Hints: Question 1

(d) Define the first component distribution as the one with the larger mean.

- This is the component with the smaller variance
- Low values of $\bar{x}$ are consistent with component 2
- As $\bar{x}$ increases so does the (posterior) probability of component 2 (until around $\bar{x} = 16.5$)
- Show that p_1^* depends on $\bar{x}$ only through B_1 and B_2
- Express $D_2B_2^2 - D_1B_1^2$ as a quadratic in $\bar{x}$
- Show that there exists a k such that your quadratic increases as $\bar{x}$ increases ($\bar{x} > k$) and increases as $\bar{x}$ decreases ($\bar{x} < k$)

Hints: Question 2

(h) Follow solution on page 50 of the lecture notes, but with $\bar{Y}|\mu, \tau \sim N(\mu, 1/m\tau)$ due to the central limit theorem - you should get a generalised t distribution

(i) Produce plot as in part (d)

(j) Use the formula for the predictive distribution, as given in section 1.4.2 of the notes. With the hint, we have

$$f(v|\boldsymbol{x}) = \int f(v|\tau)\pi(\tau|\boldsymbol{x})d\tau,$$

where, using the hint, we have $v|\tau \sim Ga((m-1)/2, m\tau/2)$ and - as usual - $\tau|\boldsymbol{x} \sim Ga(G, H)$. The solution to Example 1.10 in the notes *might* prove useful. A bit.

(k) Sorry, not giving help here... I know, that is cruel. But sorry