Learning outcomes: Chapter 2

1. Suppose that $X_1, X_2, \ldots, X_n$ is a sequence of IID random variables with common distribution function F.

(a) You should understand that the distribution of the maximum order statistic M_n is given by $F^n(x)$;

(b) Without reference to F, you should memorise that the *Extremal Types Theorem* (ETT) shows that the distribution of rescaled maxima $(M_n - b_n)/a_n \longrightarrow G$, where G is one of either the Gumbel, Fréchet or Weibull distributions.

(c) You should memorise the distribution function for the Gumbel, Fréchet and Weibull distributions as mentioned in (b).

(d) You should know that the ETT for rescaled maxima is analogous to the Central Limit Theorem for rescaled sample means.

2. You should know that the *Generalised Extreme Value* (GEV) distribution gives a parameterisation of the distributions in the ETT which encompasses all three types.

(a) You do *not* need to memorise the distribution function of the GEV – this will be given in the exam paper;

(b) You should know that the GEV shape parameter determines the tail behaviour of the observed extremes, and that the cases when this parameter $= 0$, > 0 and < 0 give the Gumbel, Fréchet and Weibull distributions (respectively).

3. For simple examples where F, a_n and b_n are known, you should be able to show that the limiting distribution for M_n is one of the three extreme value distributions.

4. You should understand the four steps of any typical application of the GEV:

(a) Data pre-processing, and the potential problems associated with the choice of block size for determining M_n;

(b) Construction of the log-likelihood equations and the optimisation of the log-likelihood function in R, from first principles or by using the ismev package;

(c) The use of simple graphical diagnostics to assess the goodness-of-fit of the GEV, and the interpretation of the plots given in ismev;

(d) The estimation of *return levels* – you should be able to define a return level in plain English, and obtain an expression for this quantity by inversion of the GEV distribution function (it would also be worthwhile memorising the return level equation for the exam).

5. You should understand how standard errors are constructed for the GEV parameters using standard likelihood techniques, and you should be able to obtain standard errors for return levels via the *delta method* (you will not be expected to invert 3×3 matrices in the exam).

6. You should be able to construct confidence intervals for the GEV parameters and return levels, understanding the role of *profile log-likelihood* for the construction of return level confidence intervals (You will not be expected to memorise critical values from the χ^2 distribution).

7. You should be aware of the drawbacks associated with an analysis of block maxima using the GEV.