MAS2903
Problems Class 3

Dr. Lee Fawcett

Semester 2, 2018—2019

Announcements

- Assignment 2 currently being marked – will aim to get back in time for the drop-in next Thursday, if not before

Announcements

- Assignment 2 currently being marked – will aim to get back in time for the drop-in next Thursday, if not before
- Revision week next week...

Announcements

- Assignment 2 currently being marked – will aim to get back in time for the drop-in next Thursday, if not before
- Revision week next week...
  - Everything we do today will be revision

Announcements

- Assignment 2 currently being marked – will aim to get back in time for the drop-in next Thursday, if not before
- Revision week next week...
  - Everything we do today will be revision
  - Solutions to all questions in Chapter 5
  - Revision booklet and solutions – extra questions
  - Crib sheet
  - Past papers

Announcements

- Assignment 2 currently being marked – will aim to get back in time for the drop-in next Thursday, if not before
- Revision week next week...
  - Everything we do today will be revision
  - Solutions to all questions in Chapter 5
  - Revision booklet and solutions – extra questions
  - Crib sheet
  - Past papers
- Extra office hours next week, TBA

Question 18

From example 2.6 in the lecture notes, we know that the posterior precision is given by $d + n\tau$, where n is the number of observations and τ is the precision of our measurements.

Question 18

From example 2.6 in the lecture notes, we know that the posterior precision is given by $d + n\tau$, where n is the number of observations and τ is the precision of our measurements.

(a) Thus, the standard devision of the posterior distribution is $1/\sqrt{d + n\tau}$, and we want

$$\frac{1}{\sqrt{d + n\tau}} = 0.1$$

Question 18

From example 2.6 in the lecture notes, we know that the posterior precision is given by $d + n\tau$, where n is the number of observations and τ is the precision of our measurements.

(a) Thus, the standard devision of the posterior distribution is $1/\sqrt{d + n\tau}$, and we want

$$\begin{aligned} \frac{1}{\sqrt{d + n\tau}} &= 0.1 \\ \longrightarrow n &= \frac{100 - d}{\tau}. \end{aligned}$$

Question 18

From example 2.6 in the lecture notes, we know that the posterior precision is given by $d + n\tau$, where n is the number of observations and τ is the precision of our measurements.

(a) Thus, the standard devision of the posterior distribution is $1/\sqrt{d + n\tau}$, and we want

$$\begin{aligned} \frac{1}{\sqrt{d + n\tau}} &= 0.1 \\ \longrightarrow n &= \frac{100 - d}{\tau}. \end{aligned}$$

Here, we have $d = 1$ and $\tau = 1/4$, giving $n = 396$.

Question 18

(b) When $n = 100$ the posterior standard deviation is

$$\frac{1}{\sqrt{d + 100\tau}} = \frac{1}{\sqrt{d + 25}}.$$

Question 18

(b) When $n = 100$ the posterior standard deviation is

$$\frac{1}{\sqrt{d + 100\tau}} = \frac{1}{\sqrt{d + 25}}.$$

The prior variance is $1/d$, which can never be negative; Thus, d cannot be negative. As $d \to 0$, we have $1/5 = 0.2$ for the posterior standard deviation; as d increases, the posterior standard deviation defined above will always decrease.

Question 34 (i)

From lectures (Example 2.6) we know that

$$D = d + n\tau \quad \text{and} \quad B = \frac{db + n\tau\bar{x}}{D}.$$

Question 34 (i)

From lectures (Example 2.6) we know that

$$D = d + n\tau \quad \text{and} \quad B = \frac{db + n\tau\bar{x}}{D}.$$

From the prior, we have $b = 110$ and $d = 1/0.4$.

Question 34 (i)

From lectures (Example 2.6) we know that

$$D = d + n\tau \quad \text{and} \quad B = \frac{db + n\tau\bar{x}}{D}.$$

From the prior, we have $b = 110$ and $d = 1/0.4$.
From the data, we have $n = 5$ and $\bar{x} = 109.2$.

Question 34 (i)

From lectures (Example 2.6) we know that

$$D = d + n\tau \quad \text{and} \quad B = \frac{db + n\tau\bar{x}}{D}.$$

From the prior, we have $b = 110$ and $d = 1/0.4$.
From the data, we have $n = 5$ and $\bar{x} = 109.2$.
We also know that $\tau = 1/4$. Therefore

Question 34 (i)

From lectures (Example 2.6) we know that

$$D = d + n\tau \quad \text{and} \quad B = \frac{db + n\tau\bar{x}}{D}.$$

From the prior, we have $b = 110$ and $d = 1/0.4$.
From the data, we have $n = 5$ and $\bar{x} = 109.2$.
We also know that $\tau = 1/4$. Therefore

$$D = 1/0.4 + 5 \cdot \frac{1}{4} = 3.75,$$

and

$$B = \frac{110/0.4 + 5 \cdot \frac{1}{4} \cdot 109.2}{3.75} = 109.733.$$

Question 34 (i)

From lectures (Example 2.6) we know that

$$D = d + n\tau \quad \text{and} \quad B = \frac{db + n\tau\bar{x}}{D}.$$

From the prior, we have $b = 110$ and $d = 1/0.4$.
From the data, we have $n = 5$ and $\bar{x} = 109.2$.
We also know that $\tau = 1/4$. Therefore

$$D = 1/0.4 + 5 \cdot \frac{1}{4} = 3.75,$$

and

$$B = \frac{110/0.4 + 5 \cdot \frac{1}{4} \cdot 109.2}{3.75} = 109.733.$$

Thus

$$Y|\boldsymbol{x} \sim N\left(109.733, \frac{3.75 + 1/4}{3.75/4}\right) = N(109.733, 4.267).$$

Question 34 (ii)

We require $P(Y|\boldsymbol{X} > 110)$:

$$
\begin{aligned}
P(Y|\boldsymbol{X} > 110) &= P\left(Z > \frac{110 - 109.733}{\sqrt{4.267}}\right) \\
&= P(Z > 0.129)
\end{aligned}
$$

Question 34 (ii)

We require $P(Y|\boldsymbol{X} > 110)$:

$$
\begin{aligned}
P(Y|\boldsymbol{X} > 110) &= P\left(Z > \frac{110 - 109.733}{\sqrt{4.267}}\right) \\
&= P(Z > 0.129) \\
&= 1 - P(Z < 0.129) \\
&= 1 - 0.551 = 0.449.
\end{aligned}
$$

Question 27 (a)

We have a single observation on a Bin$(100, \theta)$ distribution, giving the likelihood:

$$f(x|\theta) = \binom{100}{3} \times \theta^3(1-\theta)^{97}.$$

Question 27 (a)

We have a single observation on a Bin$(100, \theta)$ distribution, giving the likelihood:

$$f(x|\theta) = \binom{100}{3} \times \theta^3 (1-\theta)^{97}.$$

Then

$$\pi(\theta|\boldsymbol{x}) \quad \propto \quad \theta^1 (1-\theta)^{199} \times \theta^3 (1-\theta)^{97}$$

Question 27 (a)

We have a single observation on a Bin$(100, \theta)$ distribution, giving the likelihood:

$$f(x|\theta) = \binom{100}{3} \times \theta^3 (1-\theta)^{97}.$$

Then

$$\pi(\theta|\boldsymbol{x}) \propto \theta^1 (1-\theta)^{199} \times \theta^3 (1-\theta)^{97}$$

$$= \theta^4 (1-\theta)^{296},$$

and so $\theta|\boldsymbol{x} \sim Beta(5, 297)$.

Question 27 (b)

Suppose the statistician's posterior is $\theta|\boldsymbol{x} \sim Beta(A, B)$. Then

$$E[\theta] = \frac{A}{A+B} = \frac{4}{102},$$

giving $B = 24.5A$.

Question 27 (b)

Suppose the statistician's posterior is $\theta|\boldsymbol{x} \sim Beta(A, B)$. Then

$$E[\theta] = \frac{A}{A+B} = \frac{4}{102},$$

giving $B = 24.5A$. Substituting into

$$Var(\theta) = \frac{AB}{(A+B)^2(A+B+1)} = 0.0003658$$

gives

$$\frac{24.5A^2}{(25.5A)^2(25.5A+1)} = 0.0003658$$

and so $A = 4$. Therefore $B = 98$, and so we have
$\theta|\boldsymbol{x} \sim Beta(4, 98)$.

Question 27 (b)

Suppose the statistician's posterior is $\theta|\boldsymbol{x} \sim Beta(A, B)$. Then

$$E[\theta] = \frac{A}{A+B} = \frac{4}{102},$$

giving $B = 24.5A$. Substituting into

$$Var(\theta) = \frac{AB}{(A+B)^2(A+B+1)} = 0.0003658$$

gives

$$\frac{24.5A^2}{(25.5A)^2(25.5A+1)} = 0.0003658$$

and so $A = 4$. Therefore $B = 98$, and so we have $\theta|\boldsymbol{x} \sim Beta(4, 98)$.

Question 27 (b)

We know the likelihood is

$$f(x|\theta) \propto \theta^3(1-\theta)^{97},$$

and so

$$\text{Posterior} \quad \propto \quad \text{Prior} \times \text{Likelihood}$$

Question 27 (b)

We know the likelihood is

$$f(x|\theta) \propto \theta^3(1-\theta)^{97},$$

and so

$$\begin{aligned} \text{Posterior} &\propto \text{Prior} \times \text{Likelihood} \\ \theta^3(1-\theta)^{97} &\propto \theta^{a-1}(1-\theta)^{b-1} \times \theta^3(1-\theta)^{97} \end{aligned}$$

meaning that $a = b = 1$ and so $\theta \sim Beta(1,1)$ - in other words, $\theta \sim U(0,1)$!

Question 27 (b)

We know the likelihood is

$$f(x|\theta) \propto \theta^3(1-\theta)^{97},$$

and so

$$\begin{aligned} \text{Posterior} &\propto \text{Prior} \times \text{Likelihood} \\ \theta^3(1-\theta)^{97} &\propto \theta^{a-1}(1-\theta)^{b-1} \times \theta^3(1-\theta)^{97} \end{aligned}$$

meaning that $a = b = 1$ and so $\theta \sim Beta(1,1)$ - in other words, $\theta \sim U(0,1)$!

Alternatively, we could just observe that the Posterior is proportional to the likelihood, without any contribution from the prior – so the statistician must have been prior ignorant (hence the Uniform prior).

Question 35 (i)

Likelihood: $X_i \sim Po(\ell_i\theta)$, and so

$$f(x|\theta) = \prod_{i=1}^{n} \frac{e^{-\ell_i\theta}(\ell_i\theta)^{x_i}}{x_i!}$$

Question 35 (i)

Likelihood: $X_i \sim Po(\ell_i\theta)$, and so

$$
\begin{aligned}
f(x|\theta) &= \prod_{i=1}^{n} \frac{e^{-\ell_i\theta}(\ell_i\theta)^{x_i}}{x_i!} \\
&= \frac{e^{-\ell_1\theta} \times \ldots \times e^{-\ell_n\theta} \times (\ell_1\theta)^{x_1} \times \ldots \times (\ell_n\theta)^{x_n}}{x_1! \times \ldots \times x_n!}
\end{aligned}
$$

Question 35 (i)

Likelihood: $X_i \sim Po(\ell_i\theta)$, and so

$$
\begin{aligned}
f(x|\theta) &= \prod_{i=1}^{n} \frac{e^{-\ell_i\theta}(\ell_i\theta)^{x_i}}{x_i!} \\
&= \frac{e^{-\ell_1\theta} \times \ldots \times e^{-\ell_n\theta} \times (\ell_1\theta)^{x_1} \times \ldots \times (\ell_n\theta)^{x_n}}{x_1! \times \ldots \times x_n!} \\
&= \frac{e^{(\ell_1+\ldots+\ell_n)\theta} \times \ell_1^{x_1} \times \ldots \times \ell_n^{x_n} \times \theta^{x_1} + \ldots + \theta^{x_n}}{\prod_{i=1}^{n} x_i!}
\end{aligned}
$$

Question 35 (i)

Likelihood: $X_i \sim Po(\ell_i\theta)$, and so

$$
\begin{aligned}
f(x|\theta) &= \prod_{i=1}^{n} \frac{e^{-\ell_i\theta}(\ell_i\theta)^{x_i}}{x_i!} \\
&= \frac{e^{-\ell_1\theta} \times \ldots \times e^{-\ell_n\theta} \times (\ell_1\theta)^{x_1} \times \ldots \times (\ell_n\theta)^{x_n}}{x_1! \times \ldots \times x_n!} \\
&= \frac{e^{(\ell_1+\ldots+\ell_n)\theta} \times \ell_1^{x_1} \times \ldots \times \ell_n^{x_n} \times \theta^{x_1} + \ldots + \theta^{x_n}}{\prod_{i=1}^{n} x_i!} \\
&= \frac{e^{n\bar{\ell}\theta}\theta^{n\bar{x}} \prod_{i=1}^{n} \ell_i^{x_i}}{\prod_{i=1}^{n} x_i!}
\end{aligned}
$$

Question 35 (i)

Likelihood: $X_i \sim Po(\ell_i\theta)$, and so

$$
\begin{aligned}
f(x|\theta) &= \prod_{i=1}^{n} \frac{e^{-\ell_i\theta}(\ell_i\theta)^{x_i}}{x_i!} \\
&= \frac{e^{-\ell_1\theta} \times \ldots \times e^{-\ell_n\theta} \times (\ell_1\theta)^{x_1} \times \ldots \times (\ell_n\theta)^{x_n}}{x_1! \times \ldots \times x_n!} \\
&= \frac{e^{(\ell_1+\ldots+\ell_n)\theta} \times \ell_1^{x_1} \times \ldots \times \ell_n^{x_n} \times \theta^{x_1} + \ldots + \theta^{x_n}}{\prod_{i=1}^{n} x_i!} \\
&= \frac{e^{n\bar{\ell}\theta}\theta^{n\bar{x}} \prod_{i=1}^{n} \ell_i^{x_i}}{\prod_{i=1}^{n} x_i!} \\
&\propto e^{n\bar{\ell}\theta}\theta^{n\bar{x}}
\end{aligned}
$$

Question 35 (i)

The conjugate prior for the Poisson model is $\theta \sim Ga(g, h)$, as this will also give a gamma posterior. Thus,

Question 35 (i)

The conjugate prior for the Poisson model is $\theta \sim Ga(g, h)$, as this will also give a gamma posterior. Thus,

$$\pi(\theta|\boldsymbol{x}) \quad \propto \quad \theta^{g-1} e^{-h\theta}$$

Question 35 (i)

The conjugate prior for the Poisson model is $\theta \sim Ga(g, h)$, as this will also give a gamma posterior. Thus,

$$\pi(\theta|\boldsymbol{x}) \quad \propto \quad \theta^{g-1} e^{-h\theta} \times e^{n\bar{\ell}\theta} \theta^{n\bar{x}}$$

Question 35 (i)

The conjugate prior for the Poisson model is $\theta \sim Ga(g, h)$, as this will also give a gamma posterior. Thus,

$$\begin{aligned}\pi(\theta|\boldsymbol{x}) &\propto \theta^{g-1}e^{-h\theta} \times e^{n\bar{\ell}\theta}\theta^{n\bar{x}} \\ &\propto \theta^{(g+n\bar{x})-1}e^{-(h+n\bar{\ell})\theta}\end{aligned}$$

Question 35 (i)

The conjugate prior for the Poisson model is $\theta \sim Ga(g, h)$, as this will also give a gamma posterior. Thus,

$$\begin{aligned}\pi(\theta|\boldsymbol{x}) &\propto \theta^{g-1} e^{-h\theta} \times e^{n\bar{\ell}\theta} \theta^{n\bar{x}} \\ &\propto \theta^{(g+n\bar{x})-1} e^{-(h+n\bar{\ell})\theta}\end{aligned}$$

and so $\theta|\boldsymbol{x} \sim Ga(g + n\bar{x}, h + n\bar{\ell})$.

Question 35 (i)

The conjugate prior for the Poisson model is $\theta \sim Ga(g, h)$, as this will also give a gamma posterior. Thus,

$$\begin{aligned} \pi(\theta|\boldsymbol{x}) &\propto \theta^{g-1}e^{-h\theta} \times e^{n\bar{\ell}\theta}\theta^{n\bar{x}} \\ &\propto \theta^{(g+n\bar{x})-1}e^{-(h+n\bar{\ell})\theta} \end{aligned}$$

and so $\theta|\boldsymbol{x} \sim Ga(g + n\bar{x}, h + n\bar{\ell})$.
Notice the question says "Very little is known about θ"; this suggests we use a vague prior – according to **Example 3.10** of the notes, this means we let $g, h \to 0$, and so

$$\theta|\boldsymbol{x} \sim Ga(n\bar{x}, n\bar{\ell})$$

Question 35 (i)

The conjugate prior for the Poisson model is $\theta \sim Ga(g, h)$, as this will also give a gamma posterior. Thus,

$$\begin{aligned}\pi(\theta|\boldsymbol{x}) &\propto \theta^{g-1}e^{-h\theta} \times e^{n\bar{\ell}\theta}\theta^{n\bar{x}} \\ &\propto \theta^{(g+n\bar{x})-1}e^{-(h+n\bar{\ell})\theta}\end{aligned}$$

and so $\theta|\boldsymbol{x} \sim Ga(g + n\bar{x}, h + n\bar{\ell})$.
Notice the question says “Very little is known about θ"; this suggests we use a vague prior – according to **Example 3.10** of the notes, this means we let $g, h \to 0$, and so

$$\theta|\boldsymbol{x} \sim Ga(n\bar{x}, n\bar{\ell}) = Ga(28, 120).$$

Question 35 (ii)

Candidate's formula:

$$f(y|\boldsymbol{x}) = \frac{f(y|\theta) \times \pi(\theta|\boldsymbol{x})}{\pi(\theta|\boldsymbol{x}, y)}$$

Question 35 (ii)

Candidate's formula:

$$f(y|\boldsymbol{x}) = \frac{f(y|\theta) \times \pi(\theta|\boldsymbol{x})}{\pi(\theta|\boldsymbol{x}, y)}$$

- $Y|\theta \sim Po(\ell\theta)$
- $\theta|\boldsymbol{x} \sim Ga(n\bar{x}, n\bar{\ell}) = Ga(G, H)$
- $\theta|\boldsymbol{x}, y \sim ??$

Question 35 (ii)

$$\pi(\theta|\boldsymbol{x}, y) = \pi(\theta|\boldsymbol{x}) \times f(y|\theta)$$

Question 35 (ii)

$$\begin{aligned}\pi(\theta|\boldsymbol{x}, y) &= \pi(\theta|\boldsymbol{x}) \times f(y|\theta) \\ &= \frac{H^G \theta^{G-1} e^{-H\theta}}{\Gamma(G)} \times \frac{e^{-(\ell\theta)}(\ell\theta)^y}{y!}\end{aligned}$$

Question 35 (ii)

$$
\begin{aligned}
\pi(\theta|\boldsymbol{x}, y) &= \pi(\theta|\boldsymbol{x}) \times f(y|\theta) \\
&= \frac{H^G \theta^{G-1} e^{-H\theta}}{\Gamma(G)} \times \frac{e^{-(\ell\theta)}(\ell\theta)^y}{y!} \\
&= \frac{H^G \ell^y}{\Gamma(G) y!} \times \theta^{(G+y)-1} e^{-(H+\ell)\theta},
\end{aligned}
$$

Question 35 (ii)

$$\begin{aligned}\pi(\theta|\boldsymbol{x}, y) &= \pi(\theta|\boldsymbol{x}) \times f(y|\theta) \\ &= \frac{H^G \theta^{G-1} e^{-H\theta}}{\Gamma(G)} \times \frac{e^{-(\ell\theta)}(\ell\theta)^y}{y!} \\ &= \frac{H^G \ell^y}{\Gamma(G) y!} \times \theta^{(G+y)-1} e^{-(H+\ell)\theta},\end{aligned}$$

and so $\theta|\boldsymbol{x}, y \sim Ga(G+y, H+\ell)$.

Question 35 (ii)

Candidate's formula:

$$f(y|\boldsymbol{x}) = \frac{f(y|\theta) \times \pi(\theta|\boldsymbol{x})}{\pi(\theta|\boldsymbol{x}, y)}$$

Question 35 (ii)

Candidate's formula:

$$f(y|\boldsymbol{x}) = \frac{f(y|\theta) \times \pi(\theta|\boldsymbol{x})}{\pi(\theta|\boldsymbol{x}, y)}$$

- $Y|\theta \sim Po(\ell\theta)$
- $\theta|\boldsymbol{x} \sim Ga(n\bar{x}, n\bar{\ell}) = Ga(G, H)$
- $\theta|\boldsymbol{x}, y \sim Ga(G + y, H + \ell)$

Question 35 (ii)

Candidate's formula:

$$f(y|\boldsymbol{x}) = \frac{f(y|\theta) \times \pi(\theta|\boldsymbol{x})}{\pi(\theta|\boldsymbol{x}, y)}$$

- $Y|\theta \sim Po(\ell\theta)$
- $\theta|\boldsymbol{x} \sim Ga(n\bar{x}, n\bar{\ell}) = Ga(G, H)$
- $\theta|\boldsymbol{x}, y \sim Ga(G + y, H + \ell)$

Giving:

$$f(y|\boldsymbol{x}) = \frac{e^{-\ell\theta}(\ell\theta)^y}{y!}$$

Question 35 (ii)

Candidate's formula:

$$f(y|\boldsymbol{x}) = \frac{f(y|\theta) \times \pi(\theta|\boldsymbol{x})}{\pi(\theta|\boldsymbol{x}, y)}$$

- $Y|\theta \sim Po(\ell\theta)$
- $\theta|\boldsymbol{x} \sim Ga(n\bar{x}, n\bar{\ell}) = Ga(G, H)$
- $\theta|\boldsymbol{x}, y \sim Ga(G + y, H + \ell)$

Giving:

$$f(y|\boldsymbol{x}) \quad = \quad \frac{\frac{e^{-\ell\theta}(\ell\theta)^y}{y!} \times \frac{H^G\theta^{G-1}e^{-H\theta}}{\Gamma(G)}}{}$$

Question 35 (ii)

Candidate's formula:

$$f(y|\boldsymbol{x}) = \frac{f(y|\theta) \times \pi(\theta|\boldsymbol{x})}{\pi(\theta|\boldsymbol{x}, y)}$$

- $Y|\theta \sim Po(\ell\theta)$
- $\theta|\boldsymbol{x} \sim Ga(n\bar{x}, n\bar{\ell}) = Ga(G, H)$
- $\theta|\boldsymbol{x}, y \sim Ga(G + y, H + \ell)$

Giving:

$$f(y|\boldsymbol{x}) \quad = \quad \frac{\dfrac{e^{-\ell\theta}(\ell\theta)^y}{y!} \times \dfrac{H^G\theta^{G-1}e^{-H\theta}}{\Gamma(G)}}{\dfrac{(H+\ell)^{G+y}\theta^{(G+y)-1}e^{-(H+\ell)\theta}}{\Gamma(G+y)}}$$

Question 35 (ii)

Candidate's formula:

$$f(y|\boldsymbol{x}) = \frac{f(y|\theta) \times \pi(\theta|\boldsymbol{x})}{\pi(\theta|\boldsymbol{x}, y)}$$

- $Y|\theta \sim Po(\ell\theta)$
- $\theta|\boldsymbol{x} \sim Ga(n\bar{x}, n\bar{\ell}) = Ga(G, H)$
- $\theta|\boldsymbol{x}, y \sim Ga(G + y, H + \ell)$

Giving:

$$f(y|\boldsymbol{x}) = \frac{\frac{e^{-\ell\theta}(\ell\theta)^y}{y!} \times \frac{H^G\theta^{G-1}e^{-H\theta}}{\Gamma(G)}}{\frac{(H+\ell)^{G+y}\theta^{(G+y)-1}e^{-(H+\ell)\theta}}{\Gamma(G+y)}}$$

$$= \frac{\Gamma(G+y)}{\Gamma(G)\Gamma(y+1)} \times \frac{\ell^y H^G}{(H+\ell)^{G+y}}$$

Question 35 (ii)

Candidate's formula:

$$f(y|\boldsymbol{x}) = \frac{f(y|\theta) \times \pi(\theta|\boldsymbol{x})}{\pi(\theta|\boldsymbol{x}, y)}$$

- $Y|\theta \sim Po(\ell\theta)$
- $\theta|\boldsymbol{x} \sim Ga(n\bar{x}, n\bar{\ell}) = Ga(G, H)$
- $\theta|\boldsymbol{x}, y \sim Ga(G + y, H + \ell)$

Giving:

$$\begin{aligned} f(y|\boldsymbol{x}) &= \frac{\frac{e^{-\ell\theta}(\ell\theta)^y}{y!} \times \frac{H^G\theta^{G-1}e^{-H\theta}}{\Gamma(G)}}{\frac{(H+\ell)^{G+y}\theta^{(G+y)-1}e^{-(H+\ell)\theta}}{\Gamma(G+y)}} \\ &= \frac{\Gamma(G+y)}{\Gamma(G)\Gamma(y+1)} \times \frac{\ell^y H^G}{(H+\ell)^{G+y}} \\ &= \frac{1}{B(y, G)} \frac{\ell^y H^G}{y(H+\ell)^{G+y}}, \end{aligned}$$

Question 35 (ii)

Candidate's formula:

$$f(y|\boldsymbol{x}) = \frac{f(y|\theta) \times \pi(\theta|\boldsymbol{x})}{\pi(\theta|\boldsymbol{x}, y)}$$

- $Y|\theta \sim Po(\ell\theta)$
- $\theta|\boldsymbol{x} \sim Ga(n\bar{x}, n\bar{\ell}) = Ga(G, H)$
- $\theta|\boldsymbol{x}, y \sim Ga(G + y, H + \ell)$

Giving:

$$\begin{aligned} f(y|\boldsymbol{x}) &= \frac{\dfrac{e^{-\ell\theta}(\ell\theta)^y}{y!} \times \dfrac{H^G\theta^{G-1}e^{-H\theta}}{\Gamma(G)}}{\dfrac{(H+\ell)^{G+y}\theta^{(G+y)-1}e^{-(H+\ell)\theta}}{\Gamma(G+y)}} \\ &= \frac{\Gamma(G+y)}{\Gamma(G)\Gamma(y+1)} \times \frac{\ell^y H^G}{(H+\ell)^{G+y}} \\ &= \frac{1}{B(y, G)} \frac{\ell^y H^G}{y(H+\ell)^{G+y}}, \qquad G = 28, H = 120, \ell = 60, y = 0, 1, \ldots, . \end{aligned}$$