Solutions to Exercises 7

1. The number of ways of drawing 6 balls from 48 balls is

$$\begin{aligned} {}^{48}\mathrm{C}_6 &= \frac{48!}{6! \times (48-6)!} \\ &= \frac{48!}{6! \times 42!} \\ &= \frac{48 \times 47 \times 46 \times 45 \times 44 \times 43}{6 \times 5 \times 4 \times 3 \times 2 \times 1} \\ &= 12,271,512. \end{aligned}$$

   As there is only one selection that matches the 6 balls drawn, the probability of winning the jackpot in this lottery is

$$\frac{1}{12,271,512} = 0.00000008149.$$

2. This is a question about permutations as the ordering is important. The number of permutations of 4 features from 10 features is

$${}^{10}\mathrm{P}_4 = \frac{10!}{(10-4)!} = \frac{10!}{6!} = \frac{10 \times 9 \times 8 \times 7 \times 6!}{6!} = 5040.$$

   As there is only one ordering that matches my preferred ordering, the probability of choosing my preferred ordering is

$$\frac{1}{5040} = 0.0001984.$$

3. There are 10 choices for the 1st digit, and 10 choices for the second digit, and so on. Therefore the number of possible 7 digit phone numbers is

$$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10,000,000$$

   and so the probability of randomly selecting my unique telephone number is

$$\frac{1}{10,000,000} = 0.0000001.$$

4. (a) i.

$${}^{4}\mathrm{C}_0 = 1$$

   (SSSS)

   ii.

$${}^{4}\mathrm{C}_1 = 4$$

   (USSS, SUSS, SSUS, SSSU)

   iii.

$${}^{4}\mathrm{C}_2 = \frac{4 \times 3}{2 \times 1} = 6$$

   iv.

$${}^{4}\mathrm{C}_3 = \frac{4 \times 3 \times 2}{3 \times 2 \times 1} = 4$$

v.

$$^4\mathrm{C}_4 = 1$$

(b) i. $0.8 \times 0.8 \times 0.8 \times 0.8 = 0.8^4 = 0.4096$

ii. $0.2 \times 0.8 \times 0.8 \times 0.8 = 0.2 \times 0.8^3 = 0.1024$

iii. $0.2 \times 0.2 \times 0.8 \times 0.8 = 0.2^2 \times 0.8^2 = 0.0256$

iv. $0.2 \times 0.2 \times 0.2 \times 0.8 = 0.2^3 \times 0.8 = 0.0064$

v. $0.2 \times 0.2 \times 0.2 \times 0.2 = 0.2^4 = 0.0016$

(c) We need to multiply the probability of a particular sequence by the number of such sequences.

i. $1 \times 0.4096 = 0.4096$

ii. $4 \times 0.1024 = 0.4096$

iii. $6 \times 0.0256 = 0.1536$

iv. $4 \times 0.0064 = 0.0256$

v. $1 \times 0.0016 = 0.0016$

Note that these probabilities sum to 1.

(d) Let the number of unsatisfactory items be X. Then

$$\mathrm{E}(X) = 0 \times 0.4096 + 1 \times 0.4096 + 2 \times 0.1536 + 3 \times 0.0256 + 4 \times 0.0016 = 0.8.$$

(e) First we calculate $\mathrm{E}(X^2)$.

$$\begin{aligned}
\mathrm{E}(X^2) &= 0^2 \times 0.4096 + 1^2 \times 0.4096 + 2^2 \times 0.1536 + 3^2 \times 0.0256 + 4^2 \times 0.0016 \\
&= 0 \times 0.4096 + 1 \times 0.4096 + 4 \times 0.1536 + 9 \times 0.0256 + 16 \times 0.0016 \\
&= 1.28.
\end{aligned}$$

The variance is then

$$\mathrm{Var}(X) = \mathrm{E}(X^2) - [\mathrm{E}(X)]^2 = 1.28 - 0.8^2 = 0.64.$$

The standard deviation is therefore

$$\sqrt{0.64} = 0.8.$$