4. Daily rainfall accumulations (in mm) are observed at two locations on the River Ouse in Yorkshire, one upstream (location A) and one downstream (location B). The generalised Pareto distribution (GPD) is fitted to marginal threshold excesses at the two locations, giving the estimated parameters shown in the table below:

Location A			Location B		
$\hat{\sigma}_A = 7.440$	$\hat{\xi}_A = 0.184$	$\hat{\lambda}_{u_A=30} = 0.009$	$\hat{\sigma}_B = 5.342$	$\hat{\xi}_B = 0.201$	$\hat{\lambda}_{u_B=24} = 0.015$

(a) Assuming rainfall extremes occur independently at the two locations, estimate the probability that, on any particular day, rainfall accumulations exceed 32mm at both locations simultaneously.

(b) The logistic model, with distribution function

$$G(x,y) = \exp\left\{-\left(x^{-1/\alpha} + y^{-1/\alpha}\right)^{\alpha}\right\}, \quad x, y > 0,$$

with $\alpha \in (0,1)$, is used to model dependence in the rainfall extremes at the two locations.

(i) What marginal transformation must be achieved before the logistic model can be applied?

(ii) Briefly explain how the likelihood contribution for α would be obtained if, on a particular day, rainfall totals at location A exceed the marginal threshold at this location, but the same cannot be said for rainfall totals at location B. *[Do not show any calculations]*

(iii) Suppose we estimate that $\alpha = 0.623$. After applying the transformation in (i),

$$50\text{mm} = \begin{cases} 986.461\text{mm} & \text{at location A} \\ 1605.221\text{mm} & \text{at location B} \end{cases}$$

Now re–estimate the probability in part (a), having accounted for extremal dependence, and comment.

[**9 marks**]

5. Customers arrive at a shop according to a Poisson process with rate 10 per hour. Calculate:

(a) the probability of at least two customers arriving in the first quarter of an hour;

(b) the probability that the first customer arrived after the first quarter of an hour but within the first half an hour?

[**5 marks**]