Formulae and statistical tables

Combining probabilities

Addition

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

If A and B are mutually exclusive then

$$P(A \text{ or } B) = P(A) + P(B)$$

Multiplication

$$P(A \text{ and } B) = P(A)P(B \mid A) = P(B)P(A \mid B)$$

If A and B are independent then

$$P(A \text{ and } B) = P(A)P(B)$$

Binomial distribution

$$\begin{aligned} X &\sim \text{Bin}(n, p) \\ P(X = r) &= {}^nC_r p^r (1-p)^{n-r}, \qquad r = 0, 1, 2, \ldots, n \\ {}^nC_r &= \frac{n}{r} \times \frac{n-1}{r-1} \times \cdots \times \frac{n-r+1}{1} \\ \text{E}(X) &= np \\ \text{Var}(X) &= np(1-p) \end{aligned}$$

Poisson distribution

$$\begin{aligned} X &\sim \text{Po}(\lambda) \\ P(X = r) &= \frac{\lambda^r e^{-\lambda}}{r!}, \qquad r = 0, 1, 2, \ldots \\ \text{E}(X) &= \lambda \\ \text{Var}(X) &= \lambda \end{aligned}$$

Exponential distribution

$$
\begin{aligned}
X &\sim \mathrm{Exp}(\lambda) \\
f(x) &= \begin{cases} 0 & (x < 0) \\ \lambda e^{-\lambda x} & (x \geq 0) \end{cases} \\
P(X < x) &= \begin{cases} 0 & (x < 0) \\ 1 - e^{-\lambda x} & (x \geq 0) \end{cases}
\end{aligned}
$$

Normal distribution

$$
\begin{aligned}
X &\sim N(\mu,\ \sigma^2) \\
Z &= \frac{X - \mu}{\sigma} \\
Z &\sim N(0, 1)
\end{aligned}
$$

Sample statistics

Lower quartile:

$$Q_1 = \frac{(n+1)}{4}\text{th smallest observation.}$$

Median:

$$Q_2 = \frac{(n+1)}{2}\text{th smallest observation.}$$

Upper quartile:

$$Q_3 = \frac{3(n+1)}{4}\text{th smallest observation.}$$

Sample mean:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Sample variance:

$$
\begin{aligned}
s^2 &= \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 \\
&= \frac{1}{n-1}\left\{\sum_{i=1}^{n} x_i^2 - n(\bar{x})^2\right\}
\end{aligned}
$$

Confidence intervals for the mean

Case 1: Population variance known

If the population variance σ^2 is *known*, then a confidence interval for μ is given by

$$\bar{x} \quad \pm \quad z \times \sqrt{\sigma^2/n},$$

where z is a critical value form the standard Normal distribution (i.e. $z =$ 1.645, 1.96 and 2.576 for a 90%, 95% and 99% interval).

Case 2: Population variance unknown

If the population variance σ^2 is *unknown*, then a confidence interval for μ is given by

$$\bar{x} \quad \pm \quad t \times \sqrt{s^2/n},$$

where t is a critical value form the t distribution with $\nu = n - 1$ degrees of freedom.

Hypothesis tests

Type of test	Test statistic	Other comments
Tests for one mean		
σ known	$z = \frac{\lvert\bar{x}-\mu\rvert}{\sqrt{\sigma^2/n}}$	
σ unknown	$t = \frac{\lvert\bar{x}-\mu\rvert}{\sqrt{s^2/n}}$	$\nu = n - 1$
Tests for two means		
σ_1, σ_2 known	$z = \frac{\lvert\bar{x}_1-\bar{x}_2\rvert}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$	
σ_1, σ_2 unknown	$t = \frac{\lvert\bar{x}_1-\bar{x}_2\rvert}{s\times\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$	$s = \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}}$ $\nu = n_1 + n_2 - 2$
Goodness–of–fit tests	$X^2 = \sum \frac{(O-E)^2}{E}$	$\nu =$ (no. of categories after pooling) $-$(no. of estimated parameters) $- 1$
Tests of independence	$X^2 = \sum \frac{(O-E)^2}{E}$	$E = \frac{\text{row total} \times \text{column total}}{\text{overall total}}$ $\nu =$ (no. of columns $- 1$) $\times$ (no. of rows $- 1$)

Correlation and linear regression

The sample correlation coefficient is given by

$$r = \frac{S_{XY}}{\sqrt{S_{XX} \times S_{YY}}},$$

where

$$\begin{aligned} S_{XY} &= \left(\sum xy\right) - n\bar{x}\bar{y}, \\ S_{XX} &= \left(\sum x^2\right) - n\bar{x}^2, \qquad \text{and} \\ S_{YY} &= \left(\sum y^2\right) - n\bar{y}^2. \end{aligned}$$

The simple linear regression equation is given by

$$Y = \alpha + \beta X + \epsilon,$$

where $\{\epsilon_i\}$ are independent $N(0, \sigma^2)$ random variables and α and β can be estimated using

$$\begin{aligned} \hat{\beta} &= \frac{S_{XY}}{S_{XX}} \qquad \text{and} \\ \hat{\alpha} &= \bar{y} - \hat{\beta}\bar{x}. \end{aligned}$$

Statistical tables

				p		
	One–tailed test	10%	5%	2.5%	1%	0.5%
	Two–tailed test	20%	10%	5%	2%	1%
	1	3.078	6.314	12.706	31.821	63.657
	2	1.886	2.920	4.303	6.965	9.925
	3	1.638	2.353	3.182	4.541	5.841
	4	1.533	2.132	2.776	3.747	4.604
	5	1.476	2.015	2.571	3.365	4.032
	6	1.440	1.943	2.447	3.143	3.707
	7	1.415	1.895	2.365	2.998	3.449
	8	1.397	1.860	2.306	2.896	3.355
	9	1.383	1.833	2.262	2.821	3.250
	10	1.372	1.812	2.228	2.764	3.169
	11	1.363	1.796	2.201	2.718	3.106
	12	1.356	1.782	2.179	2.681	3.055
	13	1.356	1.782	2.179	2.650	3.055
	14	1.345	1.761	2.145	2.624	2.977
ν	15	1.341	1.753	2.131	2.602	2.947
	16	1.337	1.746	2.120	2.583	2.921
	17	1.333	1.740	2.110	2.567	2.898
	18	1.330	1.734	2.101	2.552	2.878
	19	1.328	1.729	2.093	2.539	2.861
	20	1.325	1.725	2.086	2.528	2.845
	21	1.323	1.721	2.080	2.518	2.831
	22	1.321	1.717	2.074	2.508	2.819
	23	1.319	1.714	2.069	2.500	2.807
	24	1.318	1.711	2.064	2.492	2.797
	25	1.316	1.708	2.060	2.485	2.787
	26	1.315	1.706	2.056	2.479	2.779
	27	1.314	1.703	2.052	2.473	2.771
	28	1.313	1.701	2.048	2.467	2.763
	29	1.311	1.699	2.045	2.462	2.756
	⋮	⋮	⋮	⋮	⋮	⋮
	∞	1.282	1.645	1.960	2.326	2.576

Table 1: This table contains values of t for which $\Pr(T > t) = p$, where $T \sim t_\nu$. In a two–tailed test, the tabulated values correspond to $\Pr(|T| > t)$ $= p$

		p				
		50%	10%	5%	1%	0.1%
	1	0.45	2.17	3.84	6.63	10.83
	2	1.39	4.61	5.99	9.21	13.82
	3	2.37	6.25	7.82	11.34	16.27
	4	3.36	7.78	9.49	13.28	18.47
	5	4.34	9.24	11.07	15.09	20.52
	6	5.35	10.64	12.59	16.81	22.46
	7	6.35	12.02	14.07	18.48	24.32
ν	8	7.34	13.36	15.51	20.09	26.13
	9	8.34	14.68	16.92	21.67	27.88
	10	9.34	15.99	18.31	23.21	29.59
	12	11.34	18.55	21.03	26.22	32.91
	15	14.34	22.31	25.00	30.58	37.70
	20	19.34	28.41	31.41	37.57	45.32
	25	24.34	34.38	37.65	44.31	52.62
	30	29.34	40.26	43.77	50.89	59.70

Table 2: This table contains values of x for which $\Pr(X^2 > x) = p$, where $X^2 \sim \chi^2_\nu$

z	-0.09	-0.08	-0.07	-0.06	-0.05	-0.04	-0.03	-0.02	-0.01	0.00
-2.9	0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019
-2.8	0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026
-2.7	0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035
-2.6	0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047
-2.5	0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062
-2.4	0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082
-2.3	0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107
-2.2	0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139
-2.1	0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179
-2.0	0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228
-1.9	0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287
-1.8	0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359
-1.7	0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446
-1.6	0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548
-1.5	0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668
-1.4	0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808
-1.3	0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968
-1.2	0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151
-1.1	0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357
-1.0	0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587
-0.9	0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841
-0.8	0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119
-0.7	0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420
-0.6	0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743
-0.5	0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085
-0.4	0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446
-0.3	0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821
-0.2	0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207
-0.1	0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602
0.0	0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

Table 3: This table contains values of $\Pr(Z < z)$, where $Z \sim N(0,1)$

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