Inference for Non-Homogeneous Poisson Processes: Models for Repairable Systems

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Abstract

Application of Non-homogeneous Poisson processes to model successive failures of a repairable system undergoing minimal repair is well known. If a system, upon failure, is repaired in such a way that it's age remains the same as it was just prior to the failure, then we say that the system is minimally repaired. This occurs frequently in a very natural way, particularly when a system is large. In such a case, upon failure of the system, repair/replacement of only a small part may change the state of the system from failed to working without affecting the system materially. In this paper, we assume minimal repair, and that such repairs are instantaneous.

Let $N(s)$ be the number of failures undergone by a repairable system under consideration, up to time s. We assume $N(s)$ to be a Non-homogeneous Poisson process (NHPP) with cumulative intensity function $\Lambda(s)$ (also known as the mean value function), giving the expected number of failures up to time s. It is well known that an NHPP is characterized by $\Lambda(s)$ or the corresponding intensity function $\lambda(s)$ (the time derivative of $\Lambda(s)$). The intensity function of a process gives the instantaneous rate of change of the expected number of failures with respect to time, and hence may be interpreted as a measure of the wear-out (or improvement) of the system. The shape of $\lambda(s)$; increasing, decreasing or otherwise; provides information about the changes in the reliability of the system over time. Throughout the paper, we shall assume that $\Lambda(s) \to \infty$ as $s \to \infty$, and that it is differentiable, *i.e.*, $\lambda(s)$ exists. Such NHPPs are frequently used for modeling repairable systems (see *e.g.*, Ascher and Feingold, 1984).

A survey of the literature relating to inference for repairable systems reveals that, the existing work in this area, in general, either presupposes

a parametrically specified model, or, in case such a model is not specified, availability of infinite/large number of copies of the system is assumed. Further, in the latter case, the inferential procedures derived for the system are applicable only over a finite time interval. This, in fact, is a limitation since, a repairable system unfolds itself as time passes, and hence it is important to study (and be able to infer about) its behavior over time. Moreover, in practice, a large number of identical copies of a repairable system may not often be available for analysis, due either to the nature of the system or the repair, or due to size and cost constraints.

A new approach to modeling repairable systems was recently proposed by Deshpande *et. al.* (1999), which takes care of some of these problems in an elegant and meaningful way. They considered, specifically, a testing problem involving two NHPPs wherein the solutions suggested by them have desirable properties valid for a large time t, and do not require large number of copies, *i.e.*, they are based on either a single realisation or only finitely many copies of the system.

In this work, we present an unified and comprehensive strategy for inference relating to repairable systems, broadly based on the approach given by Deshpande *et. al.* (1999). In particular, we consider some very general and frequently encountered testing problems relating to repairable systems, and propose non-parametric inferential procedures for the same. Such inference will obviously depend on the method of sampling, and we consider both the sampling scheme under which data for such a system are collected over a fixed time interval, giving rise to "time truncated" data, and the scheme where a fixed number of failures are observed, which leads to "failure truncated" data.

Further, the existing test procedures are typically derived after conditioning on the number of failures observed in $(0, t]$. Thus, under such conditioning, what is meant by asymptotics is not quite comprehensible, since for such regular processes, the number of failures in a finite interval cannot be increased at will by the experimenter. In our setup, we do not impose any such conditioning, and since we study the properties of the tests developed by us for large t, we have a setup where asymptotics are more meaningful. We are not aware of any other work where unconditional tests (like the ones attempted here) have been developed for repairable systems. Though we derive our asymptotic inference as $t \to \infty$, we also note that the above asymptotics actually depend on $E(N(t)) = \Lambda(t)$ being large, and therefore our asymptotics can also be considered as $\Lambda(t) \to \infty$ asymptotics.

The paper is divided into six sections. In section two we consider the typical testing problem for a single system, which is essentially a goodness-of-fit test. In section three, we present mathematical preliminaries which would be useful for the development in the rest of the paper. In the fourth and fifth sections, testing problems for a pair of systems are considered with the help of results in the third section. In the final section, we illustrate the inferential procedures developed by us, by applying them on a real life data set.