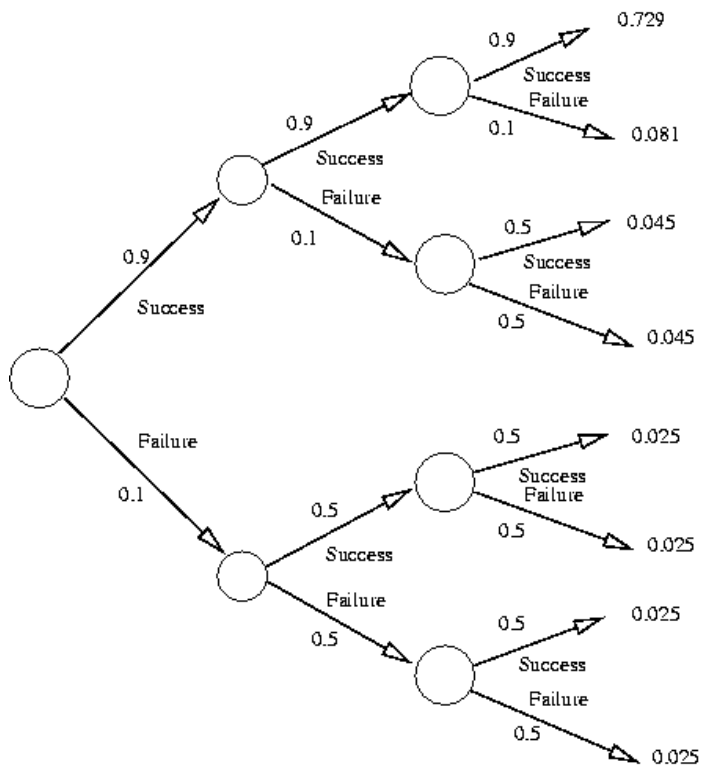# Solutions to Exercises 6

1. The probability tree is

Therefore

(a) $P(SSS) = 0.9 \times 0.9 \times 0.9 = 0.729$.
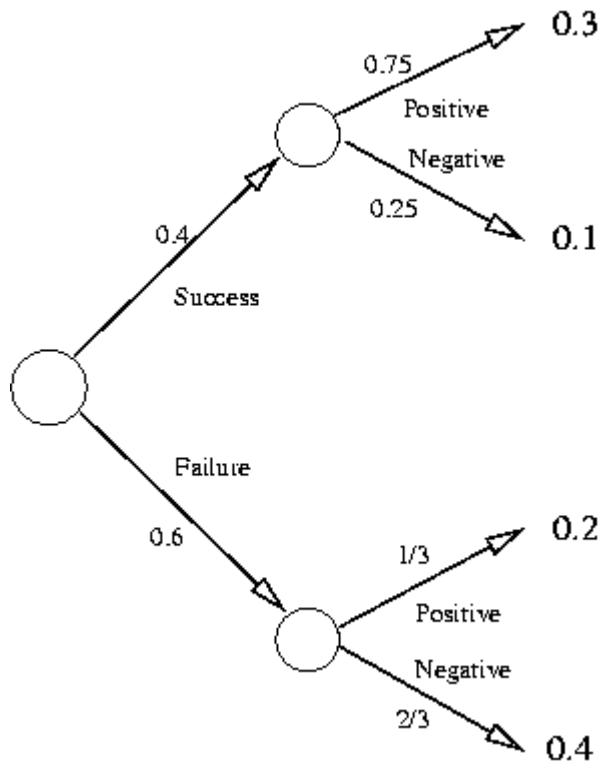
(b) $P(FSS) = 0.1 \times 0.5 \times 0.5 = 0.025$.

(c)

$$
\begin{aligned}
P(\text{only one } S) &= P(SFF \text{ or } FSF \text{ or } FFS) \\
&= 0.9 \times 0.1 \times 0.5 + 0.1 \times 0.5 \times 0.5 + 0.1 \times 0.5 \times 0.5 \\
&= 0.045 + 0.025 + 0.025 \\
&= 0.095.
\end{aligned}
$$

(d) $P(FFF) = 0.1 \times 0.5 \times 0.5 = 0.025$.

2. (a) The probability tree is as follows.

i. $P(\text{positive}) = 0.3 + 0.2 = 0.5$.

ii. $P(\text{negative}) = 0.1 + 0.4 = 1 - P(\text{positive}) = 0.5$.

iii.

$$
\begin{aligned}
P(\text{success } | \text{positive}) &= \frac{P(\text{success and positive})}{P(\text{positive})} \\
&= \frac{0.3}{0.5} = 0.6
\end{aligned}
$$

iv.

$$
\begin{aligned}
P(\text{failure } | \text{positive}) &= 1 - P(\text{success } | \text{positive}) \\
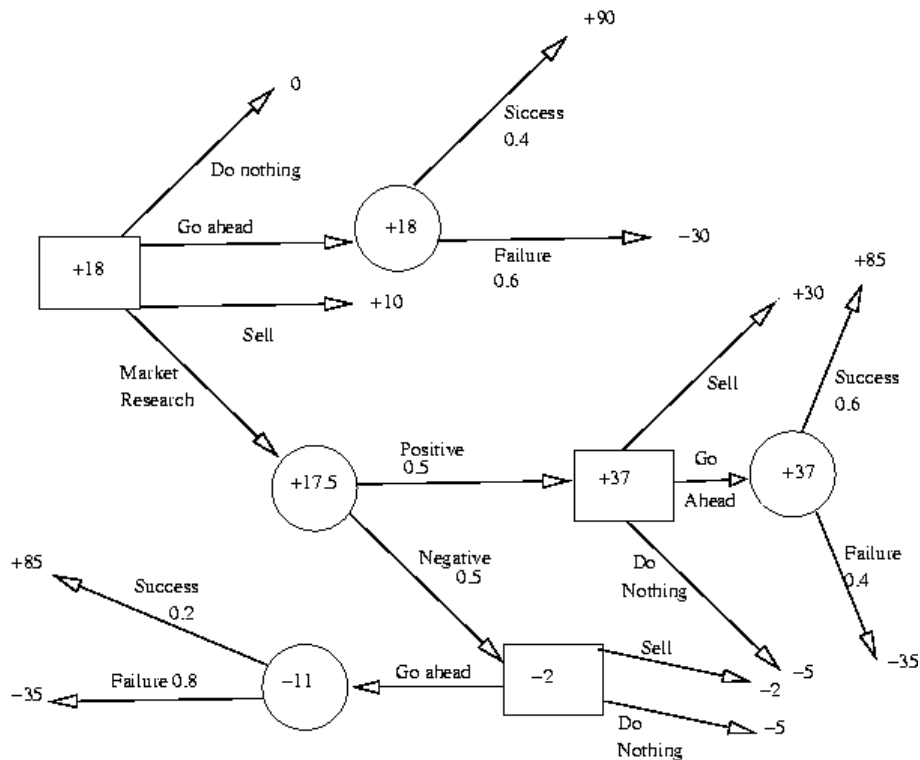&= 1 - 0.6 = 0.4
\end{aligned}
$$

v.

$$
\begin{aligned}
P(\text{success } | \text{negative}) &= \frac{P(\text{success and negative})}{P(\text{negative})} \\
&= \frac{0.1}{0.5} = 0.2
\end{aligned}
$$

vi.

$$
\begin{aligned}
P(\text{failure } | \text{negative}) &= 1 - P(\text{success } | \text{negative}) \\
&= 1 - 0.2 = 0.8
\end{aligned}
$$

(b) The tree diagram is as follows.

+90

0

Siccess
0.4

Do nothing

Go ahead   +18   −30

+18   Failure   +85
0.6   +30

+10

Sell

Market
Research   Sell   Success
0.6

Positive
0.5   Go   +37
+17.5   +37   Ahead

+85   Negative   Do   Failure
0.5   Nothing   0.4

Success
0.2   Sell
−35
Failure 0.8   −11   Go ahead   −2   −5
−35   Do   −35
Nothing   −5

(c) Market research done, at a cost of £5000. Result positive.

- Sell: $£35,000 − £5,000 = £30,000$.
- Go ahead: Value of success is $£90,000 − £5,000 = £85,000$. Value of failure is $−£30,000 − £5,000 = −£35,000$. The expected value is then

$$0.6 × £85,000 − 0.4 × £35,000 = £37,000.$$

- Do nothing more: Value is $−£5,000$.

In these circumstances she should **go ahead** because this gives the greatest expected value.

(d) Market research done, at a cost of £5000. Result negative.

- Sell: $£3,000 − £5,000 = −£2,000$.
- Go ahead: Value of success is $£90,000 − £5,000 = £85,000$. Value of failure is $−£30,000 − £5,000 = −£35,000$. The expected value is then

$$0.2 × £85,000 − 0.8 × £35,000 = −£11,000.$$

- Do nothing more: Value is $−£5,000$.

In these circumstances she should **sell** because this gives the greatest expected value (in this case, the smallest expected loss).

(e) The expected value for the option "Market Research" is therefore

$$
\begin{aligned}
& P(\text{positive}) × \text{Expected value given positive} \quad + \\
& P(\text{negative}) × \text{Expected value given negative} \\
= \quad & 0.5 × £37,000 − 0.5 × £2,000 = £17,500.
\end{aligned}
$$

(f) The expected values for the other three options are as follows.

- Do nothing: £0.
- Go ahead without market research:

$$0.4 \times £90,000 - 0.6 \times £30,000 = £18,000.$$

- Sell: £10,000

(g) The owner's best strategy is therefore to **go ahead without market research**. This gives an expected monetary value of £18,000.