MAS2317/3317:
Chapter 5 – Question 2

Student presentation

Semester 2, 2014–2015

Chapter 5, question 2

Let A denote the event of a driver having at least one accident in any one year.

Then we have $Pr(A|X) = 0.02$, $Pr(A|Y) = 0.04$, $Pr(A|Z) = 0.10$, and $Pr(X) = 0.3$, $Pr(Y) = 0.6$, $Pr(Z) = 0.1$.

(i) We have

$$\begin{aligned}
Pr(Z|A) &= \frac{Pr(A|Z)Pr(Z)}{Pr(A|X)Pr(X) + Pr(A|Y)Pr(Y) + Pr(A|Z)Pr(Z)} \\
&= \frac{0.10 \times 0.1}{0.02 \times 0.3 + 0.04 \times 0.6 + 0.10 \times 0.1} \\
&= 0.25.
\end{aligned}$$

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(ii) Let A_n^c denote Mrs Brown not having an accident for n years.

Then, since the number of accidents are independent over years

$$Pr(A_n^c|X) = Pr(A^c|X)^n = \{1 - Pr(A|X)\}^n = 0.98^n.$$

Similarly

$$Pr(A_n^c|Y) = Pr(A^c|Y)^n = \{1 - Pr(A|Y)\}^n = 0.96^n.$$

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We require n such that

$$
\begin{aligned}
Pr(X|A_n^c) > Pr(Y|A_n^c) & \\
\implies \quad & \frac{Pr(A_n^c|X)Pr(X)}{Pr(A_n^c)} > \frac{Pr(A_n^c|Y)Pr(Y)}{Pr(A_n^c)} \\
\implies \quad & 0.98^n \times 0.3 > 0.96^n \times 0.6 \\
\implies \quad & \frac{0.98^n}{0.96^n} > 2 \\
\implies \quad & n \log\left(\frac{0.98}{0.96}\right) > \log 2 \\
\implies \quad & n > \log 2 \Big/ \log\left(\frac{0.98}{0.96}\right) = 33.62
\end{aligned}
$$

and so we need $n \geq 34$.