Solutions to Exercises 8

1. (a) Assuming that calls are answered independently, $X \sim Bin(20, 0.85)$.

   (b) The mean and variance are

$$E(X) = np = 20 \times 0.85 = 17$$
$$Var(X) = np(1-p) = 20 \times 0.85 \times 0.15 = 2.55$$

   and so

$$SD(X) = \sqrt{Var(X)} = \sqrt{2.55} = 1.597.$$

   (c)

$$\begin{aligned} P(X=9) &= {}^{n}\text{C}_{r} p^{r} (1-p)^{n-r} \\ &= {}^{20}\text{C}_{9} \times 0.85^{9} \times (1-0.85)^{20-9} \\ &= \frac{20!}{9! \times 11!} \times 0.85^{9} \times 0.15^{11} \\ &= \frac{20 \times 19 \times \cdots \times 12}{9 \times 8 \times \cdots \times 1} \times 0.85^{9} \times 0.15^{11} \\ &= 167960 \times 0.85^{9} \times 0.15^{11} \\ &= 0.0000336. \end{aligned}$$

   (d)

$$\begin{aligned} P(X<2) &= P(X=0) + P(X=1) \\ &= {}^{20}\text{C}_{0} \times 0.85^{0} \times (1-0.85)^{20-0} + {}^{20}\text{C}_{1} \times 0.85^{1} \times (1-0.85)^{20-1} \\ &= 0.15^{20} + 20 \times 0.85 \times 0.15^{19} \\ &= 3.3255 \times 10^{-17} + 3.76862 \times 10^{-15} \\ &= 3.802 \times 10^{-15}. \end{aligned}$$

2. (a) $X \sim Po(10)$

   (b) The mean and variance are

$$E(X) = \lambda = 10$$
$$Var(X) = \lambda = 10$$

   and so

$$SD(X) = \sqrt{Var(X)} = \sqrt{10} = 3.16.$$

   (c)

$$\begin{aligned} P(X=12) &= \frac{\lambda^{r} e^{-\lambda}}{r!} \\ &= \frac{10^{12} \times e^{-10}}{12!} \\ &= 0.09478. \end{aligned}$$

(d)

$$\begin{aligned}P(X \leq 2) &= P(X=0)+P(X=1)+P(X=2)\\ &= \frac{10^0 \times e^{-10}}{0!}+\frac{10^1 \times e^{-10}}{1!}+\frac{10^2 \times e^{-10}}{2!}\\ &= e^{-10}+10e^{-10}+50e^{-10}\\ &= 0.0000454+0.0004540+0.0022700\\ &= 0.00277.\end{aligned}$$

3. Let Y be the total sales in the 5-day period.

   (a) $\mathrm{E}(Y)=5\times 2=10$. So $Y \sim \mathrm{Po}(10)$.

   (b)

$$\begin{aligned}P(Y>12) &= 1-P(Y\leq 12)\\ &= 1-\{P(Y=0)+P(Y=1)+\cdots+P(Y=12)\}\\ &= 1-\left\{\frac{e^{-10}10^0}{0!}+\frac{e^{-10}10^1}{1!}+\cdots+\frac{e^{-10}10^{12}}{12!}\right\}\\ &= 1-e^{-10}\left\{\frac{1}{0!}+\frac{10}{1!}+\frac{10^2}{2!}+\cdots+\frac{10^{12}}{12!}\right\}\\ &= 1-e^{-10}\{1+10+50+\cdots+2087.6757\}\end{aligned}$$

   (The first term $(Y=0)$ is 1. The second $(Y=1)$ is $10/1$ times the first. The third $(Y=2)$ is $10/2$ times the second and so on. Alternatively you can use Minitab).

$$\begin{aligned}P(Y>12) &= 1-e^{-10}\{17435.1965\}\\ &= 1-0.7916\\ &= 0.2084.\end{aligned}$$

4. You might find a probability tree helpful in answering this question.

   (a)

$$\begin{aligned}P(\text{compt accptd }|\text{machine OK}) &= P(\text{compt OK and accptd }|\text{machine OK})\\ &\quad +P(\text{compt defective and accptd }|\text{machine OK})\\ &= 0.95\times 0.97+0.05\times 0.15\\ &= 0.929.\end{aligned}$$

   (b)

$$\begin{aligned}P(\text{compt rejected }|\text{machine OK}) &= 1-P(\text{compt accptd }|\text{machine OK})\\ &= 1-0.929\\ &= 0.071.\end{aligned}$$

(c)

$$\begin{aligned} P(\text{compt accptd } | \text{machine not OK}) &= P(\text{compt OK and accptd } | \text{machine not OK}) \\ &\quad + P(\text{compt defective and accptd } | \text{machine not OK}) \\ &= 0.8 \times 0.97 + 0.2 \times 0.15 \\ &= 0.806. \end{aligned}$$

(d)

$$\begin{aligned} P(\text{compt rejected } | \text{machine not OK}) &= 1 - P(\text{compt accptd } | \text{machine not OK}) \\ &= 1 - 0.806 \\ &= 0.194. \end{aligned}$$

(e)

$$\begin{aligned} P(\text{2 accptd out of 5 } | \text{machine OK}) &= {}^5\text{C}_2 \times 0.929^2 \times 0.071^3 \\ &= \frac{5 \times 4}{2 \times 1} \times 0.929^2 \times 0.071^3 \\ &= 10 \times 0.00030889 \\ &= 0.0030889 \end{aligned}$$

(f)

$$\begin{aligned} P(\text{2 accptd out of 5 } | \text{machine not OK}) &= {}^5\text{C}_2 \times 0.806^2 \times 0.194^3 \\ &= \frac{5 \times 4}{2 \times 1} \times 0.806^2 \times 0.194^3 \\ &= 10 \times 0.0047432 \\ &= 0.047432 \end{aligned}$$

(g)

$$\begin{aligned} P(\text{2 accptd out of 5 \textit{and} machine OK}) &= P(\text{machine OK}) \\ &\quad \times P(\text{2 accptd out of 5 } | \text{machine OK}) \\ &= 0.9 \times 0.0030889 \\ &= 0.0027800 \end{aligned}$$

(h)

$$\begin{aligned} P(\text{2 accptd out of 5 \textit{and} machine not OK}) &= P(\text{machine not OK}) \\ &\quad \times P(\text{2 accptd out of 5 } | \text{machine not OK}) \\ &= 0.1 \times 0.047432 \\ &= 0.0047432 \end{aligned}$$

(i)

$$\begin{aligned} P(\text{2 accptd out of 5}) &= P(\text{2 accptd out of 5 \textit{and} machine OK}) \\ &\quad + P(\text{2 accptd out of 5 \textit{and} machine not OK}) \\ &= 0.0027800 + 0.0047432 \\ &= 0.0075232 \end{aligned}$$

(j)

$$
\begin{aligned}
P(\text{machine OK } | \text{2 accptd out of 5}) &= \frac{P(\text{machine OK } \textit{and} \text{ 2 accptd out of 5})}{P(\text{2 accptd out of 5})} \\
&= \frac{0.0027800}{0.0075232} \\
&= 0.3695
\end{aligned}
$$

Notice that, having seen the evidence of two components accepted out of five, the probability that the machine requires adjustment changes from 0.1 to $1 - 0.3695 = 0.6305$.