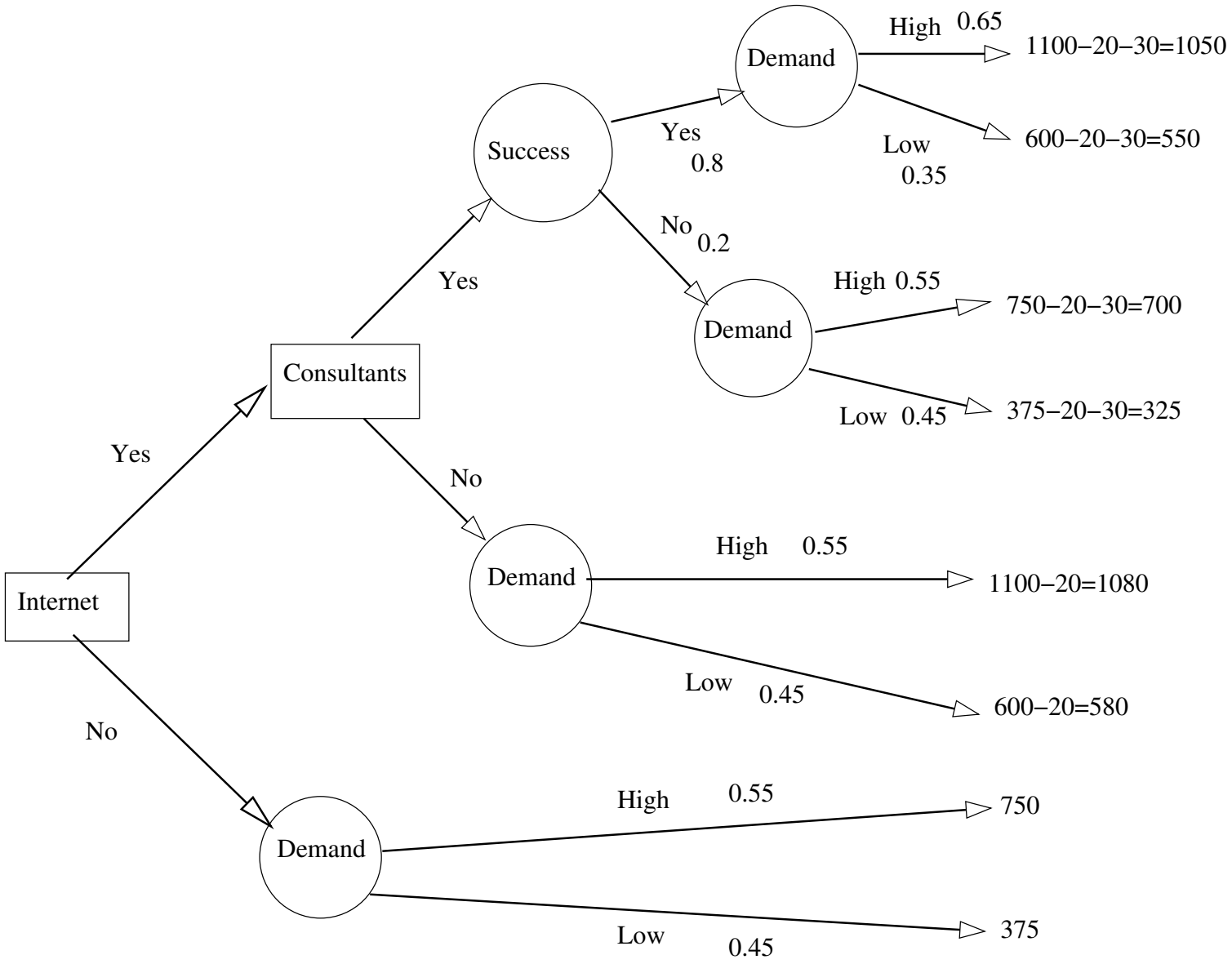
and

$$x = 2000 + 500 \times 1.555 = 2777.5.$$

The company must hold $\underline{2777.5}$ gallons.

Question B6 (a)

$$\begin{aligned} \Pr(\text{High demand given no Internet}) &= 0.55 \\ \Pr(\text{Successful campaign}) &= 0.8 \\ \Pr(\text{High demand given successful campaign}) &= 0.65 \end{aligned}$$


Internet
Yes
Consultants
Yes
Success
Yes
0.8
Demand
High 0.65
1100−20−30=1050
Low
0.35
600−20−30=550
No 0.2
Demand
High 0.55
750−20−30=700
Low 0.45
375−20−30=325
No
Demand
High 0.55
1100−20=1080
Low 0.45
600−20=580
No
Demand
High 0.55
750
Low 0.45
375

Question B6 (b)

Values in thousands of pounds. Monetary value V.

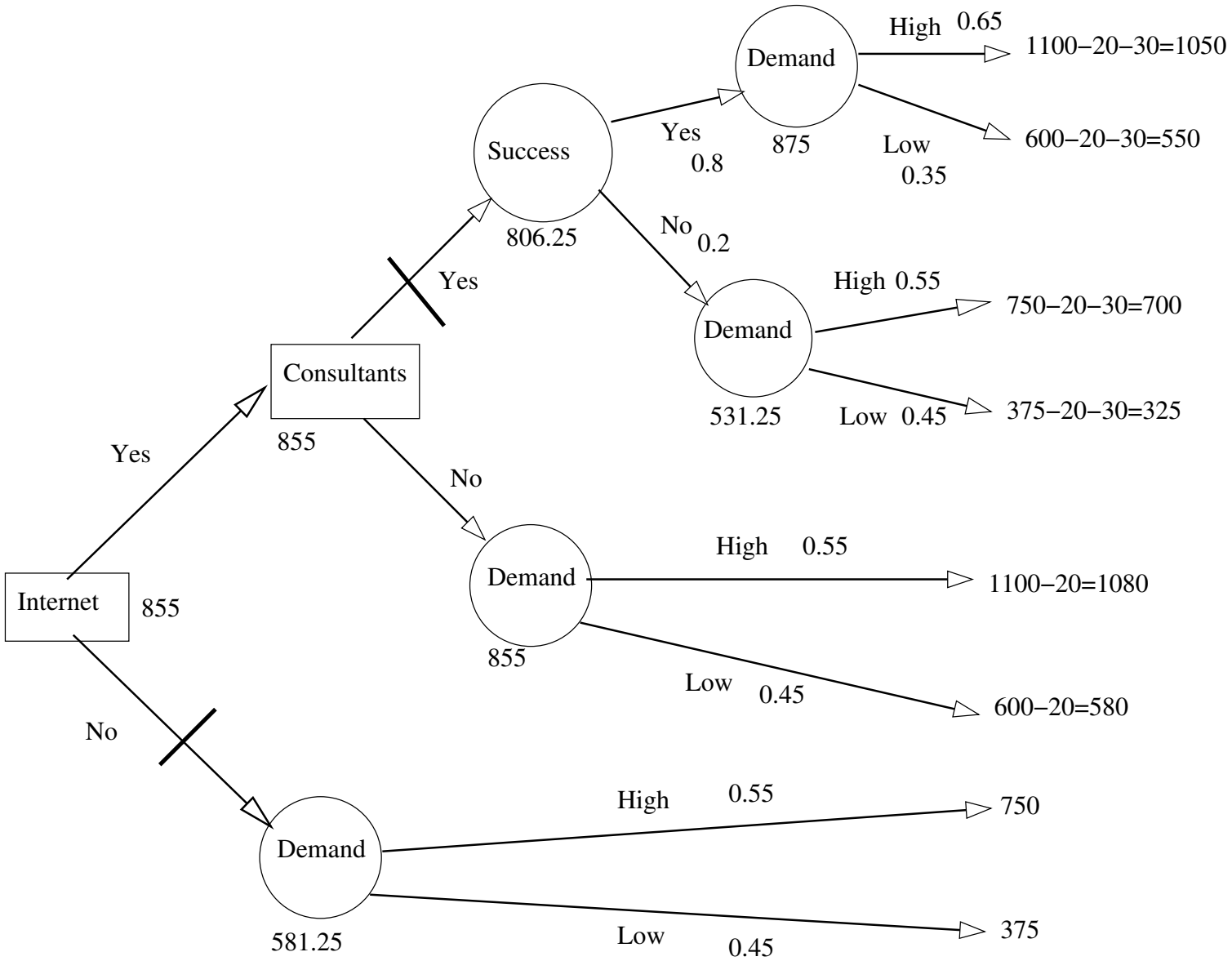
$$\begin{aligned}
&\mathrm{E}(V|\text{Internet},\ \text{Consultants},\ \text{Success}) \\
&\qquad = 0.65 \times 1050 + 0.35 \times 550 \ = \ 875 \\
&\mathrm{E}(V|\text{Internet},\ \text{Consultants},\ \text{Failure}) \\
&\qquad = 0.55 \times 700 + 0.45 \times 325 \ = \ 531.25 \\
&\mathrm{E}(V|\text{Internet},\ \text{Consultants}) \\
&\qquad = 0.8 \times 875 + 0.2 \times 531.25 \ = \ 806.25
\end{aligned}$$

$$\begin{aligned}
&\mathrm{E}(V|\text{Internet, No Consultants}) \\
&\quad = 0.55 \times 1080 + 0.45 \times 580 \;=\; 855
\end{aligned}$$

Hence we prefer not to employ the consultants.

$$\begin{aligned}
&\mathrm{E}(V|\text{No Internet}) \\
&= 0.55 \times 750 + 0.45 \times 375 \;=\; 581.25
\end{aligned}$$

Hence the optimal decision is to set up the Internet site but not to employ the consultants.


Internet
855
Yes
No
Consultants
855
Yes
No
Success
806.25
Yes
0.8
No
0.2
Demand
875
High
0.65
1100−20−30=1050
Low
0.35
600−20−30=550
Demand
531.25
High
0.55
750−20−30=700
Low
0.45
375−20−30=325
Demand
855
High
0.55
1100−20=1080
Low
0.45
600−20=580
Demand
581.25
High
0.55
750
Low
0.45
375